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THE EQUILIBRIUM ORBIT IN NINA.

by

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Introduction.

The linear theory of charged particle orbits in an A.G. Synchrotron assumes that the magnets are so shaped that the lines of equal magnetic field strength form a set of arcs of concentric circles. The equation of motion in the median plane is then of the form

$$d^2y/dx^2 + \frac{1}{r^2}(1 - n)y = 0,$$

where $r = mv/B_z e$

v = particle velocity,

m = particle mass,

B_z = magnetic field perpendicular to the median plane,

e = particle charge,

y = displacement along increasing r.

The field index n is a positive number for a defocusing (D) magnet and a negative number for a focusing (F) magnet.

In the design of NINA these ideal magnets are replaced by sets of rectangular magnet blocks, the end blocks being approximately one half the length of the others (Fig.1.). These blocks are separated by wedge-shaped spaces. At the narrow end of the wedges the separation of the blocks is arbitrarily taken to be 1/10th inch. Each magnet will have the same field length as the ideal continuous curved magnet and this field length will be arranged to be independent of r. This can be achieved by suitable design of the end blocks. In a sectored magnet the measurement of the field length is not made along the arc of a circle as in the case of an ideal magnet, but along a series of straight lines, so that in the blocks the measurement ideally runs along an iso-magnetic line. The field length is defined as

$$L(y) = \left[\int B_z(y)dx \right] / B_o \left(1 + \frac{n}{r_o} y\right),$$

and it is this that we require to be independent of y and equal to the field length of the ideal magnet. In the calculations reported here this condition on the field length is enforced.

In the sectored magnet a particle with equilibrium momentum $eB_o r_o$ will not travel along the line $B_z = B_o$ as it does in the ideal magnet but will follow a new equilibrium orbit which is displaced from this, the displacement depending on the number of blocks comprising the magnet. If this displacement becomes large it can result in a loss of aperture.

For good beam dynamics it is preferable to have a large number of blocks so as to approach as nearly as possible the ideal magnet. However it is impracticable to have blocks which are too short and the minimum block length which has been considered is about 9" in which case there are 12 full blocks and 2 end blocks in each complete magnet. Economically however it is preferable to have a small number of blocks in each magnet since the cost of glueing each block is essentially independent of length. The maximum length considered is about 12" (9 blocks and 2 end blocks in each magnet).

The purpose of the calculation is to check that the displacement of the orbit if 9 blocks are used is not sufficiently great to cause a significant loss of aperture in the machine.

To carry out the calculation some assumptions must be made about the field distribution. We assume that over the length of each magnet block the field has its true theoretical value, the fringing field at the ends of the magnet being neglected, and the field in the wedges is adjusted so that the field length, as defined above, is constant and independent of radius. This calculation the details of which are given in a later section, is easy to carry out, and most of the estimates have been made using this model. The displacements that result are very small, and to check that they are a genuine estimate of the effect it was felt advisable to compare them with the results of a second calculation made assuming a slightly different field distribution. In this second method an attempt is made to estimate the true field in the gaps between blocks while the field length is corrected by lengthening the end blocks. The fringing field is again neglected.

It should be noted that these calculations have been performed on a single magnet without considering the continuity of the orbit between consecutive magnets. However, as the displacements predicted by the calculations are small, it is expected that little error will result from this approximation.

Equations of Motion.

Only the motion in the median (x,y) plane is considered and x is measured from one end of a magnet block, being directed along the magnetic axis defined by $B_z = B_o$, $B_x = B_y = 0$. The y-axis is perpendicular to this with origin at the position where $B_z = B_o$.

The variation of B_z with y is taken to be of the form :

$$B_z = B_o\ (1 + ny/r_o),$$

where r_o = magnetic radius of the synchrotron,
and $n = r_o/B_o \times dB_z/dy$ = field index.

It should be noted that according to this definition, the field index has the opposite sign to that employed in the case of the ideal magnet.

Neglecting terms of the order of $(y')^2$ and higher, the equation of motion is (1)

$$r_o y'' + ny/r_o + 1 = 0,$$

where $y' = dy/dx$
and $y'' = d^2y/dx^2$.

