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INFLUENCE OF THE GAPS BETWEEN THE BLOCKS ON THE B-LENGTH OF THE MAGNET.

by

V. W. HATTON.

National Institute for Research in
Nuclear Science,

ELECTRON LABORATORY,

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Introduction.

Nina will be constructed with sectored magnets consisting of rectangular blocks of laminations spaced round the arc of a circle to approximate to an ideal curved magnet (see for example reference 1). It is desired that the B-length of the magnet shall be correct and independent of radial distribution. It is the purpose of the present calculation to obtain an estimate of the effect of the gaps between magnet blocks on these quantities. If the effect of the gaps is small, it can be corrected for quite simply by suitable design of the end blocks. We are also interested in the difference in properties of a straight magnet and a sectored magnet.

General Description.

The gap between any two blocks is shown in figure 1 for a given air core size. This is an approximation to the physical case in the case of a wedge shaped gap between the blocks. The figure shows only the upper half of the system, the lower being its mirror image reflected in the median plane.

The median plane is considered as the zero potential whilst the magnet profile has some arbitrary maximum potential V. The median plane and the pole profile are taken to be a polygon with B, D and AA' at infinite points. By means of the Schwarz-Christoffel Transformation, this polygon may be transformed into the w-plane in which the polygon is opened out onto the real axis and the equipotential lines are straight lines.

Inserting the boundary conditions the various constants in the relationship between z and w may be obtained in terms of the gap width and the air core size. The field in the z plane is obtained as a function of the same two factors and for the B-length obtained by integration of the field from the middle of one block to the middle of the next, a similar function is obtained.

Mathematical Deviation of the Expression for the Decrease in B-length

The magnet blocks are separated by a gap of 2T which is constant for a given air core size. We wish to transform the system from the z plane (see figure 1) into the w-plane in which the equipotential lines are represented by straight lines radiating from a fixed point, the origin in the w-plane.

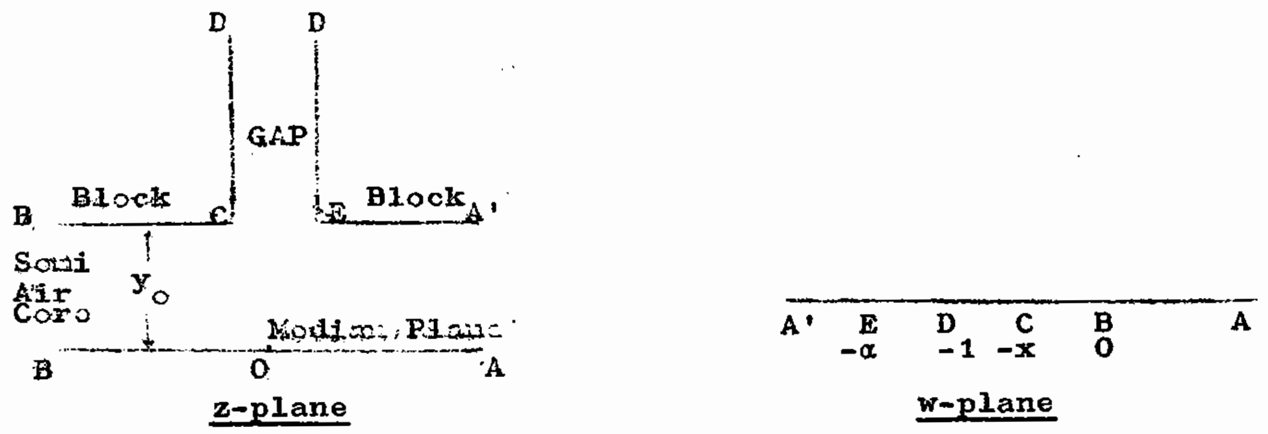

FIGURE 1.

The points B, D and AA' are considered to be at $\pm\infty$ in the z plane and we consider ABCDEA' as a polygon which, opened at AA', is transformed by means of the Schwarz-Christoffel Transformation to the real axis in the w-plane.

