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Minimal Supersymmetric Unification Predictions

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Abstract

We re-examine the predictions for the gauge couplings in the minimal extension of the standard model which result from the assumption that the model comes from a unified theory with a single scale of breaking. The effects of thresholds are considered both at the unification scale and at the low-energy scale of the new supersymmetric states. The latter are found to be particularly important and are analysed using the supergravity spectrum corresponding to common scalar and gaugino masses plus other soft supersymmetry breaking terms corrected by gauge and Yukawa radiative corrections. The analysis is extended to include the prediction for the W boson mass coming from the radiatively induced spontaneous symmetry breaking of the standard model. The requirement that this be within the measured value without fine tuning strongly limits the magnitude of the supersymmetry breaking scale and the resulting predictions for the detailed mass spectrum of the supersymmetric states and the Higgs bosons. The bottom quark mass is also determined using the full two-loop renormalisation group equation with the unification prediction $m_b = m_\tau$ at the compactification scale.

1 Introduction

The predictions for the gauge couplings of the standard model in Grand Unified or superstring theories apply at a large scale, M_X, and large logarithmic corrections must be added before comparing with the experimental measurements of these couplings at laboratory energy scales [1]. The resulting prediction in $SU(5)$ [2] and supersymmetric $SU(5)$ [3] has recently been compared [4] with the precision measurements for the weak couplings from LEP and from neutrino scattering experiments together with the accurately known electromagnetic coupling and the somewhat less accurately measured value for the strong coupling [5]. The $SU(5)$ prediction is ruled out at the seven standard deviation level and, moreover, the best value for M_X, although very large, still leads to an unacceptably fast rate of proton decay. By contrast the supersymmetric $SU(5)$ prediction is in excellent agreement with experiment with an acceptable value for M_X. In addition the precision is now so good that it is possible to estimate the mass the new supersymmetric states must have if their radiative effects are to bring the prediction into agreement with experiment. Amaldi et al. [4] have found that the squarks, sleptons and gauginos should have an average mass in the range 100 GeV to 10 TeV.

On the basis of these results it is tempting to argue that there is evidence both for new forms of (relatively) light supersymmetric matter *and* an underlying unified theory. This conclusion is so dramatic that it needs to be evaluated with considerable caution. The most obvious reservation is that it relies on the extrapolation of the standard model, albeit extended to its supersymmetric version, some twelve orders of magnitude beyond the energy scale at which it has been tested. No theory has proved to be so robust in the past so it is understandable if one views this extrapolation with some caution. Even given the framework of Grand Unification there is considerable uncertainty for the relative values of the couplings may differ in different unification schemes. Within the (large) class of theories which give the $SU(5)$ predictions there is further uncertainty coming from virtual states with mass $\geq O(M_X)$ which have not been included in the analysis and which can affect the predictions substantially. The predictions are even more sensitive to additional *light* states beyond those needed for the minimal supersymmetric standard model. Furthermore if the GUT should not break immediately to the standard model there will be several intermediate scales of breaking and again the results will change substantially. Despite these caveats we *do* find it remarkable that the simplest possible supersymmetric extension of the standard model, coupled with the simplest assumption about Grand Unification yields predictions in detailed agreement with experiment provided the new supersymmetric states are very light.

In this paper we re-examine these predictions for the minimal supersymmetric standard model (MSSM) applicable to both Grand Unification and superstring unification. In Section 2 we consider the effects of massive states with mass of $O(M_X)$, of Kaluza Klein states in compactified theories with mass of order the compactification scale, and also the effect of replacing a common SUSY mass by a spectrum for the new light supersymmetric states. The latter is expected in models which unify the fundamental interactions with gravity and the spectrum follows from the assumption of common (flavour independent) masses of gravitational origin for the gauginos and the scalars, together with radiative corrections which cause these masses to "run" from M_X to low scales. In Section 3 we add an analysis of the electroweak breaking which is generated though these radiative corrections driving the Higgs boson mass *squared* negative causing spontaneous symmetry breaking of the standard model. This leads to a prediction of the W boson mass in terms of M_X and the top quark Yukawa coupling which complements the predictions for the gauge couplings and provides a further strong test of minimal supersymmetric unification. Remarkably we find that the predictions for the gauge couplings and for the electroweak breaking scale are in agreement with experiment provided there are very light supersymmetric states, close to the present experimental limits. The strongest constraint on the supersymmetric mass scale does not come from the requirement of an acceptable unification prediction but from the need to avoid the mass hierarchy problem. This bound may be expected to apply even in non-minimal unification schemes and encourages us in the hope that the next generation of accelerators will be able to find evidence for low-energy supersymmetry. Finally we compute the spectrum of states for the preferred value of the supersymmetry breaking scale. This gives a detailed mass spectrum for the new supersymmetric states together with predictions for the Higgs scalars. We also compute the bottom quark mass, m_b, at low scales in terms of its unification value including the full two loop renormalisation group evolution. The resulting bottom mass is in excellent agreement with experiment if $m_b = m_\tau$ at the unification scale. In Section 4 we present a discussion of our results and our conclusions.

