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TRAPPING OF ELECTRON BUNCHES

IN THE ELECTRON SYNCHROTRON N.I.N.A. (WITHOUT BEAM LOADING)

by

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Trapping of Electron Bunches in the Electron Synchrotron N.I.N.A. (without beam loading)

The investigation will be solely concerned with the R.F. system except insofar as the results obtained will influence the design specification of the Linac injector.

The parameters concerning the magnet and vacuum vessel are already fixed. Parameters which are fixed or almost certain are given below:

Orbit compaction factor = $\alpha = \frac{dL}{L} / \frac{dp}{p} = 0.0453$.

Injection energy = 40 MeV.

Equilibrium energy gain per turn $\doteq$ 74 KeV at injection.

Allowed energy deviation for synchrotron oscillations $\doteq$ 2%.

Let us first consider the case of phase oscillations under conditions of continuous interaction with an R.F. field.

We shall start from the equation of motion given by Livingood in his book "Cyclic Accelerators".

$$\frac{1}{w_s} \frac{d}{dt} \left[\frac{E_s}{w_s h \Gamma} \frac{d\phi}{dt} \right] = \frac{N q V_m}{2\pi} (\sin \phi - \sin \phi_s)$$

where w_s = synchronous angular revolution frequency

h = Harmonic number of R.F.

$\Gamma = \left[\alpha - \frac{1}{\gamma_s^2} \right] \frac{\gamma_s^2}{\gamma_s^2 - 1}$

N = Number of R.F. accelerating gaps

V_m = Maximum cavity voltage

E_s = Synchronous energy of particles

q = Charge of particle

ϕ_s = Synchronous phase angle

ϕ = Angular position of particle with respect to R.F. phase

E_o = rest energy of electron

$\gamma_s = \frac{E_s}{E_o}$

As E_s = 40 MeV and $E_o \doteq 0.5$ MeV, $\gamma_s \doteq 80$ $\therefore$ $\Gamma \doteq \alpha$

If $\frac{dE_s}{dt}$ is very small compared with $\frac{d^2\phi}{dt^2}$

then

$$\frac{d^2\phi}{dt^2} = \frac{h\, w_s^{\,2}\, \alpha\, N\, q\, V_m}{2\pi E_s} \quad (\sin\phi - \sin\phi_s)$$

Now $$\frac{d^2\phi}{dt^2} = \frac{d}{dt}\,\frac{d\phi}{dt} = \frac{d\phi}{dt}\,\frac{d}{d\phi}\,\frac{d\phi}{dt}$$

Then $$\int \frac{d^2\phi}{dt^2}\, d\phi = \frac{d\phi}{dt}\,\frac{d\phi}{dt} - \int \frac{d}{d\phi}\,\frac{d\phi}{dt}\,\frac{d\phi}{dt}\, d\phi + \text{const.}$$

$$= \left(\frac{d\phi}{dt}\right)^2 - \int \frac{d^2\phi}{dt^2}\, d\phi + \text{const.}$$

$$\therefore \quad 2\int \frac{d^2\phi}{dt^2}\, d\phi = \left(\frac{d\phi}{dt}\right)^2 + \text{const.}$$

$$= \frac{2h\, w_s^{\,2}\, \alpha\, N\, q\, V_m}{2\pi E_s} \quad (-\cos\phi - \phi\sin\phi_s) + \text{const.}$$

$$\therefore \quad \left(\frac{d\phi}{dt}\right)^2 - \frac{2\Omega^2}{\cos\phi_s} \quad (\cos\phi + \phi\sin\phi_s) = \text{const.}$$

where $$\Omega^2 = \frac{-N\, q\, V_m\, h\, w_s^{\,2}\, \alpha\, \cos\phi_s}{2\pi E_s} \qquad \frac{\varepsilon\, q\, h\, \omega^2 \alpha}{2\pi E_s}$$

If $\frac{d\phi}{dt} = \left(\frac{d\phi}{dt}\right)_i$ when $\phi = \phi_i$

$$\left(\frac{d\phi}{dt}\right)^2 - \left(\frac{d\phi}{dt}\right)_i^2 - \frac{2\Omega^2}{\cos\phi_s} \quad (\cos\phi + \phi\sin\phi_s - \cos\phi_i - \phi_i\sin\phi_s) = 0.$$

We shall first find the limits of phase oscillations for a given ϕ_s.

