RL-83-057

Science and Engineering Research Council
Rutherford Appleton Laboratory
CHILTON, DIDCOT, OXON, OX11 0QX

RL-83-057

Beat-Wave Laser Accelerators: First report of the R.A.L. study group

J D Lawson

June 1983

RL 83 057

BEAT-WAVE LASER ACCELERATORS

FIRST REPORT OF THE R.A.L. STUDY GROUP

J. D. Lawson

Participants	Field of Interest
J E Allen*	Plasma Physics
R Bingham	Plasma Physics
J Butterworth	Particle Beam Transport
F E Close	High Energy Physics
R G Evans	Plasma Physics and Lasers
J D Lawson	Accelerators
G H Rees	Accelerators
R D Ruth+	Accelerators

* University of Oxford.

+ Lawrence Berkeley Laboratory and CERN.

ABSTRACT

An attempt is being made to see what is involved in constructing a high energy accelerator using the laser beat-wave principle of Tajima and Dawson. An initial study of possible parameters for a 5 TeV machine has already been made by Ruth and Chao. The present study follows the same path, but looks at further problems such as how to produce the plasma, and how to arrange 'staging'. Factors which determine the luminosity in colliding beam operation are also examined. An attempt is being made to develop a more realistic model of the physical beat-wave process. Work is at an early stage, but already serious difficulties are evident; inventive suggestions of how to overcome these are needed.

Rutherford Appleton Laboratory
Chilton
Didcot
Oxon OX11 OQX

June 1983

1. INTRODUCTION

There has recently been considerable interest in exploring new concepts for accelerating particles to very high energies [1,2]. It is evident that existing types of synchrotron and linear accelerator are nearing their economic limits, and that some new concept is required if the exponentially growing increase of available energy over the past fifty years is to be maintained during the 21st century.

One candidate, that promises a very high accelerating gradient, and hopefully, a very high energy from a machine of reasonable size is the 'beat-wave' accelerator suggested by Tajima and Dawson [3]. The basic theory has been supported to some extent by simulations and experiments [4]. A preliminary examination of the beat-wave phenomenon as the basis for an accelerator with energy of 5 TeV has been made by Ruth and Chao [5]. They used a very much simplified model for the process, and studied the inter-relation of the basic parameters, arriving at a design which appears to be near the optimum with respect to their assumed constraints.

At Rutherford Appleton Laboratory a part-time study group has been formed with the aim of further investigating the idea. The analysis of Ruth and Chao has been taken as a starting point, and their reference design forms the basis for ours. An additional constraint on the range of phase available for acceleration has been pointed out by Ruth since the original publication of ref.5; this is discussed in section 5.4 and incorporated in the new reference design.

Following the work of Ruth and Chao, three types of question may be asked. First, how good an approximation to the plasma behaviour does their very simple idealized analysis represent? Second, what is the effect of constraints that they have not yet considered? Third, can we invent new features that circumvent the perceived constraints?

With regard to the second question it is important to distinguish between fundamental constraints and thos which arise from present day technology. If we are planning for devices to be commissioned in the next century, due allowance must be made for technical advances. Meeting positional tolerances and exercising phase control of the lasers can fairly be expected to be less of a problem in the future, whereas limitations arising, for example, from energy balance or the physics of the beat wave phenomenon will remain unchanged.

After a brief comment on the requirements for high energy physics, the beat-wave phenomenon is described, and the way that the parameters were chosen by Ruth and Chao is reviewed. A modified reference design is then presented, for which other parameters are then calculated and practical problems considered.

2. REQUIRED ACCELERATOR PARAMETERS

Because of the increasing energy advantage of colliding beams in the relativistic regime, ($\sqrt{2\gamma}$ for identical particles) it is assumed that any future accelerator must operate in the colliding beam mode. Since radically new techniques will demand a long development time it is reasonable to expect that there will be a further round of machines using 'conventional' techniques that might lead to electron and proton energies respectively of 300 GeV (linear collider) and 20 TeV (the 'Desertron')(6). Beyond these energies linear colliders will be needed for electrons (to avoid synchrotron radiation), and for protons (to avoid prohibitive size and magnet cost). It is as components of linear colliders, therefore, that laser accelerators must be considered.

Acceleration both of protons and electrons needs consideration. An important difference is that for electrons at high energies the synchrotron radiation loss is very restrictive; even very small curvature of the orbits produces unacceptable energy loss rates. There is one advantage of this effect, however, in that it enables the use of 'damping rings' to produce very low emittance electron beams for injection into the laser accelerator.

Conventional machines are planned that should reach 1 TeV per beam for hadron beams and 0.1 TeV per beam for e^- or e^+. The maximum energies thought ever to be practically achievable are of order 0.4 TeV per beam for e^-e^+ and 20 TeV per beam for hadrons. The luminosity might be of order 10^{33} cm^{-2} sec^{-1}, though whether such expensive machines will ever be built remains an open question. As these represent 'the end of the dinosaurs' they are the lowest energies at which new devices should be considered. We now consider the luminosities that are needed to make machines at such energies useful. Electrons and hadrons are considered separately.

Electrons appear to be pointlike, and the e^-e^+ annihilation cross-section decreases with the square of the energy. At a centre of mass energy of 1 TeV $\sigma(e^-e^+ \rightarrow \mu^- \mu^+)$ is 10^{-37} cm^2. Even if there is some resonant enhancement, like the Z^o, with a cross section 5000 times this, a luminosity of $10^{30}cm^{-2}sec^{-1}$ would only give 1000 events per month. (Implied also is an energy resolution small compared with the resonance width).

The situation with hadrons is different; these are extended objects and their total cross-sections rise with energy. At a centre of mass energy of 10 TeV the p-p cross-section will be of order of $10^{-25}cm^2$, so that a luminosity of only $10^{25}cm^{-2}sec^{-1}$ gives an event rate of 1 per second. Interesting events will be rarer, but it will be worth waiting for them. Assuming a minimum statistical sample in a month of 2000 events, this luminosity permits the study of processes with cross section as small as $10^{-25}cm^2$. There are interesting events seen at this level at the CERN collider, so that the relatively low value of $10^{25}cm^{-2}sec^{-1}$ can be considered interesting for protons.

No colliding beam facilities yet exist involving electron-proton collisions. There are plans to build such a device (HERA) with 30 GeV electrons and 800 GeV protons and a luminosity between 10^{31} and 10^{32} cm^{-2} sec^{-1}. This is equivalent to a 50 TeV electron beam hitting a static target. It is possible that a single 10 Tev electron beam of the type discussed in Section 5, colliding with static nuclear targets would compete favourably with the HERA type of collider.

A 1 TeV beat wave electron-proton collider would probably have no competitor of the 'dinosaur' variety. At a luminosity of 10^{30} cm^{-2} sec^{-1} acceptable event rates would emerge for real photon-proton interactions using the bremsstrahlung photons from the electron beam. The problems involved in meeting these luminosities, particularly the very high value required for electrons, are considered in section 5.6.

