The R.F. System

An outline of the R.F. system is shown in figure (1).

Ten R.F. cavities are situated symmetrically as shown, the electron orbit passing through the centre of each.

Each cavity is connected by a stub of wave guide to the waveguide ring. The electrical length of the stub is Θ wavelengths. Coupling irises A and B couple power between the stub and the cavity and between stub and waveguide ring.

If the R.F. harmonic number = h = 300, the distances in orbit between successive cavities will be 30 wavelengths. The cavities should therefore be in phase with each other. The electrical length of the section of waveguide ring between cavities should therefore be an even number of wavelengths

$\beta \ell = n (2\pi)$ when n is an integer.

The input power is fed into the system through a coupling iris C in a tee junction placed symmetrically between cavities 1 and 10 if n is even, or slightly offset n is odd.

Following N.I.R.N.S.Y. Reports A 2.65, A.2171 and A 2.75 a programme for a digital computor (Mercury Autocode) has been used to find the voltage amplitudes throughout the system, for variations in cavity mismatch, coupling holes A, B, C, frequency variation and electrical length Θ.

Methods of Calculation

The R.F. cavities will be represented as shunt R.L.C. circuits. This will be a good model provided no other resonances of the cavities are close to the 408 Mc/s R.F. frequency, and there are no resonances close to the linac frequency of 7 x 408 Mc/S. The design of the cavities will have to take account of these instances.

The power input to the system will be fed to the ring via a waveguide isolator The impedance looking back along the input waveguide will equal that of the wave-guide, and in any voltage current representation the R.F. drive may be represented as either a constant current generator shunted by the waveguide impedance, or a constant voltage generator in series with the waveguide impedance (see appendix (a)).

There are two methods open to us by which we may calculate the phase and energy deviations during synchrotron oscillation.

(a) Runge-Kutta solution of the differential equations of motion, assuming continuous interaction with an R.F. travelling wave.

(b) Step by step calculation, calculating the voltage in each cavity each time an electron passes through. From a knowledge of the acceleration received and the magnet parameters a time may be calculated when it will pass through the next cavity.

Due to the period of the synchrotron oscillations the time intervals needed for the Runge-Kutta solution will be of the same order as the time of flight between cavities. In view of this, method (b) will be adopted being a much more realistic model of the actual system.

Approximations

To each R.F. cycle 7 bunches of electrons will be injected. For one turn injection and R.F. harmonic number of 300, 2100 bunches will be injected. Each bunch will have a very small phase spread which may be ignored, but its energy spread will be quite a large part of the total allowed energy deviation. During oscillation this energy spread will become a phase spread and the bunches will fairly quickly become a continuum losing their separate identity. It is evidently impossible to calculate the dynamics of 10^{12} electrons separately, so some approximation must be used.

1st approximation

The complete ring system is used with 10 cavities. The dynamics of 2100 particles are calculated. Each particle having the charge and mass of an electro bunch concentrated to a point in phase space. To simulate the energy spread each 7th particle is injected with identical phase with respect to the driving source, but with different energies. For the 2100 particles there will be 7 different phase angles at injection, for each phase angle 300 different energies. After injection 2100 values of both phase and energy will have to be followed.

This calculation would be very long so a 2nd approximation will be considered.

2nd approximation

The voltages in each cavity are assumed not to be very different in amplitude and phase from one another (this will be checked in all calculations) so that the mean voltage and phase around the ring may be calculated and assumed to be present at each cavity.

The 210 particles in flight between cavities 2 and 3 are assumed to be identical in phase and energy with the 210 particles in flight between cavities 1 and 2, 4 and 5, etc.

The method used in allocating phase and energy at injection will be similar to that of the first approximation.

The time taken for calculation will be approximately 1/10th that of the first approximation.

3rd approximation

This will be used for quick calculations to see whether it is worth calculating by the other two approximations. It corresponds to the model used by D.E.S.Y. in their analogue study.

7 particles are considered, so computing times are approximately 1/30th of the times of our 2nd approximation.

The model is a single cavity model such as our 2nd approximation. The dynamics of the first 7 particles entering the cavity are calculated. 203 particles will follow these through the cavity before the first 7 particles re-enter. These 203 particles in groups of 7 are considered to be identical in phase and energy with the first 7, i.e. particle 1 is identical with particle 8, 15, 22, 29, etc.

If the electron bunches had negligible energy spread this approximation would be quite a good one for the first few orbits, but in use its limitations must be remembered.

Programme for 2nd approximation

Introduction

At this moment the programme for the ring system is just being tested in Fortran, as it was previously in Mercury Autocode, and is not yet incorporated in the beam loading calculations.

The model is a single cavity fed from a matched waveguide source. The actual ring system source can be programmed with practically no alteration.

The cavity is represented by figure (2).

