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NOTE ON THE MATCHING OF THE LINAC BEAM TO THE SYNCHROTRON.

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MATCHING OF THE LINAC BEAM TO THE SYNCHROTRON.

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INTRODUCTION.

The linac beam is introduced into the synchrotron via a beam transport system and an inflector. The non-collimation, finite cross-section and finite energy spread of the linac beam give rise to a similar beam in the synchrotron. Matching is required to minimise the amplitudes of the resulting betatron oscillations.

The injection system maintains the symmetry about the synchrotron median plane. Oscillations in the median plane (radial oscillations) and those in the plane perpendicular to it (vertical oscillations) may then, in the linear approximation, be treated separately.

MOMENTUM MATCHING.

The main elements of the injection scheme are shown in Fig. 1. For electrons travelling on the ideal path from the linac (the linac axis), the bending magnets M_1 and M_2 must

1) displace those with synchronous momentum so that after passing through the inflector 1 they travel on the synchronous orbit in the synchrotron

2) provide the momentum dispersion for those with non-synchronous momentum to be placed on their displaced equilibrium orbits in the synchrotron.

There will then be no contribution to the betatron amplitudes from the energy spread. Since the betatron oscillations are, in the linear approximation, momentum independent only synchronous electrons are discussed below.

PHASE SPACE MATCHING.

1) Synchrotron Acceptance.

Electrons which leave the inflector off the synchronous orbit in one of the independent planes perform betatron oscillations about it in that plane. The general equation of these oscillations is

$$y\,(s) \;=\; A\,\beta\,(s)^{\frac{1}{2}}\,\sin\left(\phi\,(s) + B\right) \qquad -(1)$$

s is distance along the orbit, y the displacement. $\beta(s)$ is the amplitude function with the periodicity of the magnet structure and $\phi\,(s)$ is the phase function, which changes by $2\pi Q$ per revolution, given by

$$\phi(s) = \int_0^s \frac{ds}{\beta(s)}$$

A and B are constants depending on the initial conditions. By differentiating (1) and eliminating $\phi(s)$ between the equation obtained and (1)

$$\frac{1}{\beta(s)}\left(y^2(s) + \alpha\,(s)\,y(s) + \beta(s)\,y^1(s)\right)^2 = A^2 \qquad (2)$$

where

$$y^1(s) = \frac{dy}{ds}\Big|_s \quad \text{and}\ \alpha\,(s) = -\frac{1}{2}\;\frac{d\beta}{ds}\Big|_s$$

(2) is the equation of an ellipse in phase space of area πA^2. The co-ordinates of an electron at any point on the orbit will lie on an ellipse on each orbit, the movement around the ellipse depending on the betatron phase shift per orbit. Since A is constant for given initial conditions and (2) holds for any point on the orbit, the ellipse transforms with constant area on moving along the orbit. The orientation of the ellipse in terms of α and β is shown in Fig. 2. The maximum value of y occurs when β is a maximum (and α = o) as can be seen from (2). The maximum value is then $A \ \beta_{max}^{\frac{1}{2}}$.

It is usually necessary to limit the betatron oscillation amplitudes to less than or equal to a . This limits $A \leq A_o = a/\beta_{max}^{\frac{1}{2}}$. The ellipse with equation (2) defined at the injection point with $A = A_o$ is the acceptance ellipse and its area ($\pi A_o^2 = \pi a^2/\beta_{max}$) is the synchrotron acceptance. Electrons must be injected with conditions such that they lie within the acceptance ellipse if they are subsequently to perform betatron oscillations with amplitudes less than or equal to a .

The β functions in the two independent planes at the injection point will not be identical, giving differently orientated acceptance ellipses.

2) Linac Emittance.

Emittance ellipses may be constructed for the linac, i.e. ellipses in phase space for the two independent directions which contain all or a specified fraction of the linac electrons. They may, in practice, be defined by the positions and sizes of apertures placed after the linac. The areas of these ellipses are the linac emittances. Values of α, β and γ may be assigned to the ellipses from their orientations and areas.

3) Matching.