3. THE BEAT-WAVE CONCEPT

The beat-wave phenomenon has been extensively described elsewhere[2,3], here an outline description is given. First, we note that a plasma is capable of propagating a transverse plane electromagnetic wave of frequency ω , with hyperbolic dispersion relation $c^2k^2 = \omega^2 - \omega_p^2$, where ω_p is the plasma frequency. For $\omega \gg \omega_p$, this gives phase and group velocities $c\left(1+\omega_p^2/2\omega^2\right)$ and $c\left(1-\omega_p^2/2\omega^2\right)$ respectively. If two waves with frequencies ω_1 and ω_2 are propagated in the same direction in plasma, in which the plasma frequency is equal to $\omega_1 - \omega_2$, then it is evident that a beat wave with frequency $\Delta\omega$ and wave number $\Delta k = k_2 - k_1$ travels through the medium. This excites a Langmuir wave with <u>phase</u> velocity $\Delta\omega/\Delta k$ which, when $\Delta\omega$ and Δk are small, is equal to $\partial\omega/\partial k$, the <u>group</u> velocity of the original waves. In a practical system the two electromagnetic waves would be produced by lasers and may be considered in first approximation to be plane. The beat-wave excites a longitudinal plasma wave, which can be seen to arise as follows. The transverse electric fields accelerate electrons in the transverse direction; this transverse motion interacts with the magnetic field of the wave to produce longitudinal velocities. The sign of the velocity at a given instant depends on the position of the electron, and this leads to a density modulation in the direction of propagation. The force on the electrons that produces the charge separation is a 'ponderomotive' force, with amplitude proportional to the gradient of $\langle E^2 \rangle$, where E is the peak electric field of the laser wave[9].

At this point two fundamental properties of the beat wave may be stated. The first, already mentioned is that

$$v_g = c\,(1 - \omega_p^2/2\omega^2) \tag{1}$$

This wave group velocity is the same as the velocity of a particle with $\gamma \approx \omega/\omega_p$. The second property is that the maximum value of E_z, the longitudinal accelerating field is (in gaussian units)

$$E_z = \alpha m_0 c\, \omega_p / e \tag{2}$$

where m_0 is the electron mass, and α is a quantity of order unity. In ref 5 it is shown that to avoid the undesirable trapping and acceleration of cold background plasma electrons α must be less than 0.5. The assumption of a cold plasma is unrealistic, but saturation effects (section 5) will almost certainly ensure that α is below 0.5. The question of the extent to which electrons are then trapped needs investigation.

It is immediately evident that there is a trade-off to be considered in choosing ω_p. From Eq.(1) a low value is required in order to minimize the phase slip of extremely relativistic particles with respect to the wave, whereas from Eq.(2) a high value is required to maintain a high acceleration rate. Knowing the plasma wavelength and $1 - v_g/c$ it is possible to determine the phase slip, and hence the distance over which a particle can be accelerated without a phase jump in the accelerating field. Clearly, the maximum shift cannot exceed π. In ref. 4 it is shown that $5\pi/8$ is a preferable figure, since near zero and π the acceleration is small and inefficient, and furthermore, there will in practice be particles spread over a range of phases. Subsequently Ruth has shown that only half this range is available. The reason for this is that the plasma is not infinite, but bounded in the transverse direction, and consequently there is a radial electric field in quadrature with the longitudinal field, which produces a strong defocusing force over the first half of the accelerating phase range and a strong focusing force over the second half. This results in an acceptable range δ of only $5\pi/16$. (See Fig.1). The accelerator must therefore be split into 'stages' of length $(5/32)\lambda_p/(1-v_g/c)$. For the parameters in Table 1 this leads to a staging length of 5 metres. For the technical reasons of cost and efficiency, a CO_2 laser operating at a wavelength 10.6 microns would be greatly preferable to the 1 micron wavelength chosen here. To maintain the same accelerating field, however, this would require the unreasonably short stage length of 5 cms.

If, then, the section length is determined by the phase slip, the laser beam diameter can be determined; the radial profile is assumed to be gaussian in form, the focal waist being at the centre of the stage. The pulselength is short and the plasma very underdense, so that self-focusing effects will not be significant. In order to maximize the average power density over the whole length L of the stage the Rayleigh length, $\pi\sigma_0^2/\lambda$, where σ_0 is the RMS radius of the beam is taken as L/2. (Note that in particle beam transport theory λ is equivalent to $\pi\epsilon$ and R to β.) The value of σ_0 is then determined; for the chosen values of ω and ω_p it is of the order of 1 mm, comfortably larger than the value of 0.26 mm for the plasma wavelength.

Having determined the profile of the beam, it is now possible to relate the laser field E in the beam to the laser power and the pulse length required to excite the plasma wave to the amplitude E_z given in Eq(2). To do this requires a model of the build-up process. Taking the simplest possible model, linear build-up with no competing processes in a cold collisionless plasma requires pulse energy proportional to ω^3/ω_p^4, which again, depends only on the choice of ω and ω_p and not on the laser field strength. For the values of ω and ω_p chosen earlier, it is found that of order 8.5 KJ per section is needed. For a 5 TeV machine this requires 200 lasers, with total energy of 1.7 MJ per pulse. Having determined the energy, it is necessary to fix the pulse length. A long pulse length requires many cycles for the beat wave to build up to full amplitude, and hence tight tolerances on ω_p; a short pulse implies high peak power and a more expensive laser. For the reference design a time of 100 psec is chosen. The reasons for this choice are given in Section 5.1.

4. A REFERENCE DESIGN

As explained in the previous section, once ω and ω_p are fixed the maximum accelerating electric field, and the phase slip per unit length are determined. Assuming that the length of the stages is chosen to allow phase slip per stage of $5\pi/16$, and the laser beam is of gaussian form, then the energy of the lasers is determined. Using the arguments outlined in the previous section, and set out in detail in ref. 4, the parameters of Table 1 can be assembled.

5. REQUIREMENTS FOR A PRACTICAL ACCELERATOR

Table 1 shows outline parameters and others that can be deduced from them. Many other parameters are of course required for a practical accelerator, and some of these are considered in this section. The discussion is within the framework of the simplified beat-wave theory, the validity of which is considered in Section 5.6 and ref.7.