R is given by a parallel combination of the cavity shunt resistance and the generator shunt resistance.

Ig represents the driving current and Ib the beam current. Ig is sinusoidal waveform at w = 408 Mc/s. Ib is a succession of delta functions corresponding to the circulating current.

Response of parallel resonant circuit to train of delta functions

If X(t) is the value of the parameter X as a function of t, then X(p) is the Laplace Transform of the parameter X.

$$V(p) = \frac{I(p)}{Y(p)} \qquad (1)$$

$$Y(p) = pc + \frac{1}{R} + \frac{1}{pL} = \frac{C}{p}\left(p^2 + \frac{p}{RC} + \frac{1}{LC}\right) \qquad (2)$$

$$V(p) = \frac{I(p)}{}\cdot\frac{p}{c}\;\frac{1}{(p + a + jw_o)(p + a - jw_o)} \qquad (3)$$

$$\text{Where} \qquad a = \frac{1}{2RC} \qquad w_o^{\,2} = \frac{1}{LC} - a^2 \qquad (4)$$

Response to a delta function of magnitude Io

$$= V(p) = Io.\frac{p}{c}\;\frac{1}{(p + a + jw_o)(p + a - jw_o)}$$

$$v(t) = \frac{Io}{C}\left\{\frac{(-a - jw_o)}{-2jw_o}\,e^{(-a - jw_o)t} + \frac{(-a + jw_o)}{2jw_o}\,e^{(-a + jw_o)t}\right\} \qquad (5)$$

$$= \frac{Io}{C}\left\{\frac{1}{2jw_o}\left[a\,e^{-jw_o t} - a\,e^{jw_o t}\right] + \frac{1}{2}\left[e^{-jw_o t} + e^{jw_o t}\right]\right\} e^{-at} \qquad (6)$$

$$= \frac{Io}{C}\left(\cos w_o t - \frac{a}{w_o}\sin w_o t\right) e^{-at} \qquad (7)$$

$$= \frac{Io}{C}\,e^{-at}\left[1 + \left(\frac{a}{w_o}\right)^2\right]^{\frac{1}{2}} \cos(w_o t - \phi) \qquad (8)$$

When $\tan\phi = \frac{a}{wc}$

$$V(t) = A\,e^{-at}\cos(w_o t - \phi) \qquad (10)$$

$$\text{Where } A = \frac{Io}{C}\left[1 + \left(\frac{a}{w_o}\right)^2\right]^{\frac{1}{2}} \qquad (11)$$

For ease of calculation later let us expand this into a complex expression, the real part of this complex expression being identical to $A e^{-at} \cos(w_o t - \phi)$.

$$V(t) = A e^{-at} e^{j(w_o t - \phi)}$$

We should now check this expression in a simple case. Suppose delta functions of current are incident on the circuit at frequency f, value $w_o = 2\pi f$. Io being given by the identity $Ib \equiv Io. \mathcal{f}$

$$\text{Now } V(t-\tau) = Ae^{-at} e^{j(w_o t - \phi)} . e^{a\tau} e^{-jw_o\tau} \quad (12)$$

$$= V(t) e^{(a - jw_o)\tau} \quad (13)$$

After N delta functions (where N is very large)

$$V(t) = Ae^{-at} e^{j(w_o t - \phi)} \left\{ 1 + e^{(a - jw_o)\frac{1}{f}} + e^{(a - jw_o)\frac{2}{f}} .. + e^{(a - jw_o)\frac{(N-1)}{f}} + e^{(a - jw_o)\frac{N}{f}} \right\} \quad (14)$$

$$= Ae^{-at} e^{j(w_o t - \phi)} \left\{ e^{a\frac{N}{f}} . e^{a\frac{(N-1)}{f}} ... + e^{a/f} + 1 \right\} \quad (15)$$

If we take a new time zero from the moment that the Nth delta function is incident.

$$t' = t - \frac{N}{f} \quad (16)$$

$$V(t') = A e^{-at'} e^{-a\frac{N}{f}} e^{j(w_o t' - \phi)} e^{jw_o\frac{N}{f}} \left\{ e^{a\frac{N}{f}} = e^{a\frac{(N-1)}{f}} .... e^{a/f} + 1 \right\} \quad (17)$$

$$= A e^{-at'} e^{j(w_o t' - \phi)} \left\{ 1 + e^{-a/f} ... e^{-(N-1)\frac{a}{f}} + e^{-\frac{Na}{f}} \right\} \quad (18)$$

$$= \frac{A e^{-at'} e^{j(w_o t' - \phi)}}{1 - e^{-a/f}} = \frac{A e^{-at'} e^{j(w_o t' - \phi)}}{1 - (1 - \frac{a}{f})}$$

$$= \frac{A e^{-at'} e^{j(w_o t' - \phi)}}{} \quad \frac{\mathcal{f}}{a} \quad (19)$$