Some of this work overlaps with several recent papers [6,7,8,9] which consider the effects of non-degenerate supersymmetric spectrum and the predictions for the bottom quark mass in the light of the recent LEP results. Broadly our analysis agrees where there is overlap although these papers have not addressed the question of electroweak breaking and the associated naturalness problem in such detail. We have also sought fully to explore the relevant portion of the soft supersymmetry breaking parameter space.

2 The Renormalisation Group Analysis

We start with the renormalisation group equations in the $\overline{MS}$ regularisation scheme for the $SU(3) \otimes SU(2) \otimes U(1)$ gauge couplings.

$$\frac{d\alpha_i^{-1}(Q^2)}{d\ln(Q^2)} = \beta_i^0 - \frac{1}{8\pi^2}\,\beta_{ij}^1\,\alpha_j(Q^2) \;+\; O(\alpha^2) \tag{2.1}$$

where β_i^0 and β_{ij}^1 are constants determined by the light particle content with masses below the scale Q^2 [3]. Equation (2.1) determines the gauge couplings at a scale Q^2 in terms of the initial values at a scale M_X^2 summing the leading and next to leading logarithmic terms in $\ln(Q^2/M_X^2)$. The initial values are determined by the Grand Unified theory or compactified (string) theory at M_X. The implications of these equations for the supersymmetric threshold was explored [3,4] by using in equation (2.1) the non-supersymmetric and supersymmetric β-functions below and above the threshold respectively. In this paper we wish to explore more thoroughly the inclusion of such threshold effects which take account of particle masses both at the unification scale and at the supersymmetric scale [6].

2.1 Threshold effects at M_X.

At the two loop order of precision non-logarithmic corrections due to states with mass of $O(M_X)$ are also important. These "threshold" corrections can be analytically computed [10] and included in the boundary value at the scale M_X^2

$$\alpha_i^{-1}(M_X^2) = k_i\,\alpha_X^{-1} + \frac{1}{6\pi}\frac{1}{2}[\,(t_{iF}^H)^2 + 4(t_{iF}^H)^2\ln(\frac{M_X}{M_F}) + \frac{1}{2}(t_{iS}^H)^2\ln(\frac{M_X}{M_S})] \tag{2.2}$$

Here α_X is the universal gauge coupling following from a simple Grand Unified theory or from a compactified superstring theory and k_i depend on the normalisation chosen for the gauge couplings. Henceforth we choose a normalisation such that in $SU(5)$ $k_i = 1$, as is also true in $SO(10)$ and $E(6)$ theories. In four dimensional compactified string theories k_i is the Kac-Moody level with $k_i = 1$ the usual choice. The remaining terms of eq(2.2) give the threshold corrections due to massive states; M_X, M_F and M_S are the masses of the massive gauge, fermion and scalar fields respectively and t^H_{iV}, t^H_{iF} and t^H_{iS} are the matrices which represent the generators of the gauge group on the superheavy vector, fermion and scalar fields respectively. In general we expect a spectrum of heavy fermion and scalar fields in which case there will be several fermion and scalar terms contributing to eq(2.2). Since some of these fields acquire mass through the Yukawa couplings in the theory and their mass may be substantially different from M_X.