5.1 Plasma Column

The central feature of the accelerator is a plasma column with well defined density. For the chosen laser pulse length of 100 psec, (justified later in this section) 115 cycles are required for the build up of the beat-wave, as shown in Fig. 2. This means that to maintain resonance the density, which is proportional to the square of the plasma frequency and the reciprocal of the beam radius, must remain within about 1/(2 x 115), (say 0.005) of its nominal value over the length of 5 metres. It is difficult to produce a discharge plasma with such well controlled properties. The laser pulse will

TABLE 1

REFERENCE DESIGN PARAMETERS

Chosen Parameters	Symbol	Value	Eq. no in Ref 5
Laser radian frequency	ω	$1.78\mathrm{x}10^{15}\ \mathrm{sec}^{-1}$	
Plasma radian frequency	ω_p	$7.2\ \mathrm{x}10^{12}\ \mathrm{sec}^{-1}$	
Plasma density	n_o	$1.6\ \mathrm{x}\ 10^{16}\ \mathrm{cm}^{-3}$	
Phase shift per section	δ	$5\pi/16$*	48,53
Phase form factor	$\sin(\frac{1}{2}\delta)/\frac{1}{2}\delta$	0.85	
Trapping Parameter	α		10
Transverse laser field	E_o		44
Derived Parameters			
Wave-particle velocity difference	$1-\frac{v_g}{c} = \frac{1}{2}\frac{\omega_p^2}{\omega^2}$	$8\ \mathrm{x}\ 10^{-6}$	29
Effective peak accelerating field.	$E_z = \left(\frac{m_oc}{e}\right)(0.85\alpha)\omega_p$	5 GeV/m	55
Pulse time	$\tau = \left(\frac{8m_o^2c^2}{e^2}\right)\left(\frac{\alpha}{E_o^2}\right)\left(\frac{\omega^2}{\omega_p}\right)$	100 psec*	43
Pulse energy	$W\tau = \left(\frac{m_o^2c^5}{e^2}\right)\alpha\delta\left(\frac{\omega^3}{\omega_p^4}\right)$	8,500J	60
Stage length	$L = 2c\delta\,\omega^2/\omega_p^3$	5m*	49
Spot area/π	$\sigma_o^2 = 2c^2\delta\,\omega/\omega_p^3$	(0.9mm) *	59
$\Delta\gamma$/stage	$2\times0.85\alpha\delta\,\omega^2/\omega_p^2$	25 GeV	57
λ_p/σ_o	$\pi(2/\delta)^{1/2}(\omega_p/\omega)^{1/2}$	0.28*	
Beam loading limit to number of particles per pulse.	$N=\frac{10\pi m_oc^3}{128e^2}\left(\frac{\alpha\eta}{0.85}\right)\left(\frac{\omega}{\omega_p^2}\right)$	$3\times10^{12}\eta$ (η=beam loading efficiency)	65
Cycles in beat-wave build-up	$\frac{\omega_p\tau}{2\pi} = \left(\frac{2m_oc}{e}\right)^2\left(\frac{\alpha}{E_o^2}\right)\left(\frac{\omega}{\omega_p}\right)^2$	115*	
Peak laser field	$E_o = \left(\frac{4}{\sigma_o}\ \frac{W\tau}{\tau}\right)^{1/2}$	$4.8\mathrm{x}10^8$ GeV/m*	

*Denotes different from ref. 5.   Reasons given in text.

readily produce 100% ionization of cold gas at the operating pressure, so that it is proposed to rely on the ionization produced by the laser beam in hydrogen gas at the required pressure.

It is of interest to calculate the radius of the plasma column so produced. The RMS widths of the gaussian beam is σ , and since the power density is proportional to the square of the field amplitude it depends on radius as $P_0 \exp\,(r^2/\frac{1}{2}\sigma^2)$, where $P_0 = W/\frac{1}{2}\pi\sigma^2$, so that

$$P = (2W/\pi\sigma^2)\exp\,(-2r^2/\sigma^2) \tag{3}$$

If P_i is power density required to ionize the gas, then the radius of the column is, from Eq. (3)

$$r = \sigma\left\{\tfrac{1}{2}\ln\left(2W/\pi\sigma^2 P_i\right)\right\}^{1/2} \tag{4}$$

At the centre of a stage $\sigma = \sigma_0 = 0.09$ cm, and $W = W\tau/\tau = 8.5 \times 10^{13}$ watts, and assuming a value of 3×10^{13} for P_i gives $r = 1.65\sigma = 0.15$ cm. At the ends of the column, where $\sigma = \surd 2\sigma_0$, the radius is about 0.2 cm.

The ionized gas will rapidly expand, giving a corresponding decrease in density. Resonance can therefore be preserved only if the light pulse is sufficiently short that the gas has not enough time to expand during its passage. Assuming (somewhat arbitrarily!) a 2 mm radius beam and expansion velocity of 10^7 cm per sec, 0.5% expansion will occur after a time $0.005 \times 0.2/10^7 = 10^{-10}$ sec. Short laser pulses of given energy imply higher power, and are therefore more expensive. We therefore take the value 10^{-10} sec for the laser pulse length. (C.f. 140 psec in ref.5), but note that in order to avoid competing processes in the plasma a much shorter value may be needed[9].

The optical path between the laser and the plasma must be evacuated, but any window in the vessel containing the gas will be destroyed by the light pulse. It is necessary, therefore, to operate in an open ended system with continuously flowing gas, and hence there will be a density gradient near the ends of the vessel. Owing to this density gradient, only over that region where the density is constant within the figure of 0.3%,calculated above,can resonant build up of the beat-wave occur. Whether this degree of uniformity can be achieved over the majority of the 5 metre staging distance needs to be calculated.

5.2 Laser and Associated Optics

For a laser of given pulse energy, the peak power, and therefore cost, decreases with increasing pulse length. For this reason a long pulse is desirable. We assume that the production of a 100 psec light pulse, in the form of a gaussian beam will present no fundamental difficulty. A mirror system is envisaged to produce a gaussian waist in the plasma column. The minimum size of mirror that can be used is determined by considerations of surface damage. For a 100 psec pulse of 1 micron radiation about 10 joules per cm^2 can be tolerated. This requires 850 cm^2, giving a radius of 16 cm. The associated diffraction angle θ_D is about $\lambda/D = 10^{-4}/32 = 3 \times 10^{-6}$. To produce a spot size of radius of 0.09 cm requires a focal length $2\sigma/\theta_D \approx 600$ metres. The mechanical arrangement of the mirrors and plasma columns requires careful consideration. It seems difficult to make the stages collinear; if they are not, then magnets will be needed to deflect the particle beams.

The minimum requirement is that the mirrors should not intercept beams from other mirrors, and that only one beam should be present in each stage. Assuming mirrors with co-planar axes the deflection angle must certainly exceed the convergence angle $0.16/600 = 2.7 \times 10^{-4}$ radian. The ends of the stages must be separated to allow the insertion of a bending magnet system, and this further increases the minimum angle. The geometry is shown in Fig. 3; it is assumed that the beams within a radius of 2σ must not intercept within the plasma region. As explained in section 3, the stage length L=2R where R is the Rayleigh length. The beam radius σ at a distance NR length from the centre of the plasma column is $2\sigma_0(1 + N^2)^{1/2}$. If, then, the spacing between the ends of the plasma columns is 2(N-1)R, the angle θ is $2\sigma/(N-1)R$. Substituting for R and σ

$$\theta = 4\sigma_0(1+N^2)^{1/2}/(N-1)L \tag{5}$$

If the spacing is equal to L = 2R, then N = 2 and $\theta = 4\sqrt{5}\,\sigma_0/L$ = 1.8×10^{-3} radians. For larger values of N, θ decreases to a lower limit of $4\sigma_0/L$.