For a high Q circuit we may ignore

$(\frac{a}{w_o})^2$ and ϕ

$$\text{then } V(t) = \frac{Io}{C} e^{-at'} \_ e^{j w_o t'} \quad (20)$$

$$= \frac{Ib}{f} . \frac{1}{C} . \frac{\mathcal{f}}{a} e^{-at'} e^{jw_o t'}$$

$$= \frac{Ib}{Ca} e^{(-a + jw_o)t'}$$

$$= \frac{Ib}{} . \frac{W_o R}{Q} . \frac{2Q}{w_o} . e^{(-a + jw_o)t'}$$

$$= 2 Ib R e^{(-a + jw_o)t'} \quad (21)$$

where $a = \frac{wo}{2Q}$ $\quad C = \frac{Q}{W_o R}$

Programme

The input data is as follows:-

Q_L	Loaded Q of cavity
ψ	Measure of detuning of cavity where $\tan \psi = 2Q_L \frac{SW}{W}$
Ib	Mean circulating D.C. current
Go	Shunt conductance of generator source
Gc	Shunt conductance of cavity
δ	Angular phase of R.F. voltage at time of injection of first particle
Es	Synchronons energy at injection
DEs	Synchronons energy increase for each cavity transit
Ig	Peak generator current
Epsilon	Maximum allowed energy deviation
β	Gap factor of cavity, i.e. allowance made for transit time across the cavity, and non ideal electric field distribution at centre line of cavity

The energy spread at injection will also feed in as input data.

First the parameters which will be needed in the calculation are set up, for example cavity rise - decay times, initial times of entry of particles and their energies.

f = $4.078816 \cdot 10^8$

w = $2\pi f$

a = $w / 2QL$

SW = $\frac{W}{2QL} \tan \psi$

w_o = w - SW

α' = orbit compaction factor = 0.0453

C = $\frac{QL}{W}$ (Go + Gc)

A = $7 \frac{Ib}{c\,\beta} \left\{ 1 + \left(\frac{a}{w_o}\right)^2 \right\}^{\frac{1}{2}}$

T(1) = time of injection of first particle = δ / W

τ = time intervals between successive particles at injection

= $1/7 f$

Vo = $\frac{Ig \cos \psi}{G_o + G_c}$

T(i) = $T(i-1) + \tau$ for i = 2(1) 210

DE(i) to be set up for i = 1(1) 210

ES set to ES at injection

Parameters for this calculation are printed out and control is passed to a sub programme where the calculation proper is carried out. The values of the constant parameters are transferred to the sub programme along with the initial values of T and DE for the 210 particles.

First a constant is set up for each particle $EX(i) = 1.0 \quad i = 1(1)\ 210$. Various expressions contain this constant, and if a particle has too large an energy deviation such as it would strike the walls of the vacuum vessel and be lost, the EX value for this particle is set to zero so that its effects are no longer felt by the system. At the instant this occurs in the calculation a record will be printed out to the effect that the particle is lost.

Basis of calculation of effect of beam current

The total voltage in the cavity is the vector sum of the drive voltage at angular frequency w, and the voltage induced by the electron bunches passing through the cavity.

$$V = V_o\, e^{jwt} - V(t)$$

The minus sign is used since we shall take positive voltages as accelerating voltages.

At $t = 0$ the voltage is that induced by the drive current.

At T(1) the first particle enters. The voltage induced by the beam

$$V(1) = \frac{Ib}{C\,\gamma\,\beta} \left\{ 1 + \left(\frac{a}{W_o}\right)^2 \right\}^{\frac{1}{2}} = A$$

For T(i) $\qquad i = 2(1)\ 210$

$$V(i) = V_{(i-1)} e^{-aT}\, e^{jWoT} + (A \times EX(i))$$

$$DE(i) = (V_o\, e^{jwt(i)} - V(i) - DES + DE(i))\ EX(i)$$

here T = time interval between transit of ith and $(i-1)^{th}$ particle, and the DE(i) in the R.H.S. of the DE(i) equation represents the previous value of DE(i).

$$T(i) = \frac{30\ (2\pi)\ \alpha'\ DE(i)}{E_s\ \ w} + T(i)$$

In the T(i) equation above T is reset to zero after the time that would be taken for a synchronons particle to pass between cavities. DE(1) and T(1) are reset in a similar fashion.

If $/DE(i)//E_s$ exceeds epsilon, EX(i) is set to zero and the ith particle no longer has any effect in the calculation. A print out occurs naming the particle that is lost.

E_s is reset to E_s + DES.

The time interval between 2nd transit of the 1st particle and the 1st transit of the 210th particle is calculated. From this we get V(1) again so the 210 bunches and so on.