The beam transport system transforms the emittance ellipses with constant area. For complete matching (i.e. the amplitudes of the betatron oscillations to be minimised) the emittance ellipses transformed to the injection point should have the same orientations as their respective admittance ellipse. This means that each pair of emittance and acceptance ellipses should have the same values of α, β, and γ at this point. Since the emittances are likely to be much less than the acceptances complete matching may not be required. Quadrupole triplets are used to provide flexibility in matching (1)

FORMULATION OF MATCHING PROBLEM.

The matching procedure is as follows:-

1) The injected beam is required to clear the stray field of the magnet proceeding the straight containing the inflector (2). The parameters of this inflector must be decided.

2) The paths of electrons with small energy deviations are traced back from their displaced orbits at the injection point to a point outside the magnet ring where the first bending magnet may be placed.

3) The parameters of the bending magnets to bring these paths back to the linac axis are calculated. The position of the linac relative to the inflector must previously be decided.

4) The admittance ellipses are traced back through the inflector and the momentum dispersion system to a point beyond them.

5) The quadrupole systems to match the emittance ellipses into the transformed acceptance ellipses are calculated (3) The linac characteristics must previously be known. The appendix following gives the matching method.

(1) CERN 60-26
(2) NINA EFC/1.
(3) W.I.B.Smith. CEA-90.

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APPENDIX PHASE - SPACE MATCHING.

A. Focal Length of Quadrupole.

The transfer matrix for passage through quadrupole into free space is

$$M = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi & \frac{\sin \phi}{K^{\frac{1}{2}}} \\ -K^{\frac{1}{2}} \sin \phi & \cos \phi \end{bmatrix} \qquad \phi = K^{\frac{1}{2}}L \qquad K = \frac{k}{B\rho}$$

L is the length of the quadrupole, i.e. $\begin{pmatrix} x_2 \\ \dot{x}_2 \end{pmatrix} = M \begin{pmatrix} x_1 \\ \dot{x}_1 \end{pmatrix}$

k the field gradient in the quadrupole,

Bp the magnetic rigidity of the beam,

s is distance in free space.

For a collimated beam ($\dot{x}_1 = 0$), $x_2 = 0$ when

$$\cos \phi - s K^{\frac{1}{2}} \sin \phi = 0$$

$$\text{i.e. } f = \frac{1}{K^{\frac{1}{2}} \tan \phi}$$

If ϕ is small, $\tan \phi \approx \phi$ and

$$f = \frac{1}{KL}$$

B. Transfer Matrix of Quadrupole in terms of f.

$$T.M = \begin{bmatrix} \cos \phi & \frac{\sin \phi}{K^{\frac{1}{2}}} \\ -K^{\frac{1}{2}} \sin \phi & \cos \phi \end{bmatrix} = \begin{bmatrix} \cos \phi & L \sin \phi / \phi \\ -KL \sin \phi / \phi & \cos \phi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & L \\ -KL & 1 \end{bmatrix} \text{ if } \phi \text{ is small.}$$

If L itself is small compared with 1,

$$T.M = \begin{bmatrix} 1 & 0 \\ P & 1 \end{bmatrix} \quad \text{where } P = \frac{-1}{f} \qquad (3)$$

The thin lens approximation (L small) gives a quadrupole which affects only the angular displacement.

C. <u>Transfer matrix for α, β and γ.</u>

The general equation of an ellipse in phase space is

$$\frac{1}{\beta}\left\{ y^2 + \overline{\alpha y + \beta y^1}^{\,2} \right\} = A^2$$

i.e. $$\gamma y^2 + 2\alpha y y^1 + \beta y^{12} = A^2$$

or $$f y^2 + 2h y y^1 + g y^{12} = A^2$$

This may be written

$$(y, y^1) \begin{bmatrix} f & h \\ h & g \end{bmatrix} \begin{pmatrix} y \\ y^1 \end{pmatrix} = A^2 \qquad fh - h^2 = 1$$

Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the normal transfer matrix between two points

$(ad - bc) = 1.$

i.e. $$\begin{pmatrix} y2 \\ y^1 2 \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} y1 \\ y^1 1 \end{pmatrix}$$