For the extreme values of ϕ during phase oscillation $\frac{d\phi}{dt} = 0$.

If electrons are injected at this extreme value and

$$(\Delta E)_i = 0 = \left(\frac{d\phi}{dt}\right)_i$$

$$\left(\frac{d\phi}{dt}\right)^2 = \frac{2\Omega^2}{\cos\phi_s} \quad (\cos\phi + \phi\sin\phi_s - \cos\phi_i - \phi_i\sin\phi_s)$$

For the limits of oscillation $\frac{d\phi}{dt} = 0$

$$\therefore \quad (\cos\phi + \phi\sin\phi_s - \cos\phi_i - \phi_i\sin\phi_s) = 0$$

$\cos\phi = (-\phi \sin\phi_s + \cos\phi_i + \phi_i \sin\phi_s)$

$\cos\phi = -\phi A + B$

where $A = \sin\phi_s$

$B = +\cos\phi_i + \phi_i \cos\phi_s$

Graphs of $\cos\phi$ and $-\phi A + B$ against ϕ are shown in Figure (1).

If the particle is injected with angle ϕ corresponding to position A, it will change in phase until position B is reached and $\frac{d\phi}{dt}$ again equals zero. It will then return to position A.

Similarly if injected with ϕ corresponding to position B, the other limit of oscillation will correspond to position A.

For constant ϕ_s and other angles of injection the limits of oscillation may be found by lines drawn parallel to AB.

A limiting case CD is found where the line is tangential to the cosine curve at C. Beyond this the particle is lost to the stable region.

It will be found that the value of ϕ corresponding to position C is given by $\phi = (\pi - \phi_s)$.

For CD to be tangential at C

$$\frac{d}{d\phi}(\cos\phi) = -\sin\phi_s$$

$$\therefore -\sin\phi = -\sin\phi_s$$

$$\phi = (\pi - \phi_s)$$

If we inject at $\phi_i = \pi - \phi_s$ the other limit of oscillation may be found by the expression

$$\cos\phi = -\phi\sin\phi_s - \cos\phi_s + (\pi - \phi_s)\sin\phi_s.$$

Maximum Values of $\frac{\Delta E}{E}$

From Livingood $\frac{\Delta E}{E_s} = \frac{1}{w_s r h}\frac{d\phi}{dt}$

The maximum value of $\frac{d\phi}{dt}$ occurs when $\phi = \phi_s$ i.e. when $\frac{d^2\phi}{dt^2} = 0$.

$$\therefore \frac{\Delta E}{E_s}(\max) = \frac{1}{hw_s\alpha}\Omega\left[\frac{2}{\cos\phi_s}(\cos\phi_s + \phi_s\sin\phi_s - \cos\phi_i - \phi_i\sin\phi_s)\right]^{\frac{1}{2}}.$$

For the case of a particle which is just stable

$$\frac{\Delta E}{E_s}(\max) = \frac{1\Omega}{hw_s\alpha}$$

$$\left[\frac{2}{\cos\phi_s}\left(\cos\phi_s+\phi_s\sin\phi_s+\cos\phi_s-(\pi-\phi_s)\sin\phi_s\right)\right]^{\frac{1}{2}}.$$

$$= \frac{\Omega}{hw_s\alpha}\left[\frac{2}{\cos\phi_s}\left(2\cos\phi_s+(2\phi_s-\pi)\sin\phi_s\right)\right]^{\frac{1}{2}}.$$

$$= \frac{1}{hw_s\alpha}\left[-\frac{N\,q\,V_m\,hw_s^2\alpha}{2\pi E_s}\;2\left(2\cos\phi_s+(2\phi_s-\pi)\sin\phi_s\right)\right]^{\frac{1}{2}}.$$

$$= \left[-\frac{N\,q\,V_m}{h\alpha\pi E_s}\left(2\cos\phi_s+(2\phi_s-\pi)\sin\phi_s\right)\right]^{\frac{1}{2}}.$$

<u>Calculation of Injection Parameters</u>

For values of ϕ_s between 92° and 178° we shall calculate the following.