Now that the 210 particles have traversed the cavity for the first time the voltage included by the beam is calculated. The reflection factor at the input is also calculated.

Vb = V(210)

Vb is also printed out in normalized form, being normalized to its maximum possible value of

$$/Vb/ = /\frac{VG1}{2} \frac{(Go + Gc)}{Ib}$$

The total admittance is calculated as $\frac{Ig}{Vc} = Y$.

The input admittance as $\frac{Ig}{Vc} - Gc = Yin$

The reflection coefficient as $\frac{Go - Yin}{Go + Yin}$

The calculation is returned to t=0 with V(1) set and the process is repeated until a set number of orbits is completed.

A suitable distribution of input energies may be obtained as follows:-

DE(i) = (sign of Sin i) x (sin(i) - 1) x ES x DES1

where i = 1(1).210.

DES1 = maximum energy spread at injection as percentage of ES.

The distribution should be random in time but should give a greater concentration near low energy deviations.

Programme for the 3rd approximation

The basis for this programme is slightly different from the previous case. The basic calculations are done in terms of currents not voltages; in terms of an R.F. sinusoidal beam current not delta functions, and in a stationary vector form.

The drive current Ig varies as exp (jwt) and instead of calculating the exponential as in the previous case all time varying expressions will have the exp (jwt) removed, as though a rotating co-ordinate system was used.

The effect of the induced beam voltage will be produced by a ficticious current which would give rise to that voltage.

210 particles are injected. The cavity voltage is assumed constant while the 210 particles pass through. The cavity voltage is now calculated and remains constant while the 210 particles pass through again, and so on.

This allows the 210 particles to be considered as 30 groups of 7 particles. The $(N + i)^{th}$ particle having identical energy and phase to the $(M + i)^{th}$ particle where N = 0(7) 203

M = 0(7) 203

i = 1(1) 7

hence only the energy and phase of the first 7 particles need be calculated.

Particles 1, 8, 15, 22 ... etc. will be incident on the cavity at angular frequency w. These may be represented as an R.F. current of magnitude $\frac{2IG}{7}$ with phase as given by particle 1 and at frequency w. Similarly with particles 2, 9 .. etc., 3, 10 ... etc.

The contributions from these 7 groups are added vectionally and will give a resultant current at frequency w when oscillation has commenced.

The voltage in the cavity will however only build up according to the Q factor of the cavity. For convenience a ficticious current is calculated which would immediately give rise to the correct voltage.

~~Using 2 suffixes for IG i.e. Ib (m,n) with suffix m = 1 representing~~ current before transit of 210 particle, m = 2 representing current after transit of 210 particle.

n = 1 representing value calculated for phase of bunches

n = 2 representing value which would give rise to correct voltage instantaneously

Then IG (2,2) = Ib(1,2) + Ib(2,1) − Ib(1,2) $(1-e^{-a\tau})$

where a is the decay (or rise) factor of the cavity and τ is the time of flight of the particles between cavities.

This value Ib(2,2) is added vectorically to Ig the generator drive current to give Ic the total cavity current from which Vc the cavity voltage is calculated according to shunt resistance and detuning of the cavity.

The phase angles of each group of particles is calculated, hence the energy increment at each transit and time of arrival for the next transit.

The energy and phase of the 7 particles, together with the effective beam current, the cavity voltage, and reflection coefficient of the cavity are printed out after each transit. A mechanism similar to that used in the 2nd approximation, exists for eliminating particles with too great an energy deviation.

/veg

Microwave Impedance and use of an isolator

In waveguide and cavity structures the concept of impedance is different from that of lower frequencies.

At low frequencies where voltage and current along wires can be measured impedance is defined by

$$Z = \frac{V}{I}$$ where V is given in volts

I " " " amps

Z " " " ohms

In a waveguide or cavity the current flow is in a sheet of metal and varies in position and direction. The voltage between 2 sheets of metal is given by the strength of the E field between the points in question, and the distance between them $V = \int E.dl$

Microwave impedance in waveguides and cavities and free space is defined by

$$Z = Z_w = \frac{Ex}{Hy} \times \text{constant}$$

when the wave is propagating in the z direction, the constant varying according to convenience. For instance in free space

$$Z = \xi = \frac{Ex}{Hy} = \sqrt{\frac{\mu\mu_o}{\varepsilon\varepsilon_o}}$$ in M.K.S. units

For modes in waveguide 2 wave impedances are defined, one for H modes and one for E modes

$$Z_h = \xi \frac{\lambda_g}{\lambda_o}$$

$$Z_e = \xi \frac{\lambda_o}{\lambda_g}$$

being the ratio $\frac{Ex}{Hy}$ or $-\frac{Ey}{Hx}$ throughout the waveguide.