The Schwarz-Christoffel Transformation gives

$$\frac{dz}{dw} = A \frac{(w + \alpha)^{\frac{1}{2}}(w + \gamma)^{\frac{1}{2}}}{w\,(w + 1)}$$

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The constant A may be shown to be $y_o/\pi(\alpha\gamma)^{\frac{1}{2}}$ Using the substitution

$$w = \frac{\alpha - \gamma t^2}{t^2 - 1}$$

and expressing the function of t thus obtained in partial fractions, upon integrating, the following relationship between t and z is obtained

$$z = -\frac{2y_o}{\pi}\frac{1}{(\alpha\gamma)^{\frac{1}{2}}}\left[\tfrac{1}{2}\log\frac{t-1}{t+1} + \frac{(\alpha\gamma)^{\frac{1}{2}}}{2}\log\frac{(\alpha/\gamma)^{\frac{1}{2}} + t}{(\alpha/\gamma)^{\frac{1}{2}} - t} + (\alpha - 1)^{\frac{1}{2}}(1-\gamma)^{\frac{1}{2}}\tan^{-1}\left\{\frac{1-\gamma}{\alpha - 1}\right\}t\right] + C \qquad (1)$$

where C is an arbitrary constant of integration.

Evaluation of the constants C, α and γ.

a) For the range of w, $0 \leqslant w \leqslant \alpha$, two ranges from $-\infty$ to $+\infty$ and is real.

The corresponding range of t is $1 \leqslant t \leqslant (\alpha/\gamma)^{\frac{1}{2}}$

Hence Im (C) = 0.

b) As $w \rightarrow -\alpha^-$, $z \rightarrow T + iy_o$.

However in the limit $w \rightarrow -\alpha^-$, $t \rightarrow 0^+$

Thus $T + iy_o = \frac{2y_o}{\pi}\frac{1}{(\alpha\gamma)^{\frac{1}{2}}}\left\{\pm\tfrac{1}{2}i\pi\right\} + C.$

Hence C = T

$\alpha = 1/\gamma$

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Inserting these values in equation (1), the relationship between z and w becomes

$$z = -\frac{2y_o}{\pi}\left[\tfrac{1}{2}\log\frac{t-1}{t+1} + \tfrac{1}{2}\log\frac{\alpha + t}{\alpha - t} + \frac{\alpha - 1}{\alpha^{\frac{1}{2}}}\tan^{-1} t/\alpha^{\frac{1}{2}}\right] + T. \qquad (2)$$

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c) In the limit $w \rightarrow -\gamma^+$, $z \rightarrow -T + iy_o$

As $w \rightarrow -\gamma^+$, $t \rightarrow \infty$

Hence $-T + iy_o = \frac{2y_o}{\pi}\frac{i\pi}{2} - \frac{2y_o}{\pi}\frac{\alpha - 1}{\alpha^{\frac{1}{2}}}\frac{\pi}{2} + T$

Thus $2T = y_o \dfrac{\alpha - 1}{\alpha^{\frac{1}{2}}}$

$$\frac{\alpha - 1}{\alpha^{\frac{1}{2}}} = \frac{2T}{y_o}$$

and $$\alpha = 1 + \tfrac{1}{2}\left(\frac{2T}{y_o}\right)^2 \pm \left(\frac{2T}{y_o}\right)\left\{1 + \frac{1}{4}\left(\frac{2T}{y_o}\right)^2\right\}^{\frac{1}{2}} \qquad (3)$$

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Field Relationships.

The field in the w-plane is given by

$B_w = \text{grad}_w V$ where V is the potential in the w-plane[1)]

Hence $B_w = \dfrac{iy_o}{\pi} \dfrac{1}{w}$

The field in the z-plane is given by B_z where

$$B_z = \overline{\frac{dw}{dz}} \cdot \text{grad}_w V$$

$$= i \frac{\bar{w} + 1}{(1 + \alpha\bar{w})^{\frac{1}{2}}(1 + \bar{w}/\alpha)^{\frac{1}{2}}} \qquad (4)$$

When $w \rightarrow 0$, $B_z \rightarrow i$

i.e. the field in the magnet block has magnitude unity and direction perpendicular to the median plane.

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The Maximum Deviation of the Field in the gap from the field inside the blocks.

The minimum field, by symmetry, occurs midway between the blocks. The value of w on the median plane corresponding to x = 0 is found to be w = 1.

The minimum field is therefore

$$B_z = \frac{2i}{(2 + \frac{1}{\alpha} + \alpha)^{\frac{1}{2}}}$$

$$\frac{2i}{\left[2+2\left\{1+\frac{1}{2}\left(\frac{2T}{y_o}\right)^2\right\}\right]^{\frac{1}{2}}}$$

Using the relationship (3)

$$\frac{1}{\alpha} + \alpha = 2\left[1 + \tfrac{1}{2}\left(\frac{2T}{y_o}\right)^2\right]$$

$$\frac{2i}{\left[4+\left(\frac{2T}{y_o}\right)^2\right]^{\frac{1}{2}}}$$

$$\frac{i}{\left[1+\left(\frac{T}{y_o}\right)^2\right]^{\frac{1}{2}}}$$

and so the minimum field becomes

$$B_z = \frac{i}{\left\{1 + \left(\frac{T}{y_o}\right)^2\right\}^{\frac{1}{2}}}$$

Hence the maximum deviation of the field, assuming $T/y_o \ll 1$ becomes

$$\left(\frac{\Delta B}{B_o}\right)_{max} = -\tfrac{1}{2}\left(\frac{T}{y_o}\right)^2$$

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Evaluation of the B-length

We define the B-length as

$$\int \frac{Bdx}{B_o}$$

where B_o is the field in the magnet whose magnitude is taken as unity.