(1) Trapping angles limited by phase slip.

For ϕ_1 $\quad \phi_1 = \pi - \phi_s$

For ϕ_2 $\quad$ we solve for ϕ

$$\cos\phi = -\phi\sin\phi_s - \cos\phi_s + (\pi-\phi_s)\sin\phi_s$$

(2) Peripheral voltage.

If the synchronous energy increment per turn ΔE_s is known

$\Delta E_s = V_c \sin\phi_s$, hence $V_c = \Delta E_s/\sin\phi_s$

(3) Maximum values of ΔE for particles which are just stable

$$\frac{\Delta E}{E}\,(\max) = \left[-\frac{V_c}{h\alpha\pi E_s}\left(2\cos\phi_s+(2\phi_s-\pi)\sin\phi_s\right)\right]^{\frac{1}{2}},$$

using V_c as found above, and measuring E_s in electron volts.

(4) Injection angles such that $\frac{\Delta E}{E_s}\,(\max) = \text{EPSILON}$

where EPSILON is known.

$$\frac{\Delta E}{E_s}\,(\max)$$
$$= \left[-\frac{N\,q\,V_m}{h\alpha\pi E_s}\left(\cos\phi_s+\phi_s\sin\phi_s-\cos\phi_i-\phi_i\sin\phi_s\right)\right]^{\frac{1}{2}}.$$
$$= \left[-\frac{V_c}{h\alpha\pi E_s}\left(\cos\phi_s+\phi_s\sin\phi_s-\cos\phi_i-\phi_i\sin\phi_s\right)\right]^{\frac{1}{2}}$$

<u>Results</u>

The results are given in the Table and in Figure 2.

ϕ_s = Stable phase angle.

ϕ_1, ϕ_2 are maximum and minimum injection phases for trapping.

V_c = peak cavity voltage, for a nominal energy increase per turn of 74 KV

ΔE max. = Maximum energy excursion of the trapped particles, expressed as a fraction of the injection energy, (40 MeV).

ϕm shows the maximum and minimum injection phases for trapping and not exceeding 2% energy excursion.