Any definition of impedance in a waveguide is made to satisfy 2 requirements.

$$P = \iint (E \wedge H).\, dn.\, ds$$

and $$P = \frac{V^2}{2Z}$$

where P is the power flow

V = V(E) can be chosen to suit the problem in hand.

In rectangular waveguide supporting the H_{01} mode the usual definition is given by

$$V = E_o\, b$$ where b is the height of the waveguide.

Power in generator resistance = P_o

If the load is mismatched to the waveguide then power reflected by the load is absorbed by the isolator.

With the equivalent circuit of figure (5) but with a generalized impedance Z and reflection coefficient P

$$\text{Power in load} = I_g^2 \left(\frac{R}{Z+R} \right)^2 Z$$

for R = Z the matched condition, this is reduced to

$$P_L = \frac{I_g^2 R}{4} = P_o$$

For the general case

$$P_L = 4 P_o \frac{RZ}{(Z+R)^2} = 4 P_o \frac{\frac{Z}{R}}{\left(\frac{Z}{R}+1\right)^2}$$

Now $\frac{Z}{R} = \frac{1+P}{1-P}$ *Where P is the reflection coefficient*

$$P_2 = 4 P_o \frac{\frac{1+P}{1-P}}{\left(\frac{1+P}{1-P}+1\right)^2} = 4 P_o \frac{(1+P)(1-P)}{(1+P+1-P)^2}$$

$$= 4 P_o \frac{(1-P^2)}{4} = P_o (1-P^2)$$

For the wave formalism we get at once

$$\text{Power in load} = P_L = P_o (1-P^2)$$

$$\text{" " generator} = P_G = P_o$$

$$\text{Power in isolator} = P_i = P_o \; P^2$$

The correct conditions at the load will be obtained using the model of a constant current generator shunted by the waveguide impedance.

The conditons at the generator are however different the power output being constant. The equivalent circuit representation would give

$$P_L = \left(I_g \frac{z}{z+R}\right)^2 R$$

$$= \frac{I_g^2 R}{4} \; 4 \left(\frac{z}{z+R}\right)^2$$

$$= 4 P_o \left(\frac{\frac{z}{R}}{\frac{z}{R}+1}\right)^2 = 4 P_o \left(\frac{\frac{1+P}{1-P}}{\frac{1+P}{1-P}+1}\right) = (1+P)^2 \; P_o$$

which is incorrect.

$$E_y = E_o \sin\left(\frac{\pi x}{a}\right)$$

$$H_x = -j \frac{E_o}{Z_h} \sin\left(\frac{\pi x}{a}\right)$$

$$P = \iint (E \wedge H).\,dn.ds = \frac{E_o^2}{Z_h} \int_0^a \int_0^b \sin^2\left(\frac{\pi x}{a}\right) \,da\,db$$

$$= \frac{E_o^2 ab}{2Z_h}$$

$$\frac{(E_o^2 ab)}{2Z_h} = \frac{E_o^2 b^2}{Z}$$

$$Z = 2Z_h \frac{b}{a}$$

This definition is useful in calculating the mismatch due to change in height of the waveguide.

The shunt impedance of a cavity is similarly defined in an arbitrary way, by defining V as E.dl at the centre line of an E_{010} resonator, a shunt impedance is obtained which when a current of charged particle is shot along the centre line, gives the correct voltage value.

When connecting a waveguide to a cavity the impedance defined above are of the order of 350 - 400 Ω and 5 - 10 MΩ. The coupling hole is considered to act as an ideal transformer with the appropriate turns ratio.

Throughout discussions in this report the waveguide impedance mentioned will be considered to be that seen through the coupling transformer.

With reference to figure (3), for a matched cavity

$$n = \frac{6\ 10^6}{4\ 10^2} = 1.5\ 10^4 = 1.225\ 10^2$$

figure (2) is an equivalent circuit of this with the generator impedance multiplied by 1.5×10^4.

Use of Isolator

Let us first consider the case of a generator isolator - load combination, with a matched load and with a mismatched load. See figure (4)

Generator delivers power P_o whatever the load condition because it always looks into a matched waveguide.

If the load is matched to the waveguide there is no reflection and Power P_o is delivered to the load. The effect of the isolator is nil. The equivalent circuit is as figure (5)

Power in load $= P_o$

Power in generator resistance = P_o

If the load is mismatched to the waveguide then power reflected by the load is absorbed by the isolator.