Then $$\begin{pmatrix} y1 \\ y^1 1 \end{pmatrix} = M^{-1} \begin{pmatrix} y2 \\ y^1 2 \end{pmatrix} \qquad M^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \text{inverse of } M.$$

$$(y_2, y^1 2) = (y_1, y^1 1)\, M^1 \qquad M^1 = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \text{transpose of } M.$$

and $$(y_1, y^1 1) = (y_2, y^1 2)\,(M^1)^{-1} \qquad (M^1)^{-1} = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

Now $$(y_1, y^1 1) \begin{bmatrix} f_1 & h_1 \\ h_1 & g_1 \end{bmatrix} \begin{pmatrix} y_1 \\ y^1 1 \end{pmatrix} = A^2$$

$$(y_2, y^1 2)(M^1)^{-1} \begin{bmatrix} f_1 & h_1 \\ h_1 & g_1 \end{bmatrix} M^{-1} \begin{pmatrix} y_2 \\ y^1_2 \end{pmatrix} = A^2$$

or $$(y_2, y^1 2) \begin{bmatrix} f_2 & h_2 \\ h_2 & g_2 \end{bmatrix} \begin{pmatrix} y_2 \\ y^1 2 \end{pmatrix} = A^2$$

where $$\begin{bmatrix} f_2 & h_2 \\ h_2 & g_2 \end{bmatrix} = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix} \begin{bmatrix} f_1 & h_1 \\ h_1 & g_1 \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

or $$\begin{pmatrix} f_2 \\ h_2 \\ g_2 \end{pmatrix} = \begin{bmatrix} d^2 & -2cd & c^2 \\ -bd & ad+bc & -ac \\ b^2 & -2ab & a^2 \end{bmatrix} \begin{pmatrix} f_1 \\ h_1 \\ g_1 \end{pmatrix} \qquad (4)$$

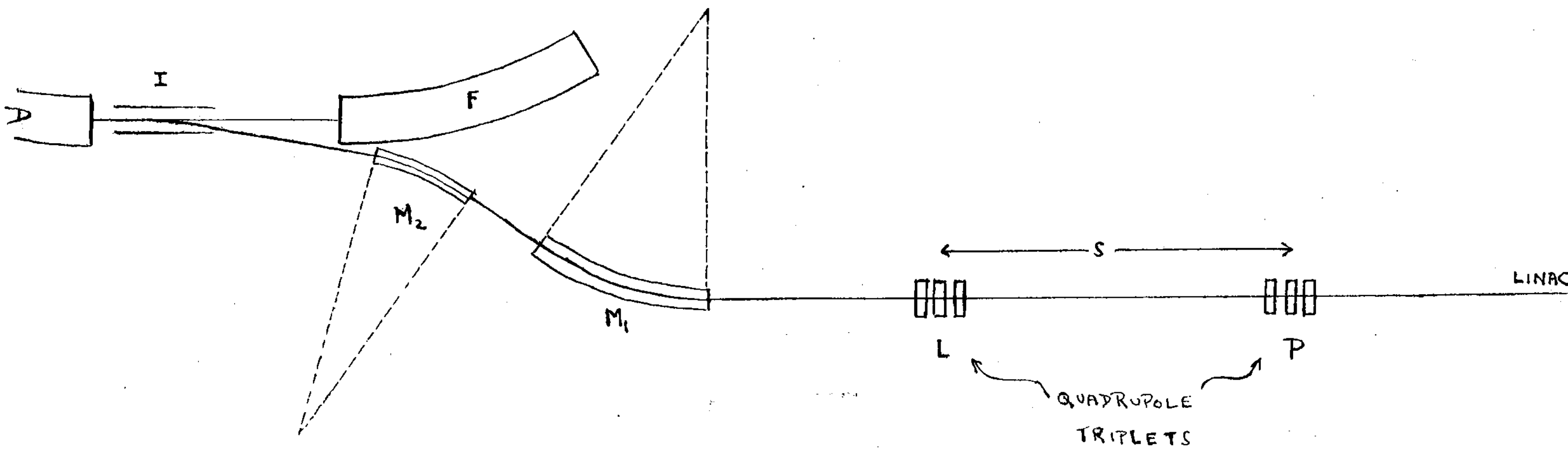

FIG 1.