This equation has solutions :

$$y = y_o \cos\frac{\sqrt{n}}{r_o}x + \frac{r_o}{\sqrt{n}}y_o' \sin\frac{\sqrt{n}}{r_o}x + \frac{r_o}{n}\left(\cos\frac{\sqrt{n}}{r_o}x - 1\right),$$ 1

and $$y = y_o \cosh\frac{\sqrt{n}}{r_o}x + \frac{r_o}{\sqrt{n}}y_o' \sinh\frac{\sqrt{n}}{r_o}x + \frac{r_o}{|n|}\left(1 - \cosh\frac{\sqrt{n}}{r_o}x\right),$$

for F and D-magnets respectively. In both expressions y_o and y_o' are the values of y and y' at x = 0.

Differentiating equations (1) gives

$$y' = -y_o\frac{\sqrt{n}}{r_o}\sin\frac{\sqrt{n}}{r_o}x + y_o'\cos\frac{\sqrt{n}}{r_o}x - \frac{1}{\sqrt{n}}\sin\frac{\sqrt{n}}{r_o}x$$ 2

and $$y' = y_o\frac{\sqrt{n}}{r_o}\sinh\frac{\sqrt{n}}{r_o}x + y_o'\cosh\frac{\sqrt{n}}{r_o}x - \frac{1}{\sqrt{n}}\sinh\frac{\sqrt{n}}{r_o}x$$

In the gaps between magnet blocks a system of cylindrical co-ordinates (R, z, Ψ) is employed (Fig.2). The magnetic field in the median plane of the gaps is assumed to be of the form :

$$B_z = B_o \left(1 + \frac{n}{r_o} y\right) \qquad 3.$$

where $y = R - a$.

The equation of motion is first obtained (2) in terms of $d\Psi/dy$ and $d^2\Psi/dy^2$, and an approximate solution for y as a power series in Ψ as far as Ψ^2 is obtained. The substitution :

$$dy/d\Psi = (a + y)\ dy/dx = (a + y)y'$$

leads to the following solution :

$$y = y_{Go} + (a + y_{Go})\ y'_{Go}\ \Psi + \frac{a + y_{Go}}{2} \left[1 - \frac{a + y_{Go}}{r_o} \left(1 + \frac{n}{r_o} y_{Go}\right)\right] \Psi^2 \qquad 4.$$

and hence

$$y' = \frac{a + y_{Go}}{a + y} \left\{ y'_{Go} + \left[1 - \frac{a + y_{Go}}{r_o} \left(1 + \frac{n}{r_o} y_{Go}\right)\right] \Psi \right\} \qquad 5.$$

In these equations y_{Go} and y'_{Go} are the values of y and y' at $\Psi = 0$ and n is positive and negative respectively for F and D-magnets.

First Method of Calculation.

Each magnet is taken to consist of m magnet blocks of length 2ℓ and end blocks of length ℓ. The magnet field in the gaps necessary to give the correct field length is calculated as follows.

The magnetic field length is :-

$$L(y) = \left[(m + 1)\ 2\ell.B_z + (m + 1)\ 2T.B_G\right] / B_z,$$

where $2T = 2T(y)$ = gap width
$B_G = B_G(y)$ = magnetic field in gap.

But the correct field length is :-

$$L = 2\pi r_o / m',$$

where m' = number of complete magnets

$$\text{so} \quad B_G(y) = \frac{\pi r_o B_z(y)}{m'(m + 1)T(y)} - \frac{\ell B_z(y)}{T}$$

It is found in the case of the NINA magnets with m = 9 or 12 that B_G varies almost linearly with y and can be represented to a good approximation by :

$$B_G(y) = B_o \left(1 + \frac{n'}{r_o}(y + y_g)\right),$$

where n' and y_g are constants.