M. DONALD

4th March, 1963

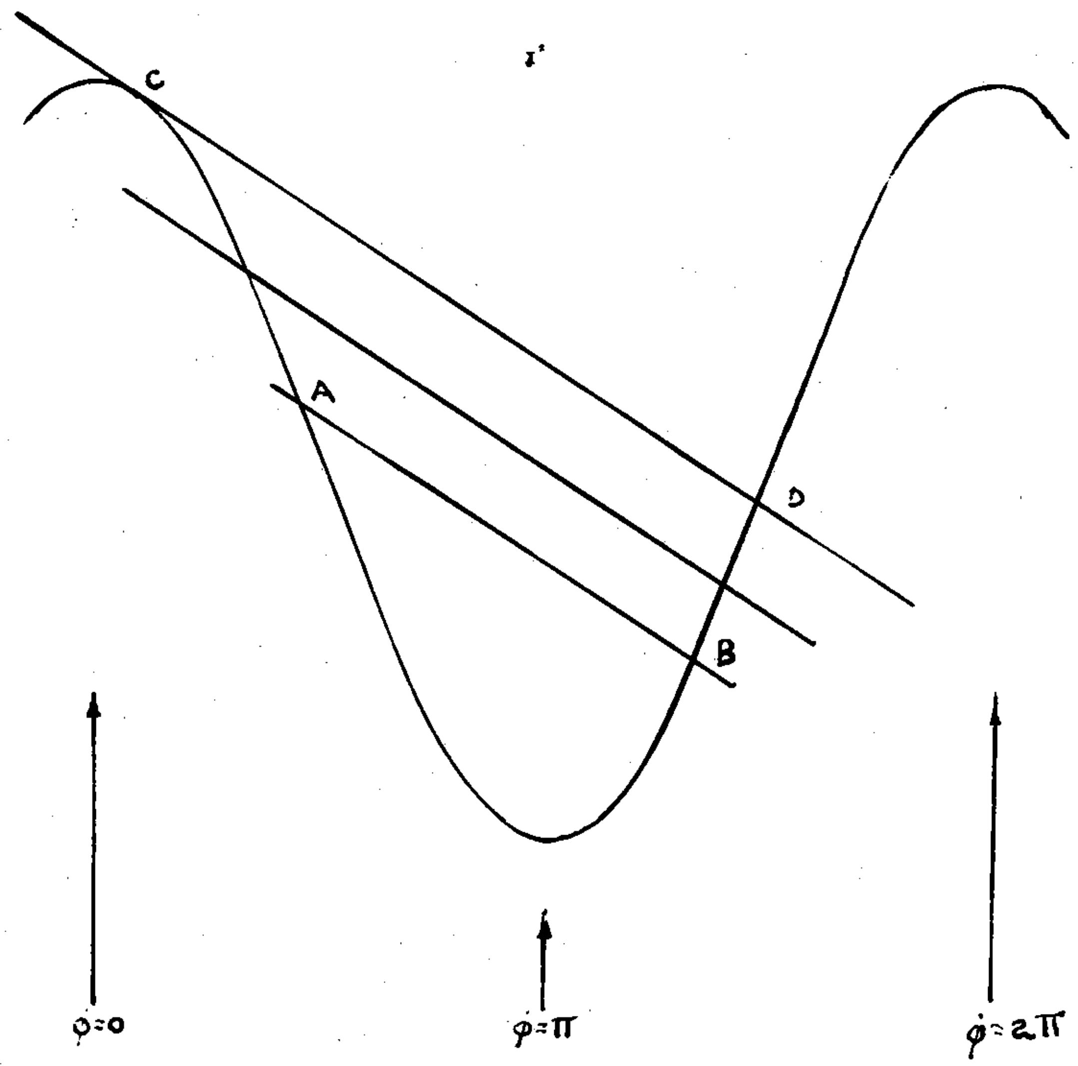
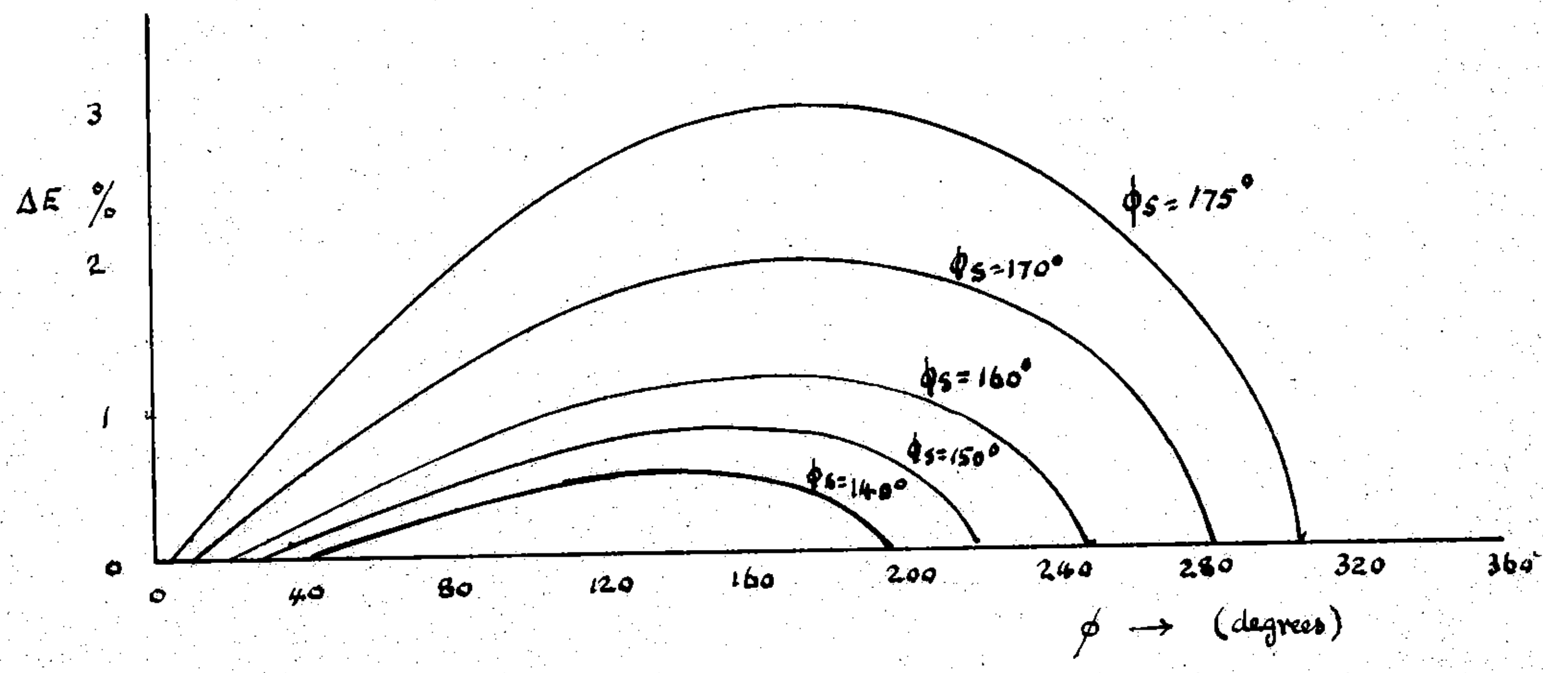