In a practical system, in the presence of a vessel to contain the gas, supports, focusing and beam bending magnets etc. it is unlikely that it will be possible to attain such a small angle.

5.3 Bending Magnet Deisgn

Two limitations constrain the design of a beam bending magnet system. First, the available field limits the angle of bend attainable. This angle, denoted by θ is 0.3B/W per metre of magnet where B is in Tesla and W in GeV. At the maximum energy of 5 TeV, for a field of 10 Tesla θ is 6×10^{-4} per metre, so that 3 metres are required to produce 1.8×10^{-3} radians calculated in section (5.2) as the minimum angle.

The second limitation applies only to electrons, but is more serious at energies above about 1 TeV. It arises from the synchrotron radiation loss in the magnetic field. The fractional energy loss in a magnet of length of S metres may be written

$$\Delta\gamma/\gamma = 1.27\times10^{-6}B^2WS \qquad (T, GeV, m) \tag{6}$$

Clearly this must be kept well below unity. For the 3 metre 10 tesla magnet, inserting values in Eq.(6) gives $\Delta\gamma/\gamma = 2$. This is unacceptably large, since it represents an energy of 10 TeV, whereas the energy gain per stage is only 25 GeV! The minimum permissible magnet length may be found by using the expression for the total angle of bend in the magnet

$$\theta = 0.3BS/W \tag{7}$$

Eliminating B between Eqs (5) and (6) yields

$$S = 1.4 \times 10^{-5} W^3 \theta^2 / (\Delta\gamma/\gamma). \tag{8}$$

An energy loss of 20% per stage corresponds to 5 GeV, or $\Delta\gamma/\gamma = 10^{-3}$ at 5 TeV. This is above the maximum that could be permitted. Even in this case the magnet length S from Eq (8) is 5.6 km! This corresponds to a value of B of only 1/200 T.

These sample calculations indicate the scale of difficulties to be encountered in a 5 TeV laser accelerator of this type. It may be noted that the steep increase of radiation loss with B and W imply that the problem is much easier at energies a factor 5 or so lower. At higher energies, of course, things get rapidly worse.

An alternative arrangement not requiring bending magnets might be to use some nested system of annular mirrors, or to arrange for the beams to pass through the plasma column at an angle. It is not clear how the former alternative might be realized; for the second, an angle less than the ratio of beam diameter to length is required. For a beam diameter of 2×10^{-3} metres, and stage length of 5 metres this angle is 4×10^{-4}. We denote this by ϕ, and the convergence angle (equal to mirror radius divided by focal length) by θ_c. This was shown in section 5.2 to be 2.7×10^{-4}.

In Fig. 4 the geometry for this arrangement is shown. If D is the spacing between plasma columns, then the beams overlap at a distance $(D + L)\phi/\theta_c$. For L = D = 5m, this is only 7.4 metres. Such overlap cannot be permitted at a distance less than the mirror distance of 600 metres. The overlap distance could be doubled by having the mirrors on alternate sides of the beam, or maybe increased by a factor 2 by having mirrors spaced azimuthally round the beam axis. Even this does not seem adequate, unless the spacing between sections is considerably increased. The increase would be by a factor $600/(2\pi \times 7.4) = 13.5$ to 68 metres. This might be worthy of consideration at energies above 1 TeV where the bending problem looks so difficult.

Inventive suggestions are required for an acceptable solution to the staging problems.

5.4 Focusing of the particle beam

An important factor in the transverse dynamics of the beam is the radial electric field which arises from the finite size of the plasma column. In order to calculate this it is necessary to know the radial dependence of the amplitude of the beat wave. Instead of being constant, the coefficient α is assumed to vary with radius as $\alpha_0(1-r^2/r_0^2)$. This gives rise to a field of the form $CE_{z0}r/r_0$ where r_0 is a length of the same order as σ_0 for the optical beam, E_{z0} is the maximum value of E_z on the axis and C is an as yet undetermined constant. The z-dependence of the radial field is harmonic, with wavelength λ, and displaced by $-\pi/2$ from E_z. The constant C may be readily determined for example by setting $\oint E dl = 0$ in a path along the axis, then radially out to a small distance r, back parallel to the axis, and then radially inward to the starting point. This yields $C = \pi\lambda r$, so that

$$|E_r| = \frac{\lambda r}{\pi r_0^2}\,|E_{z0}| \tag{9}$$

This represents a strong field which is focusing or defocusing according to the phase, as shown in Fig 3. To estimate its order of magnitude, we set $r_0 = \sigma_0 = 0.09$cm and $\lambda_p/\sigma_0 = 0.28$, which yields $|E_r| \simeq |E_z| r$, where r is in cms. It is interesting that this corresponds to a magnetic field gradient of 16 T/cm. The focusing wavelength can readily be found from Newton's law.

$$\frac{e\lambda_p E_{z0}\cos\phi}{\pi r_0^2} r = \gamma m_0 \ddot{r} = \gamma m_0 c^2 r'' \qquad (10)$$

whence the focusing wavelength Λ is given by

$$\Lambda = 2\pi\left(\frac{-\pi\gamma m_0 c^2}{\lambda_p e E_{z0}\cos\phi}\right)^{1/2} r_0 \qquad (11)$$

Setting $r_0 = \sigma_0$ and using parameters in table 1 gives $\Lambda = 2(-10^{-5}\gamma/\cos\phi)^{1/2}$ metres. Outside the range $3\pi/2 > \phi > \pi/2$ this represents strong defocusing, so that only phases within this range can be used for acceleration. The angle ϕ has been assumed earlier to vary between $\pi/2$ and $13\pi/16$, so that at the end of the first stage, where $\gamma = 7 \times 10^4$, and $\phi = 13\pi/16$, Λ is about 2 metres. This is less than the length of a stage. At 5 TeV the wavelength exceeds 20 m, so that the length of one stage is less than $\Lambda/4$.

Assuming that the input emittance is $10^{-5}/\gamma$ metre-radians the beam radius of a matched beam at 10 GeV, where $\Lambda = 1$ metre, is $(\Lambda\epsilon/2\pi)^{1/2}$, about 10 microns.

The focusing within the plasma column increases along its length as ϕ increases from $\pi/2$ to where E_r is zero to $13\pi/16$. The average focusing in the column is much stronger than the focusing which can be provided in the draft space between plasma columns. It is hardly worth attempting further study until a clearer view of the required output beam and inter-stage bending is obtained.

Owing to the finite size of the plasma column, the resonant plasma frequency ω_q is slightly less than ω_p. This has been neglected in the analysis.

5.5 Multiple Scattering in Plasma Column

It is of interest to estimate the effect of multiple scattering in the plasma. Since the RMS angle of scatter per unit length varies as γ^{-1} the effect will be greatest at the beginning of the accelerator. As in the previous section we assume 10 GeV injection with emittance $10^{-5}/\gamma$ metre radians.