INJECTION SYSTEM (SCHEMATIC)

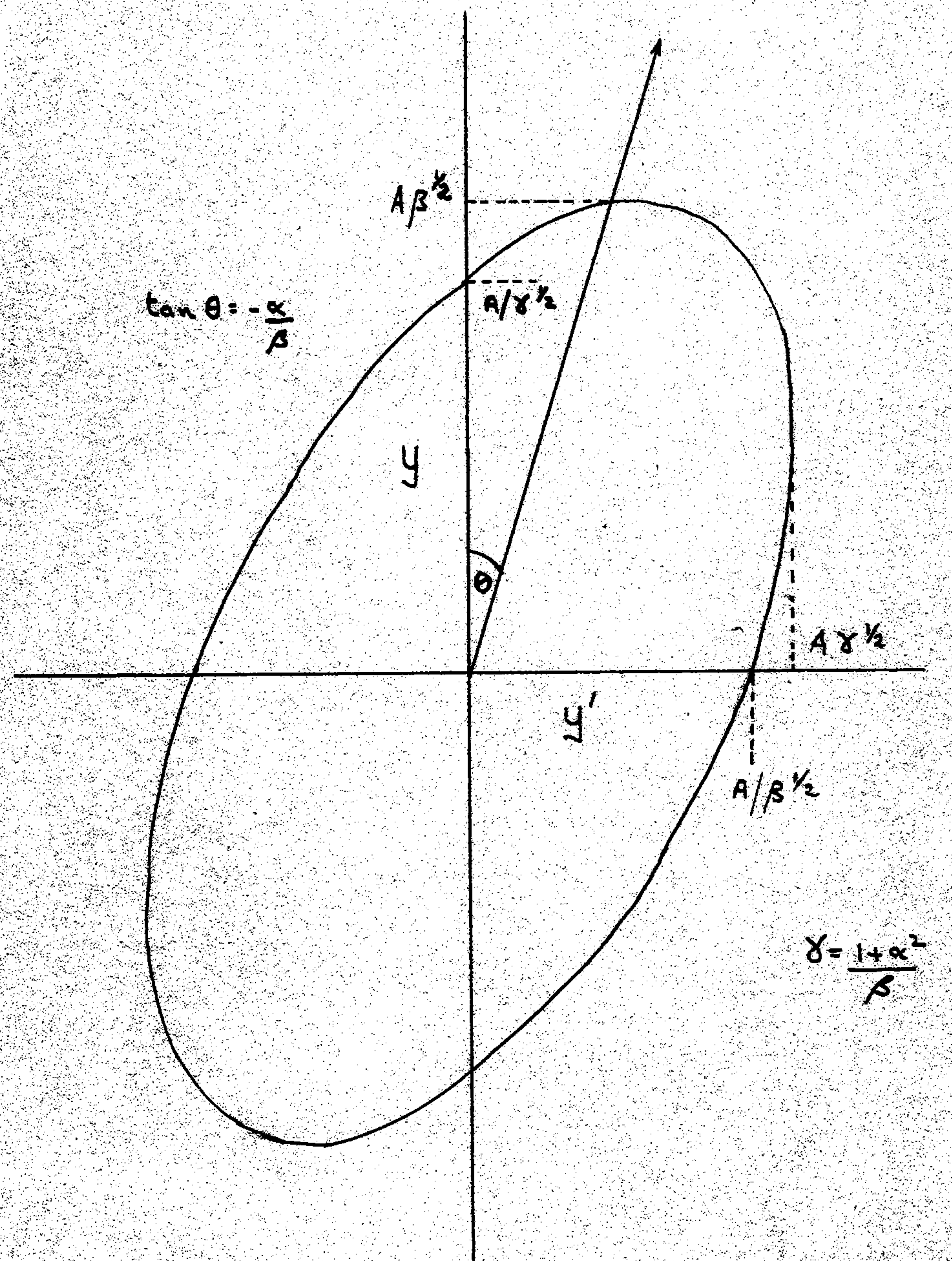
$A\beta^{1/2}$
$A/\gamma^{1/2}$
$\tan\theta = -\frac{\alpha}{\beta}$
y
θ
$A\gamma^{1/2}$
y'
$A/\beta^{1/2}$
$\gamma = \frac{1+\alpha^2}{\beta}$

FIG 2.