These effects were first studied [11] in the context of non-supersymmetric SU(5). Although individual terms are small, it was found that the totality could be large in non-minimal versions of the model in which there are scalars transforming as some high representation of $SU(5)$ giving rise to large numbers of heavy scalar fields contributing to eq(2.2). Recently the same observation has been made by Barbieri and Hall [12] in the context of supersymmetric SU(5). While we agree the possible presence of such massive states at the unification threshold introduces an inherent uncertainty in the analysis we think it still of interest to study the minimal unification possibility. Its success may indicate simplicity in the unification possible due to the absence of large representations of heavy states, as may happen in some compactified string schemes, or due to the degeneracy of such states. In any case the minimal analysis, in which the predictions are well defined, provides a benchmark to judge unification predictions.

It is also possible to calculate the "threshold" correction arising in compactified superstring models of unification due to the tower of Kaluza Klein states with mass quantised in units of the compactification scale. In such theories there is only one fundamental mass scale, the Planck scale, and the scale, M_X, at which the gauge couplings are related is determined in terms of the Planck scale by these Kaluza Klein corrections [13,14,15,16,17,7]. In a class of (2,2) orbifold four dimensional string theories the result of including these corrections has the gauge couplings given by the renormalisation group eq(2.1) with boundary values given by

$$\alpha_i^{-1}(M_X^2) = k_i\,\alpha_X^{-1} \;+\; b_i'\ln(T \mid \eta(T)\mid^4) \tag{2.3}$$

where $\eta(T)$ is the Dedekind function and T is a modulus setting the scale for compactification. Here M_X is given by

$$M_X^2 = (1.03g10^{18}\mathrm{GeV})^2/T \mid \eta(T)\mid^4 \tag{2.4}$$

and the constants b_i' depend on the particular orbifold model. (They vanish for the $Z_2 \otimes Z_2$ orbifold and for the fermionic constructions)

It is clear from this that string theories are potentially more predictive than Grand Unified theories for M_X is not a free parameter. We will return to a discussion of the string prediction for M_X at the end of this paper.

2.2 Supersymmetric threshold effects

Eqs.(2.1) and (2.2) or (2.3) determine the gauge couplings at an arbitrary scale to next to leading log order and include the effects of massive Grand Unified states or massive string states in a compactified superstring theory. They are derived assuming a single scale of breaking to the standard model gauge group and although it is straightforward to modify the analysis to take account of several scales of breaking we will not consider this possibility here, concentrating on an analysis of the minimal scheme which has proved to be so successful. In this the only remaining threshold effects which need to be computed are at the supersymmetry breaking threshold below which the beta functions β_i^0, β_i^1 change from their supersymmetric to their non-supersymmetric values, reflecting the change in the light matter content. In previous analyses [3,4] a common supersymmetry threshold was assumed so that the beta functions for all three gauge couplings change simultaneously. However, due to large radiative corrections, it is not reasonable to take all supersymmetric states to be degenerate.

2.2.1 The supersymmetric spectrum

The supersymmetric states of interest in our minimal unification analysis are just the states of the supersymmetric standard model (MSSM). Apart from the gauge supermultiplets and the matter supermultiplets needed to accommodate three families of quarks and leptons the model has just two Higgs doublet supermultiplets, H_1 and H_2. As well as the gauge couplings there are (Yukawa and associated scalar) couplings associated with the terms in the superpotential, W, needed to give masses to quarks and leptons. In addition, for an acceptable pattern of electroweak breaking [18], there must be a supersymmetric mass, μ, for the Higgsinos and Higgs bosons coming from a further term $\mu H_1 H_2$ in the superpotential. The superpotential of the MSSM has the form

$$W = h(UU^c H_2^0 - DU^c H_2^+) + \mu(H_1^0 H_2^0 - H_1^- H_2^+) \tag{2.5}$$

where, for simplicity, we have shown only the term associated with the top Yukawa coupling, h.