The increase of RMS angle (projected on to a plane through the beam axis) is given by

$$\Delta\langle\theta^2\rangle = (860/\gamma^2)\,\Delta t \tag{12}$$

where Δt is the thickness of the plasma measured in radiation lengths. In hydrogen 1 radiation length is 138 gm/cm , so that for a density of 1.6×10^{16} atoms per cm^3, 1 cm corresponds to $1.6 \times 10^{16}/138 \times 6 \times 10^{23}$ $= 2 \times 10^{-10}$ radiation lengths. The scattering in the first metre of the accelerator can be obtained roughly by putting Δt = 100 cm = 2×10^{-8} radiation lengths, and $\gamma = 2 \times 10^4$. This gives $\Delta\langle\theta^2\rangle^{1/2} = 2 \times 10^{-7}$, which compares with about 2×10^{-5}, the angle associated with the initial beam radius of 10^{-5} m and emittance of $10^{-5}/\gamma = 10^{-5}/5\times10^{-4}$ m-rad. The RMS scattering angle per unit length decreases as γ^{-1}, compared with (perhaps) $\gamma^{-\frac{1}{2}}$ for the trajectory angle; nevertheless, over the thousand metres of plasma the emittance growth is obviously an important effect. A growth in emittance can always be contained by adequate focusing, and since the ultimate spot size depends on the actual rather than normalized emittance, some growth in ϵ/γ may be tolerable. This point needs investigation.

A further effect which might cause scattering is fluctuations in the radial electric field, described in the previous section, arising from small scale non-uniformity of the plasma.

5.6 Beam Structure

The performance of the laser accelerator as a collider depends not only on the current accelerated but also very much on the beam structure. Many parameters are needed to describe this, and these are now treated in turn.

In the parameter list it is assumed that the laser pulse lasts for 100 psec, and leaves in its wake a Langmuir wave as illustrated in Fig 4. If this wave were to remain completely coherent there would be no limit to the length of pulse which could be accelerated. There would, however, be a limit due to beam loading of the total accelerated charge. Ruth and Chao estimate that a fraction $\alpha/2$ of the laser energy is deposited as kinetic energy associated with the beat wave (their Equation 63).

The real situation is likely to be very much inferior to this for the following reasons. First, the saturation amplitude is probably much less than the maximum value given in Eq.(2). This, of course, means that the particle energy is likely to be less than 30% of that assumed. Second, a great deal of laser energy is likely to be coupled into side-scattered waves, so that the efficiency of beam loading will be small. Third, the Langmuir wave may become incoherent only a few cycles after the laser wave has passed. This could perhaps be remedied by having a 'tail' to the laser pulse which provides sufficient drive to keep the wave coherent for, say, a further 100 psec, or 3 cm. This figure will be taken somewhat arbitrarily as a guess of the pulse length. In view of the energy uncertainty we take this as between $1\frac{1}{2}$ and 5 GeV for the 1 km of plasma which constitutes the accelerating region. An overall energy transfer efficiency of 5% from laser to accelerated particles for a beam with, say, 3 TeV energy, combined with 1.7 MJ of laser energy per pulse leads to about $N = 1.7 \times 10^{11}$ particles per pulse. This figure is evidently very much a guess, but the value of 10^{11} will be used for discussions of the luminosity.

The number of particles 10^{11} is of the same order as that expected in the Stanford collider. If a bunch with this number were injected into the accelerator, only a small fraction would be in the correct phase to be accelerated. This implies that for efficient acceleration pre-bunching at the plasma wavelength is required. For electrons this must be imposed at an energy much below 10 GeV, since the velocity difference between the electrons and the velocity of light is only $1/2\gamma^2$, which is about 10^{-4} at 10 GeV. This implies also that the injected electrons slip uniformly in phase during acceleration, from $\phi = \pi/2$ to $13\pi/16$ in each stage. For bunches of length ϕ_b the energy spread $\Delta\gamma/\gamma$ is given by $0.52\phi_b$. To limit the energy spread to 10% requires $\phi_b/2\pi$ of about 3%, unless non-isochronous paths for electrons of different energy are arranged between stages. In view of the conclusions on bending between stages reached in section 5.2 this is obviously difficult. Protons move more slowly; indeed, at 10 GeV γ is only 20; this is less than $\gamma_p = \omega_p/\omega$, so that the particles move more slowly than the wave. The first few stages will therefore need individual design. The rather rapid phase slip at 10 GeV ($10^{-5}\pi$ radians per wavelength, or roughly 1 radian per metre at 10 GeV) suggests that a higher injection energy is to be preferred. Limitations to magnet field strengths arising from synchrotron radiation do not apply to protons, so that there is scope for interstage compensation using non-isochronous orbits.

In view of the considerations outlined in this section, it appears very optimistic to assume the following parameters.

Table 2

Particles per bunch	10^{11}
Beam energy	3 TeV
Bunch length	1 cm
Energy spread	10%
Duty cycle within bunch	3%

Nevertheless these will be used to estimate the most favorable luminosity that might be obtained.

5.7 Luminosity

For single pass colliders the bunches are only used once, and very high densities are required. Self-fields add rather than cancel, and this causes a 'pinch', which enhances the luminosity provided that the effect is not too strong. In electron machines the orbit deflection causes strong synchrotron radiation known as beam-strahlung. This reduces the particle energy and induces energy spread. These two effects are quantified by the disruption parameter D and the 'beam-strahlung parameter' δ respectively. The condition that these should not be too large places constraints on the luminosity attainable.

Ignoring these effects, the luminosity is limited only by the beam size that can be obtained in the crossover region. This of course depends on the emittance of the final beam and the energy spread, in addition to the technical problems of accurate location. The formulae for luminosity, disruption factor, and beam-strahlung are, for an elliptical beam with vertical and horizontal semi-axes b and Rb, and length d

$$L = \frac{N^2 f E(D)}{\pi R B^2} \tag{13}$$

$$D = \frac{2 r_e n d}{\sqrt{3}(1+R)\gamma b^2} \tag{14}$$

$$\delta = \frac{32 r_e^3 N^2 \gamma}{3(1+R)^2 d\, b^2} = \frac{16 r_e^2 N \gamma^2 D}{\sqrt{3}(1+R) d^2} \tag{15}$$

where f is the repetition frequency, r_e is the classical electron radius ($r_e = 2.8 \times 10^{-13}$ cm), E(D) is an enhancement factor due to the pinching caused by disruption, and R is the aspect ratio of the bunch [6]. If $10 > D > 2$, $E(D) \simeq 6$ according to simulations done by Hollenbeek [9]. However, enhancing disruption further results in a decrease of E.