FIG. 1

ϕ_s	ϕ_1	ϕ_2	V_o $10^5 x$	$\Delta E_m/E_s$ %		ϕ_m
178	2	323	21.2	4.85	430	225
176	4	309	10.6	3.33	106	244
174	6	298	7.08	2.64	5-303 82	259
172	8	288	5.32	2.22	55	273
170	10	280	4.26	1.92	0	286
168	12	272	3.56	1.70		298
166	14	265	3.06	1.61		311
164	16	258	2.68	1.45		324
162	18	252	2.39	1.32		340
160	20	246	2.16	1.20		360
158	22	241	1.98	1.10		
156	24	235	1.82	.97		
154	26	229	1.69	.90		
152	28	224	1.58	.83		
150	30	219	1.48	.77		
148	32	214	1.39	.71		
146	34	209	1.32	.66		
144	36	205	1.26	.61		
142	38	199	1.20	.57		
140	40	194	1.15	.53		
138	42	190	1.11	.49		
136	44	185	1.09	.45	45 – 183.	
134	46	161	1.03	.41		
132	48	177	1.00	.38		
130	50	172	.97	.35		
128	52	168	.94	.32		
126	54	164	.91	.29		
124	56	159	.89	.27		
122	58	155	.87	.24		
120	60	151	.85	.22		
118	62	147	.84	.19		
116	64	143	.82	.17		
114	66	138	.81	.15		
112	68	134	.79	.13		
110	70	130	.79	.11		
108	72	126	.78	.10		
106	74	122	.77	.081		
104	76	118	.76	.066		
102	78	114	.76	.052		
100	80	110	.75	.039		
98	82	106	.75	.028		
96	84	102	.74	.018		
94	86	98	.74	.010		
92	88	94	.74	.004		



FISH DIAGRAM

$\phi_I = \pi - \phi_S$.

FIG 2.