With the equivalent circuit of figure (5) but with a generalized impedance Z and reflection coefficient P

$$\text{Power in load} = Ig^2 \left(\frac{R}{Z+R}\right)^2 Z$$

for R = Z the matched condition, this is reduced to

$$P_L = \frac{I_g^2 R}{4} = P_o$$

For the general case

$$P_L = 4\,P_o \frac{R^2 Z}{(Z+R)^2} = 4\,P_o \frac{\frac{Z}{R}}{\left(\frac{Z}{R}+1\right)^2}$$

Now $\frac{Z}{R} = \frac{1+P}{1-P}$ *Where P is the reflection coefficient*

$$P_2 = 4\,P_o \frac{\frac{1+P}{1-P}}{\left(\frac{1+P}{1-P}+1\right)^2} = 4\,P_o \frac{(1+P)(1-P)}{(1+P+1-P)^2}$$

$$= 4\,P_o \frac{(1-P^2)}{4} = P_o\,(1-P^2)$$

For the wave formalism we get at once

Power in load = $P_L = P_o\,(1-P^2)$

" " generator = $P_G = P_o$

Power in isolator = $P_i = P_o\,P^2$

The correct conditions at the load will be obtained using the model of a constant current generator shunted by the waveguide impedance.

The conditons at the generator are however different the power output being constant. The equivalent circuit representation would give

$$P_L = \left(I_g \frac{z}{z+R}\right)^2 R$$

$$= \frac{I_g^2 R}{4}\; 4\left(\frac{z}{z+R}\right)^2$$

$$= 4\,P_o\left(\frac{\frac{z}{R}}{\frac{z}{R}+1}\right)^2 = 4\,P_o\left(\frac{\frac{1+P}{1-P}}{\frac{1+P}{1-P}+1}\right) = (1+P)^2\;P_o$$

which is incorrect.

Power in generator resistance = P_o

If the load is mismatched to the waveguide then power reflected by the load is absorbed by the isolator.

With the equivalent circuit of figure (5) but with a generalized impedance Z and reflection coefficient P

$$\text{Power in load} = Ig^2 \left(\frac{R}{Z+R} \right)^2 . Z$$

for R = Z the matched condition, this is reduced to

$$P_L = \frac{I_g^2 R}{4} = P_o$$

For the general case

$$P_L = 4\,P_o \frac{R^2 Z}{(Z+R)^2} = 4\,P_o \frac{\frac{Z}{R}}{\left(\frac{Z}{R}+1\right)^2}$$

Now $\frac{Z}{R} = \frac{1+P}{1-P}$ Where P is the reflection coefficient

$$P_2 = 4\,P_o \frac{\frac{1+P}{1-P}}{\left(\frac{1+P}{1-P}+1\right)^2} = 4\,P_o \frac{(1+P)(1-P)}{(1+P+1-P)^2}$$

$$= 4\,P_o \frac{(1-P^2)}{4} = P_o\,(1-P^2)$$

For the wave formalism we get at once

Power in load = $P_L = P_o\,(1-P^2)$

" " generator = $P_G = P_o$

Power in isolator = $P_i = P_o\,P^2$

The correct conditions at the load will be obtained using the model of a constant current generator shunted by the waveguide impedance.

The conditons at the generator are however different the power output being constant. The equivalent circuit representation would give

$$P_L = \left(I_g \frac{z}{z+R}\right)^2 R$$

$$= \frac{I_g^2 R}{4} . 4 \left(\frac{z}{z+R}\right)^2$$

$$= 4\,P_o \left(\frac{\frac{z}{R}}{\frac{z}{R}+1}\right)^2 = 4\,P_o \left(\frac{\frac{1+P}{1-P}}{\frac{1+P+1}{1-P}}\right) = \frac{(1+P)^2}{} P_o$$

which is incorrect.

If there was such a thing as an ideal generator with characteristic such as curve A in figure (6), an isolator would not be needed. A typical characteristic of a Klystron amplifier is shown in curve B. With an isolator in series the effects at the load are such as if driven by an ideal generator with characteristic as curve A.