The formulas above cannot strictly be applied to the case at hand because each "bunch" consists of a train of "bunchlets" in the case of the BWA. As we shall see, it is necessary that all the bunchlets pass through each other in order to maximise luminosity. Let K be the number of bunchlets in a train; then the total number of particles is simply N=KN . If all bunches pass through each other, the luminosity formula is unaffected. On the other hand the disruption formula and beam-strahlung fraction can be replaced by the values for each individual bunchlet:

$$D_B = \frac{2r_c N_B d}{\sqrt{3}(1+R)\gamma b^2} = \frac{2r_c N d}{K\sqrt{3}(1+R)\gamma b^2} \tag{16}$$

$$\delta_B = \frac{16 r_c^2 N_B \gamma^2 D_B}{\sqrt{3}(1+R)d^2} = \frac{16 r_c^2 N \gamma^2 D_B}{K\sqrt{3}(1+R)d^2} \tag{17}$$

where all bunch dimensions are those of the bunchlet. For electrons the critical limitation is the beam-strahlung fraction, δ. For the BWA we have calculated all the parameters on the right hand side of Eq. (17) except D_B. Substituting from Table 2 and assuming an aspect ratio of 10.

$$D_B = 1{\cdot}7\times 10^{-6}\,\delta_B K \tag{18}$$

If in addition we allow $K\delta_B = 0.1$, since the beam has already a full energy spread of 0.1, then the transverse size is determined

$$b = 1{\cdot}6\times 10^{3}/K^{1/2} \quad cm. \tag{19}$$

Lastly we can substitute to find the luminosity

$$L = 1{\cdot}3\times 10^{26}\,Kf \qquad cm^{-2}\ sec^{-1} \tag{20}$$

It is evidently desirable to have a very long train of bunchlets to make K large. This spreads out the charge longitudinally which allows one to decrease the transverse size and thus increase the luminosity. To estimate K we assume (perhaps optimistically) that the plasma remains intact for a time equal to the time for the growth of the wave. This yields K = 115 and a luminosity

$$L = 1.5 \times 10^{28} \; f \; E(D) \;\; cm^{-2} \, sec^{-1} \tag{21}$$

The transverse dimensions of the beam are 1.5 and 15 microns.

To obtain high luminosity requires very high values of the repetition rate, f. This is characteristic of all electron colliders in the TeV energy range, and arises from the beam-strahlung limit. The constraint is worse for the BWA because of the short, tight bunches. If these can be smoothed out, for example by allowing the beam to drift through a long (but weak) transverse magnetic field, the luminosity might be increased by a factor of order 100. The corresponding beam radius would decrease. Whether such a scheme would be feasible requires further study.

This higher luminosity would be appropriate to protons, since beam-strahlung does not occur. The fundamental limit in this case would be the disruption parameter.

6. CONCLUSION

An attempt has been made to find a set of consistent parameters appropriate to a beat-wave linear accelerator with an energy of a few TeV which could be used as a linear collider. Since construction of such a machine, should it be practicable, is not envisaged until the next century, emphasis has been placed on what appear to be fundamental constraints rather than technical ones, such as the realization of tight dimensional tolerances and precisely controllable high power lasers at high repetition rate.

Assumptions have everywhere been optimistic, and the method of achieving some of the stringent parameters required, (such as the quality and structure of the assumed 10 GeV injected beam) has not been specified.

The detailed plasma physics of the beat-wave process is still by no means clear; both the maximum attainable field and the power requirements are not known, and assumptions made in the design have everywhere been optimistic. Indeed, because of the likely inefficiency both of the lasers and the acceleration mechanism, and the need for high repetition rate to overcome beam-strahlung limitations, the power requirements are likely to be enormous. For protons, beam-strahlung is not a problem, but higher energies are required to make the concept interesting. For energies up to a few TeV it is doubtful whether the BWA can compete in cost and conceptual (and technical) simplicity with a synchrotron.
We emphasize in conclusion that this is a very tentative exploratory study, and some of the parameters may be far from optimum. Numerous ad hoc assumptions have been made and there are guesses which may be far from correct. The study does, however, draw attention to a number of difficulties which must be faced by anyone advocating the beat-wave accelerator as a serious possibility.

REFERENCES

1. 'The Challenge of Ultra-High Energies'. Procedings of ECFA-RAL meeting, Oxford, September 1982. ECFA 83/68, published by Rutherford Appleton Laboratory.

2. J D Lawson. 'New Ideas for Accelerating Particles', Rutherford Appleton Laboratory Report RL 83 049 (1983). (Review based on Ref.1)

3. T Tajima and J M Dawson, Phys. Rev. Lett. 43, 267, (1974)
T Tajima 'Laser Acceleration by Plasma Waves for Ultra High Energies', Ref 1, p.169

4. C Joshi, 'The Plasma Beatwave Accelerator - Experiments', Ref.1 p.195 D J Sullivan and B B Godfrey 'Simulation of the Plasma Beat Wave Accelerator', Ref.1 , p 209.

5. R D Ruth and A Chao, in 'Laser Acceleration of Particles, (Los Alamos, 1982), AIP Conference Proceedings No. 91 New York (1982). Also available as Lawrence Berkeley Lab. Report 14332, (1982).

6. Keil and Skrinsky 'e^+e^- Colliders', Proc. Second ICFA Workshop on Possibilities of and Limitations of Accelerators and Detectors, Les Diablerets, 1979. CERN 1980.

7. R Bingham, R G Evans and J D Lawson, 'Possible Instabilities in the Beat Wave Accelerator', RAL Report RL 83 058.

8. F F Chen, 'Introduction to Plasma Physics', Plenum Press, New York and London, (1974), p.256.

9. C Joshi, 'The Surfatron Laser-Plasma Accelerator: Prospects and Limitations' Paper presented at the Trieste Workshop on Laser and Plasma Collective Accelerators, May-June 1983.