Equations (4) and (5) now become

$$y = y_{Go} + (a + y_{Go})y'_{Go}\Psi + \frac{a + y_{Go}}{2} \left[1 - \frac{a + y_{Go}}{r_o}\left(1 + \frac{n'}{r_o}(y_{Go} + y_g)\right)\right] \qquad 6.$$

$$\text{and} \quad y' = \frac{a + y_{Go}}{a + y} \left\{ y'_{Go} + \left[1 - \frac{a + y_{Go}}{r_o}\left(1 + \frac{n'}{r_o}(y_{Go} + y_g)\right)\right]\Psi \right\} \qquad 7.$$

The equilibrium orbit is taken to be of such a form that particles on this orbit will enter and leave each magnet in a direction parallel to the x-axis. In the present case, since the end blocks are exactly half the length of the other blocks, $y' = 0$ at the ends of the magnet and also at $x = \ell$ in the blocks and $\Psi = 0$ in the gaps. Because of this symmetry $y_{Go} = y_o$ and $y'_{Go} = -y_o$ in equations (1), (2), (6) and (7), i.e.

$$y = y_o - (a + y_o)y'_o\Psi + \frac{a + y_o}{2}\left[1 - \frac{a + y_o}{r_o}\left(1 + \frac{n'}{r_o}(y_o+y_g)\right)\right]\Psi^2 \qquad 8.$$

$$\text{and } y' = \frac{a + y_o}{a + y}\left\{- y'_o + \left[1 - \frac{a + y_o}{r_o}\left(1 + \frac{n'}{r_o}(y_o + y_g)\right)\right]\Psi\right\} \qquad 9.$$

From equations (2), if $y' = 0$ at $x = \ell$,

$$y_o = \frac{r_o}{\sqrt{n}}\, y'_o \cot\sqrt{\frac{n}{r_o}}\,\ell - \frac{r_o}{n} \qquad 10.$$

From equation (9), if $y' = 0$ at $\Psi = \phi$

$$y'_o = \left[1 - \frac{a + y_o}{r_o}\left(1 + \frac{n'}{r_o}(y_o + y_g)\right)\right]\phi \qquad 11.$$

Equations (10) and (11) can be solved for y_o and y'_o while the value at y at $x = \ell$ and $\Psi = \phi$ respectively is

$$y_\ell = y_o\cos\sqrt{\frac{n}{r_o}}\,\ell + \frac{r_o}{\sqrt{n}}\, y'_o \sin\sqrt{\frac{n}{r_o}}\,\ell + \frac{r_o}{n}\left(\cos\sqrt{\frac{n}{r_o}}\,\ell - 1\right) \qquad 12.$$

$$\text{and } y_\phi = y_o - (a + y_o)\, y'_o\phi + \frac{a + y_o}{2}\left[1 - \frac{a + y_o}{2}\left(1 + \frac{n'}{r_o}(y_o + y_g)\right)\right]\phi^2 \qquad 13.$$

Similar expressions can be obtained for a D-magnet using the second parts of equations (1) and (2).

Second Method of Calculation.

The average field B_G on the median plane in the gaps between magnet blocks is (3)

$$B_G = B_z\left(1 - \frac{2T}{\pi z}\right)$$

where $2T$ = gap width
and z = distance from the pole face to the median plane.

In the present case, B_z, T and z are all functions of y, although the expression is strictly only applicable in cases where these quantities are constant. Thus

$$B_z = B_o\left(1 + \frac{n}{r_o}\, y\right)$$

$$\text{so} \quad B_G(y) = B_o\left(1 + \frac{n}{r_o}\, y\right)\left(1 - \frac{2T}{\pi z}\right).$$

B_G is once more put in the form

$$B_G = B_o\left(1 + \frac{n'}{r_o}(y + y_g)\right)$$

where n' and y_g have different numerical values to those used in the first method.

The solution of the equation of motion in the gaps again takes the form of equation (6).

The field length is corrected by the use of end blocks of length $\ell + \Delta\ell(y)$ so that

$$L(y) = (m + 1)\left[2\ell + 2T\,\frac{B_G}{B_z}\right] + 2\Delta\ell.$$

The correct field length is approximately

$$L = (m + 1)\left[2\ell + 2T\right]$$

Substitution for B_G gives

$$\Delta\ell(y) = 2\,\left(T(y)\right)^2/\pi z(y).$$

The equilibrium orbit will be of such a shape that $y' = 0$ at the beginning and end of the magnet and by symmetry, at the centre of the central block or gap. The method used to determine the position of this orbit is to take starting values $y = y_i$ and $y' = 0$ at one end of the magnet and to calculate y and y' after traversal of successive blocks and gaps by means of equations (1), (2),(6) and (7). The starting value y_i is then adjusted to give $y' = 0$ at the centre of the central block or gap.