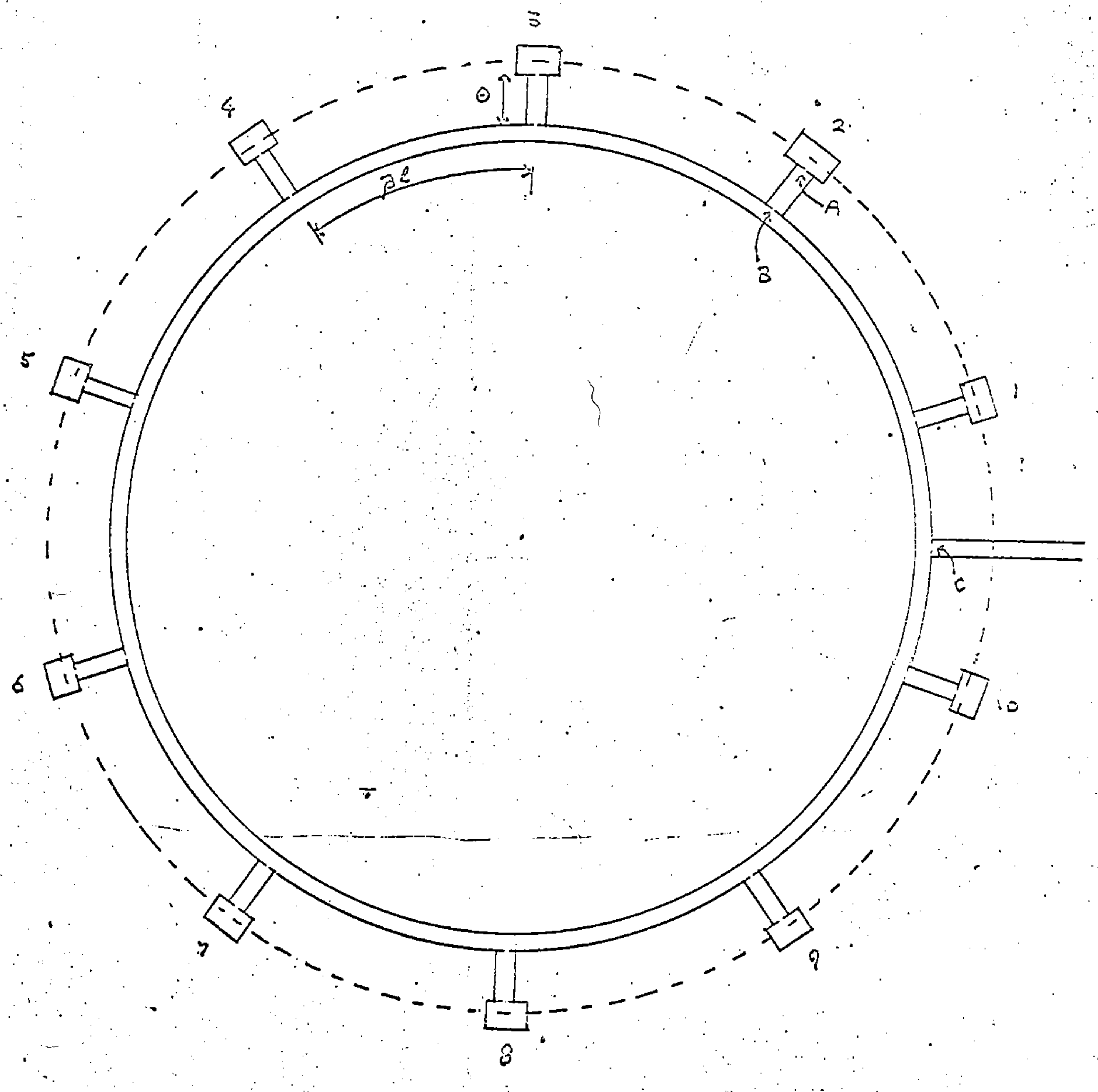

FIGURE (1)

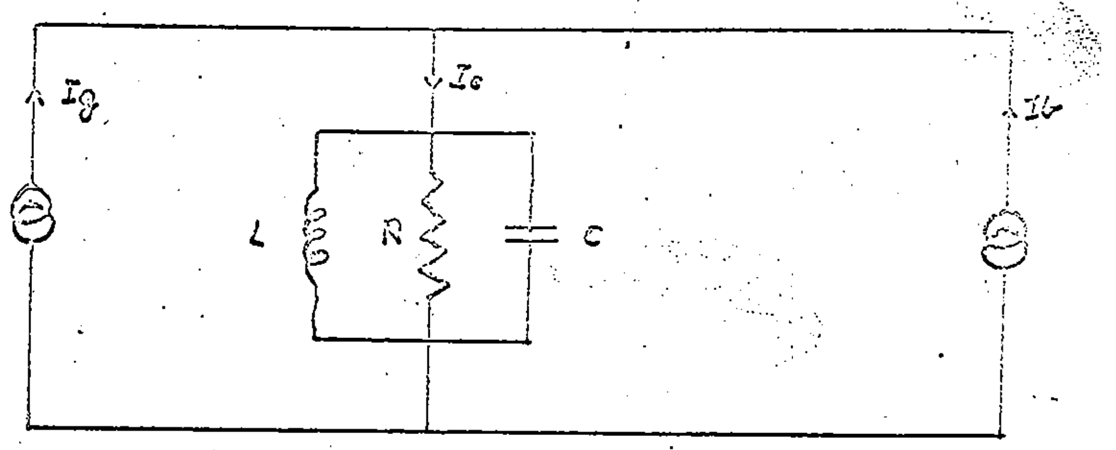

FIGURE (1)