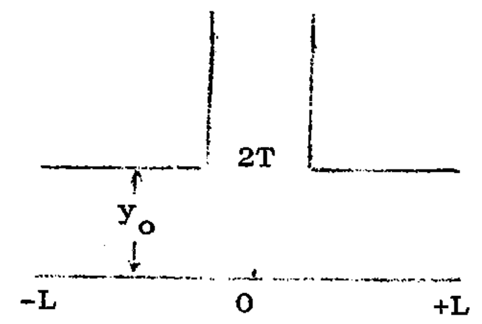

FIGURE 2.

In figure 2, the values on the median plane of $x = \pm L$ correspond to the middle of two adjacent blocks. Integrating the field over this range gives the corresponding B-length.

Integrating over the median plane

$$\text{B-length} = \int_{-L}^{L} Bdx$$

From equation 4 the value of B on the median plane is given by

$$B = i \frac{(1 + u)\alpha^{\frac{1}{2}}}{(u + \alpha)^{\frac{1}{2}}(1 + u\alpha)^{\frac{1}{2}}}$$

Also $dz = \frac{dz}{dw} dw$

Thus $Bdx = \frac{y_o}{\pi} \frac{1}{u} du$

Hence $\text{B-length} = \frac{y_o}{\pi} \Big[\log u \Big]_{x = -L}^{x = L}$

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Since x = 0 corresponds to u = 1 and the magnet blocks are symmetrical about the imaginary axis in the z-plane, we may say that

if the value $u = \rho$ corresponds to $x = L$

then the value $u = 1/\rho$ corresponds to $x = -L$

Thus $\text{B-length} = \frac{2y_o}{\pi} \log \rho$

We wish to compare this value with the actual physical length 2L of the magnet. There remains then the determination of L in terms of ρ

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Determination of L in terms of ρ.

If tp is the value of t corresponding to $u = \rho$ and $x = L$, then, from the relationship between t and z given in equation (2) one can say that

$$L = -\frac{2y_o}{\pi}\left[\frac{1}{2}\log\frac{t\rho-1}{t\rho+1} + \frac{1}{2}\log\frac{\alpha+t\rho}{\alpha-t\rho} - \frac{\alpha-1}{\alpha^{\frac{1}{2}}}\tan^{-1}t\rho/\alpha^{\frac{1}{2}}\right] + T.$$

A similar expression is obtained relating -L to $t_{1/\rho}$, the value of t corresponding to $u = 1/\rho$ (and $x = -L$).

Hence

$$2L = -\frac{2y_o}{\pi}\left[\frac{1}{2}\log\frac{t\rho-1}{t\rho+1}\,\frac{t_{1/\rho}+1}{t_{1/\rho}-1} + \frac{1}{2}\log\frac{\alpha+t\rho}{\alpha-t\rho}\,\frac{\alpha-t_{1/\rho}}{\alpha+t_{1/\rho}} - \frac{\alpha-1}{\alpha^{\frac{1}{2}}}\left\{\tan^{-1}t\rho/\alpha^{\frac{1}{2}} - \tan^{-1}t_{1/\rho}/\alpha^{\frac{1}{2}}\right\}\right]$$

From the relationship between w and t one obtains the relations

$$t_\rho^2 = \frac{\alpha+\rho}{\rho+\frac{1}{\alpha}}\ ; \quad t_{1/\rho}^2 = \alpha\ \frac{1+\alpha\rho}{\alpha+\rho}$$

from which $t_\rho^2 \cdot t_{1/\rho}^2 = \alpha^2$.

If one denotes $\bar{t} = t\rho - t_{1/\rho} < 0$

and remembering that

$$\frac{\alpha-1}{\alpha^{\frac{1}{2}}} = \frac{2T}{y_o} = \delta$$

one obtains

$$2L = \frac{2y_o}{\pi}\left\{\log\frac{\alpha-1-\bar{t}}{\alpha-1+\bar{t}} - \delta\tan^{-1}\bar{t}/2\alpha^{\frac{1}{2}}\right\}$$

and this expression is exact.