Results.

The first method has been applied to F and D-magnets with $m = 9$ and to an F-magnet with $m = 12$, while the second method has been applied to an F-magnet with $m = 9$. The parameters used in the calculations are as follows :-

$$r_o = 2077 \text{ cms},\ n\,(F) = +\,46.17,\ n(D) = -47.17.$$

For $m = 9$: $\phi = .0078540$, $a = 75.9110$ cms.

F-magnet : $\ell = 15.7166$ cms, $n' = +18.801$, $y_g = -.02372$ cms.

$\ell + \Delta\ell = 16.4610$ cms ($y = .036$ cms), $n' = +\,31.239$,

$y_g = -8.2794$ cms.

D-magnet : $\ell = 15.7166$ cms, $n' = -58.664$, $y_g = +\,.007602$cms

For $m = 12$: $\phi = .0060415$, $a = 80.7621$ cms.

F-magnet : $\ell = 12.0604$ cms, $n' = +\,20.4200$, $y_g = -.01897$cms

Using these parameters, the following results are obtained :-

m	Magnet Type	Method	y_i cms	y_o cms	y_ℓ cms	y_ϕ cms
9	F	1	-.062	-.062	+.002	-.064
9	F	2	+.036	-.054	+.005	-.056
12	F	1	-.026	-.026	+.009	-.027
9	D	1	-.038	-.038	+.021	-.040

In the second line of the table, y_o y_ℓ and y_ϕ are given for the central block of the magnet since y varies in this case from about +.036 cms. at the end blocks to +.005 cms. at the central block, y_o and y_ϕ varying in a similar manner.

Comparison of the first two rows of results indicates that the position of the orbit is not very sensitive to variations in the assumed field distribution as long as the field length is maintained at its correct value. The orbit in a D-magnet is similar to that in an F-magnet but is displaced slightly in the direction of positive y. In all cases y is small and will not result in any significant loss of aperture, so no advantage is to be gained by the use of magnets with $m = 12$ rather than $m = 9$.

References.

1. Cambridge Electron Accelerator Report, C.E.A.40, 1957.

2. CERN Symposium 1956, P295.

3. V.W.Hatton. N.I.R.N.S. Electron Laboratory, EL/TM 9.

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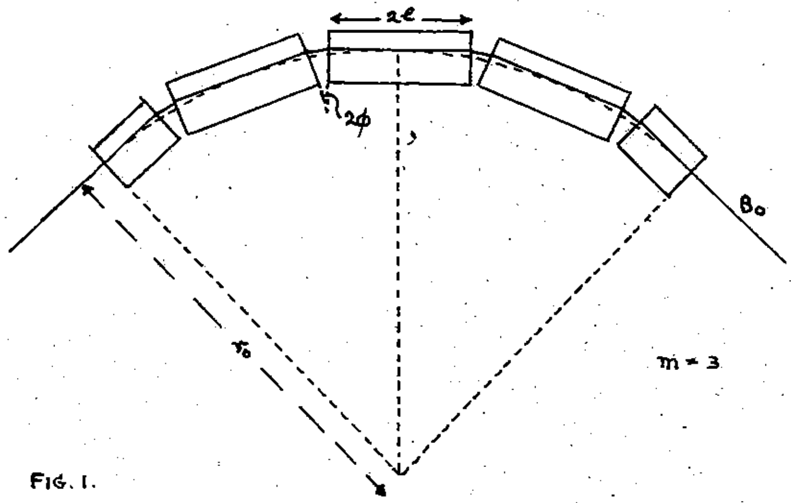

FIG. 1.