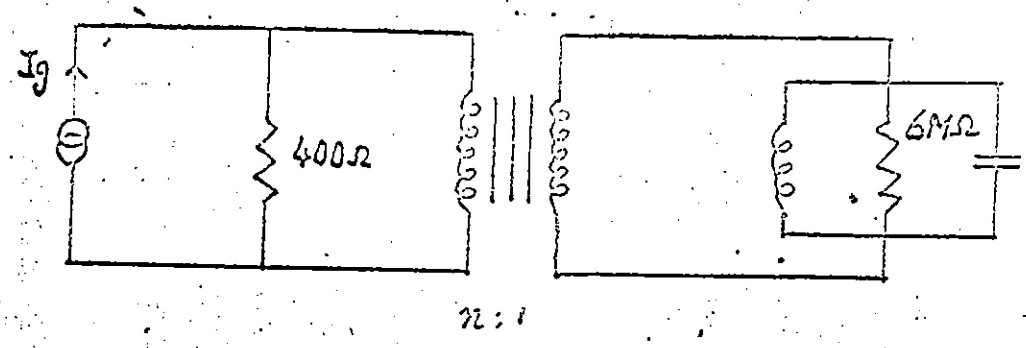

FIGURE (3)

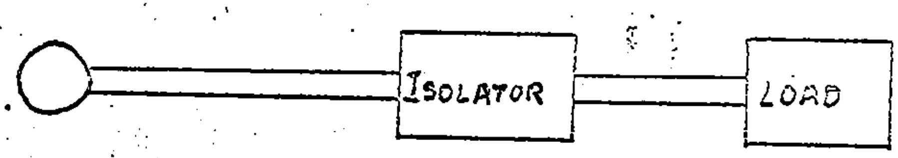

FIGURE (4)

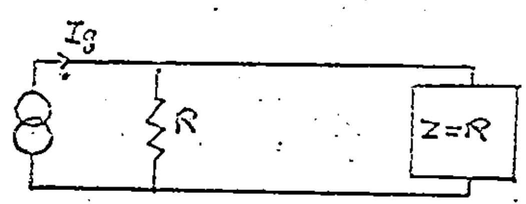

FIGURE (5)

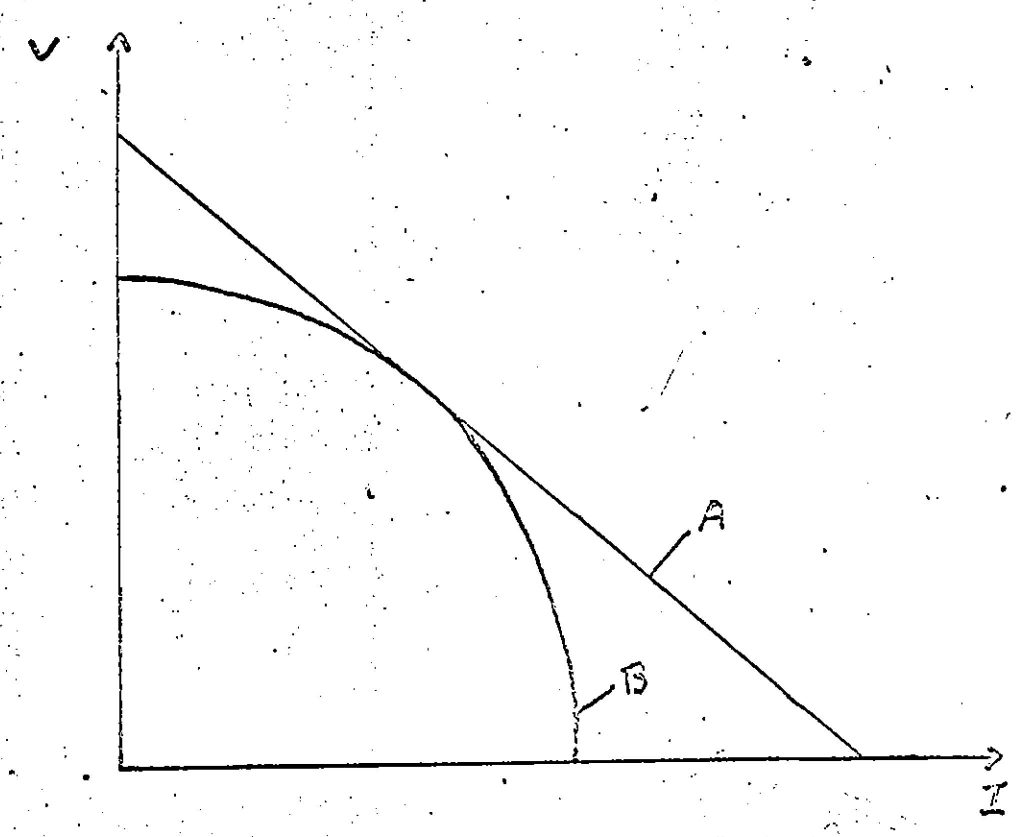

FIGURE (6)