From the above relationships it can be shown that

$$\bar{t}/\alpha^{\frac{1}{2}} = \delta\ \frac{\rho-1}{\rho+1}\ \frac{1}{\left\{1+\delta^2\ \frac{\rho}{(\rho+1)^2}\right\}^{\frac{1}{2}}}$$

and this also is exact.

To proceed further, making the approximation that δ is small and omitting 3rd order and higher terms, one obtains

$$\bar{t}/\alpha^{\frac{1}{2}} = -\delta\ \frac{\rho-1}{\rho+1}$$

from which $\log\dfrac{\delta-\bar{t}/\alpha^{\frac{1}{2}}}{\delta+\bar{t}/\alpha^{\frac{1}{2}}} = \log\rho$

and so $\dfrac{\pi L}{y_o} = \delta\tan^{-1}\dfrac{\delta}{2}\ \dfrac{\rho-1}{\rho+1} + \log\rho$

Since δ is small, ρ is large and $\dfrac{\rho-1}{\rho+1} \sim 1$

Thus $\dfrac{\pi L}{y_o} = \dfrac{\delta^2}{2} + \log\rho$

$$\frac{t_\rho t_{1/\rho} - 1 + \bar{t}}{t_\rho t_{1/\rho} - 1 - \bar{t}} \qquad \frac{t_\rho t_{1/\rho}}{\alpha^2} \qquad \frac{2T}{y_o} \qquad \frac{1}{3}\ \frac{1}{5}\ \frac{1}{27}$$

The actual physical length of the magnet

$$= 2L$$

$$= \frac{y_o}{\pi} \delta^2 + \frac{2y_o}{\pi} \log \rho$$

Since the B-length

$$= \frac{2y_o}{\pi} \log \rho$$

the difference between the B-length and the physical length

$$= - \frac{y_o}{\pi} \delta^2$$

$$= - \frac{1}{\pi} \frac{(2T)^2}{y_o}$$

The change in B-length due to the presence of the gaps

$$= - \frac{1}{\pi} \frac{(\text{total gap})^2}{(\text{Semi air core size})}$$

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Results.

In the following table are listed the change in B-length, the minimum field and the variation in B-length with radius for the F and D magnets for the cases of 12 block and 9 block magnets. One assumes that the total length of the magnet is the same in each case.

The figure given for the change in B-length is the one based on the figures for the equilibrium orbit. The angles α represent the variation in B-length over the plateau regions;

α_1 represents the variation when the blocks are parallel and so the variation is caused by the difference in air core size at the two ends of the plateau.

α_2 represents the variation which occurs when the blocks are placed around the arc of a circle and is caused both by the difference in air core size and the difference in gap at the ends of the plateau. In the case of the D magnet the two effects are in opposition and so give a much smaller angle than in the F magnet and in the parallel block case.

		y_o	2T	L_B	$\Delta B/B_o$ max	α_1	α_2
12 blocks	F Sector	1.2"	0.3600"	0.4621"	1.13%	1.51°	3.51°
	D Sector	1.5"	0.3780"	0.4077"	0.79%	1.39°	0.16°
9 blocks	F Sector	1.2"	0.4461"	0.5242"	1.73%	1.72°	3.85°
	D Sector	1.5"	0.4707"	0.4678"	1.23%	1.58°	0.19°

One can safely conclude from the results that the effect of the decrease in field caused by the gaps between the magnet blocks induces little or no difference between the magnets made up of either 12 blocks or 9 blocks. Thus one can construct the magnets with the economically preferable 9 blocks (other factors considered lead to the same conclusion[1].)

The figures quoted have been obtained from a two dimensional model. This model omits the bending of the field lines in the physical, three dimensional, case; this bending is further exaggerated by the wedge shaped air gaps between the blocks. The figures then can serve only as an indication of the order of magnitude of this effect. (In NIMROD the bending of the lines was found to increase the change in B-length by a factor of two.) The actual magnitudes must be obtained by experiment on a model magnet.

The difference between the straight and curved magnets is disturbing. It is desirable to have the effective wedge angle at the end of each magnet to be less than 1^{o}[3]. The results show, that, designing the end blocks with the measurements made on a straight magnet and using these blocks on a curved magnet could easily lead to a change in field length with radial displacement of the above magnitude - it being re-emphasized that these calculations are approximate and could easily be incorrect by a considerable factor. It is therefore important that measurements be made on gaps as they will be in the actual magnet, before the final design of the end blocks can be attempted.

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