10. R Hollenbeek, Nuclear Instruments and Methods, 184, 337, (1981).

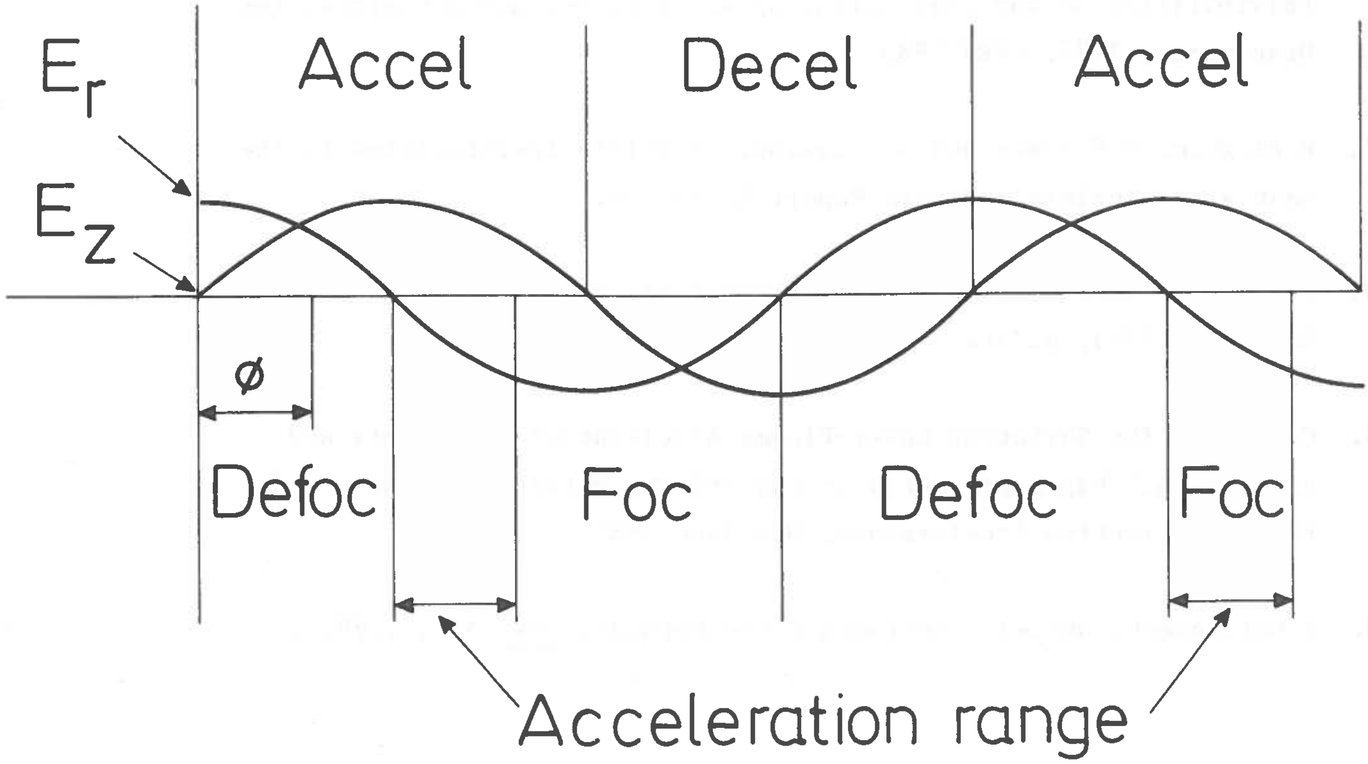
E_r
E_z
Accel
Decel
Accel
ø
Defoc
Foc
Defoc
Foc
Acceleration range