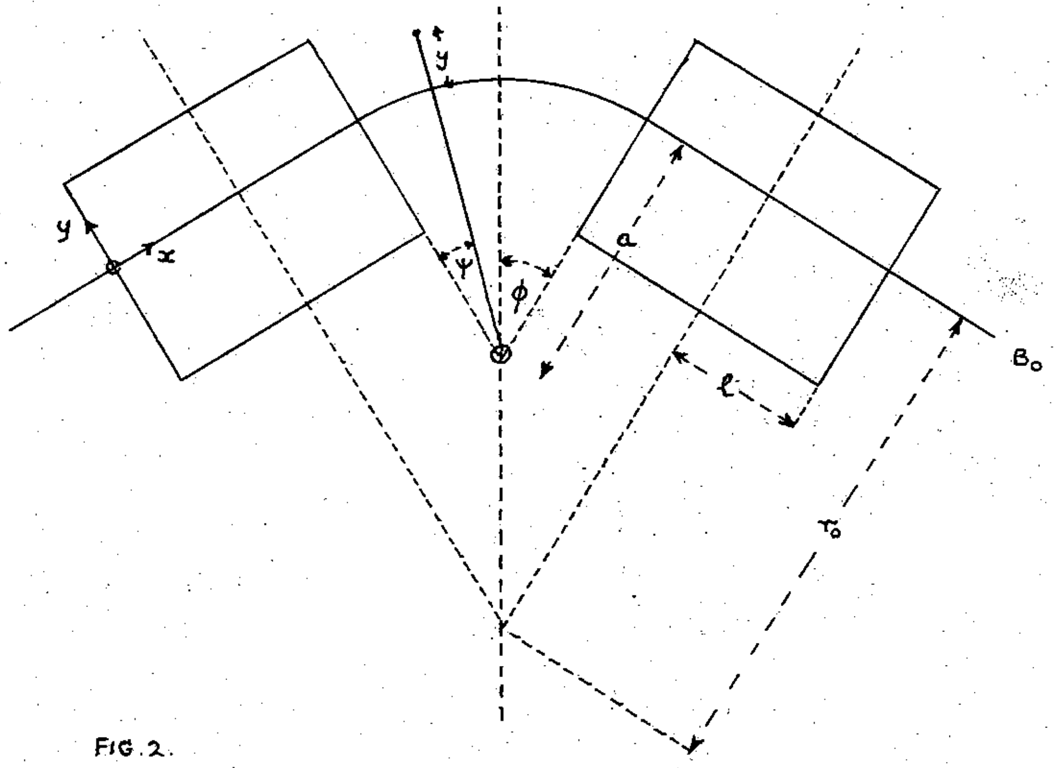

FIG. 2.

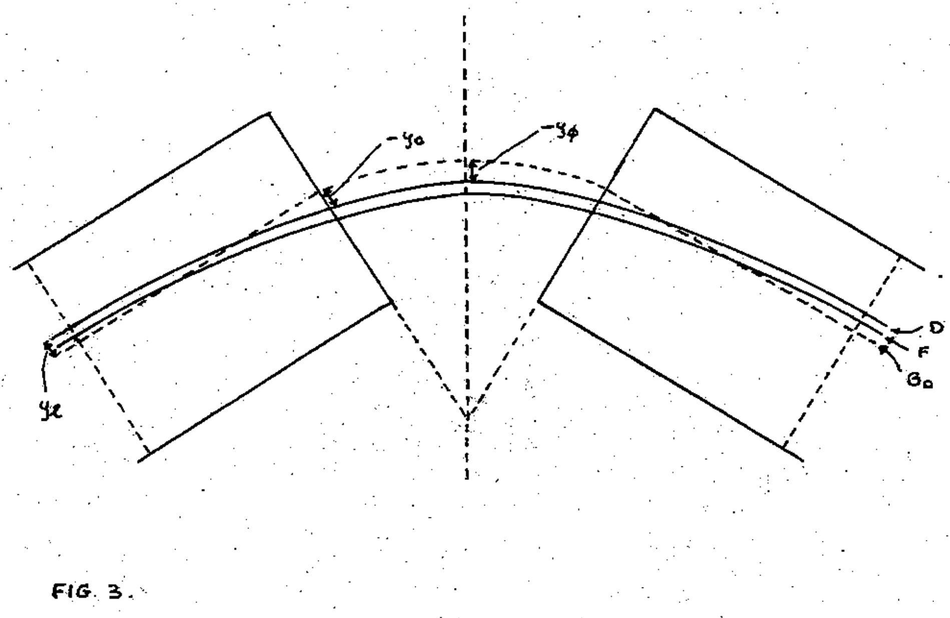

FIG. 3.