Fig 1

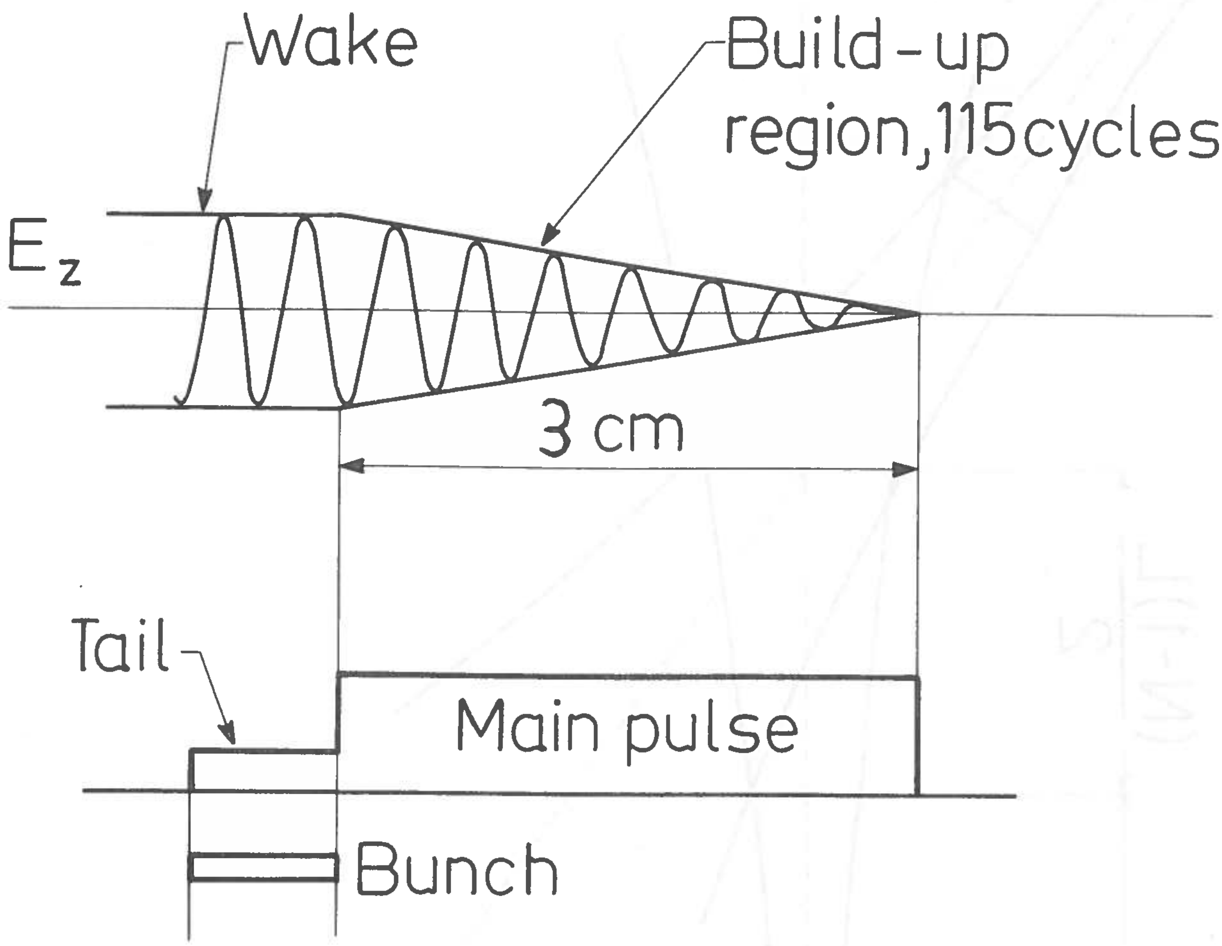

Fig 2

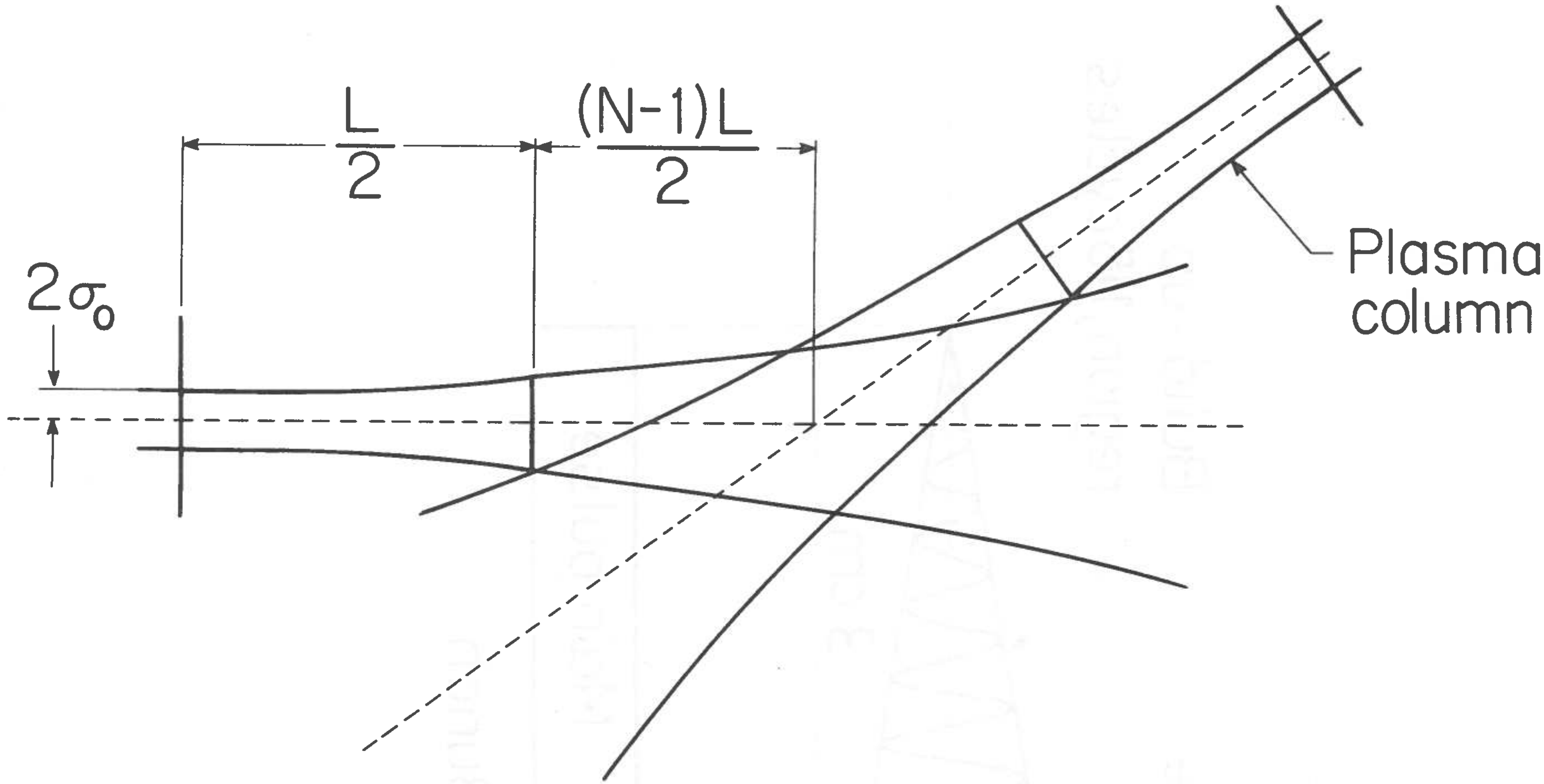

Fig 3

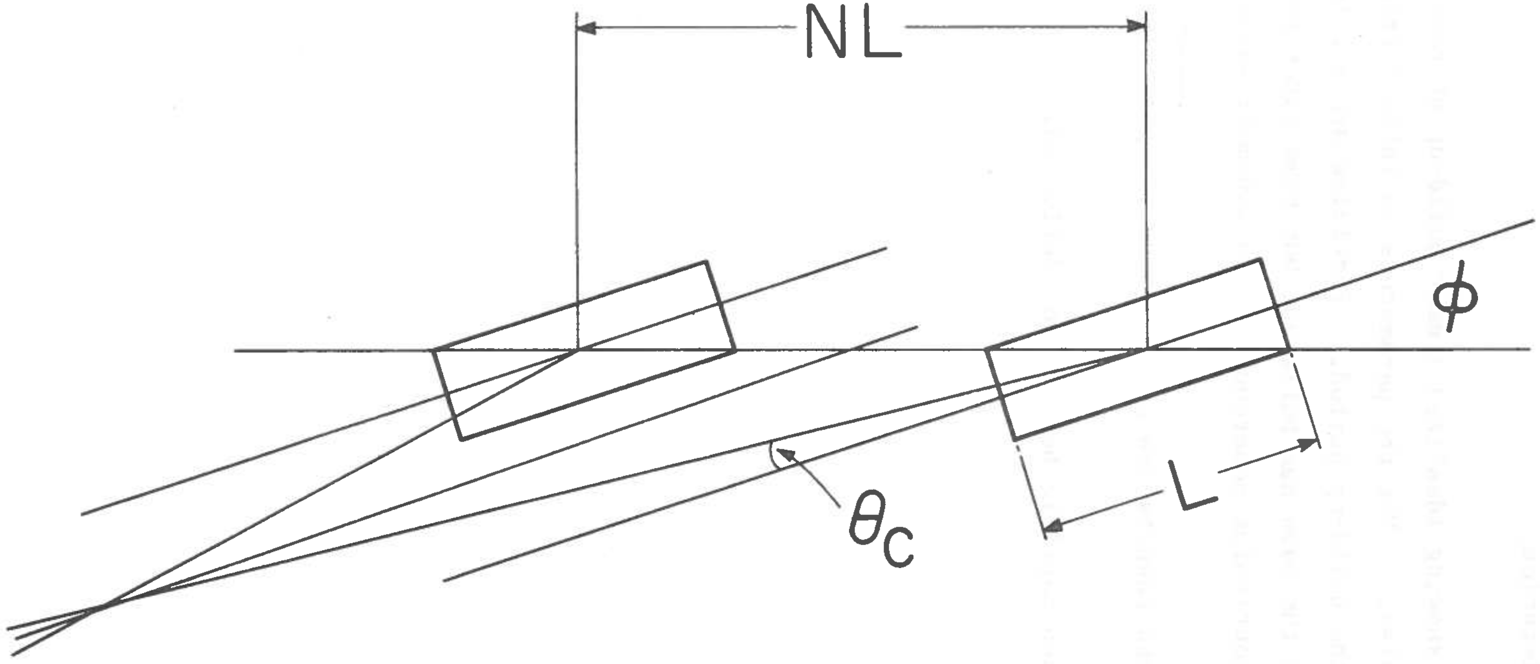

Fig 4

FIGURE CAPTIONS

Fig 1 Accelerating and focusing fields, illustrating the phase range available for acceleration.

Fig 2 Schematic diagrams showing idealized linear build-up of beat-wave within the laser pulse. For the parameters in Table 1 there are 115 cycles during the build-up period. Particles are accelerated in the 'tail' where the wave has built up, but some laser power remains to ensure continuing coherence of the Langmuir wave.

Fig 3 Staging geometry when magnets are used to bend the particle beam.

Fig 4 Staging geometry when particle beam is not deflected.