In order to quantify the effects of a non-degenerate spectrum of supersymmetric states we will use a spectrum motivated by most realistic supergravity/superstring models of supersymmetry breaking that have been extensively developed in recent years. In these models supersymmetry is broken in a sector of the theory which has only gravitational interactions with the states of the standard model. As a result supersymmetry breaking is communicated to the visible sector only via gravitational, flavour blind, interactions giving a common mass, $m_{\frac{1}{2}}$, to the gauginos and another common mass, m_0, to the scalars. The gauge bosons and fermions do not acquire mass at this stage due to residual unbroken gauge and chiral symmetries. The degeneracy of the gauginos and scalars is broken by radiative corrections involving the gauge and Yukawa couplings of the standard model and these may be calculated via the renormalisation group equations for the masses [19] using for initial values at M_X the common gaugino and scalar masses $m_{\frac{1}{2}}$ and m_0.[1] Apart from the special case that $v_2 >> v_1$ where $v_{1,2}$ are the vacuum expectation values of the Higgs fields, the only Yukawa coupling large enough to give a significant contribution to this evolution in the standard model is the one responsible for the top quark mass and this is the only one we keep in the subsequent analysis. In addition to the squark, slepton and gaugino soft mass terms there are further soft supersymmetry breaking terms that must be included. These are terms proportional to the A term of the superpotential (effectively the superpotential with the chiral fields replaced by their scalar components) with a coefficient with the dimension of mass. In this analysis there are just two such terms of significance, one we denote by A

[1]Recently it has been observed that in string theories this universality of masses may be broken if the fields have different modular weights. Although possible the necessity to avoid large flavour changing neutral currents strongly constrains the amount of such non-universality and suggests that, in a viable model, flavour blind masses at the unification scale is a good approximation [20].

associated with the trilinear term in the superpotential giving the top quark mass and one denoted by B associated with the $\mu H_1 H_2$ discussed below. The final form for the effective potential from the SUSY breaking soft terms is

$$\begin{aligned} V_{eff} &= [hA(UU^c H_2^0 - DU^c H_2^+) + h.c.] + [B\mu(H_1^0 H_2^0 - H_1^- H_2^+) + h.c.] \\ &\quad + m_{H_1}^2 \mid H_1 \mid^2 + m_{H_2}^2 \mid H_2 \mid^2 + m_L^2(\mid N \mid^2 + \mid E \mid^2) + m_{E^c}^2 \mid E^c \mid^2 \\ &\quad + m_Q^2(\mid U \mid^2 + \mid D \mid^2) + m_{U^c}^2 \mid U^c \mid^2 + m_{D^c}^2 \mid D^c \mid^2 \end{aligned} \tag{2.6}$$

The expectation for A and B depends on the metric; in the case of the no-scale theories which descend from the string the A terms, associated with trilinear couplings, vanish at the unification scale. Similarly if the μ term originates from a trilinear $\lambda H_1 H_2 N$ term where N develops a vev. $\langle N \rangle = \mu/\lambda$, then the B term will also vanish for no-scale theories. (In this case the analysis presented here, ignoring the propagating N field, is appropriate if we ignore renormalistion effects proportional to λ). Note that even if A and B vanish at the unification scale they are still generated through radiative corrections. Here we will also explore the possibility they are non-zero at the unification scale.

Using this parameterisation for the soft supersymmetry breaking mass terms at the unification scale it is now straightforward to determine the spectrum of the light states of the MSSM for the couplings and soft supersymmetry breaking terms at low energy scales may be determined in terms of their values at the unification scale using the coupled renormalisation group equations to determine the radiative corrections. As we will discuss these lead to the spontaneous breaking of the electroweak symmetry giving rise to the W, Z and quark and lepton masses. In addition there are electroweak breaking contributions to gaugino and scalar masses which are readily included in the analysis [18]. The final result is a determination of the complete mass spectrum and the gauge and Yukawa couplings at low energies in terms of the initial values at the unification scale of the parameters defining the model.

2.3 Analysis of gauge couplings

We have now assembled the formalism necessary to compare Grand Unified or compactified superstring predictions with experimental determinations of the gauge couplings of the standard model. The experimental input for the couplings $\alpha_i(M_Z)$ are chosen the same as in Amaldi et al. [4] with the exception of the strong coupling. The latter is the one most poorly determined; the present range of values extends from 0.108 to 0.118 depending on whether it is determined in deep inelastic scattering, from jets at LEP etc.[5]. [2] To allow for this we perform the analysis for any value of α_s within this range.

Before presenting the results we first review the method adopted here. In the initial analysis we ignore any possible threshold effects due to states with mass of $O(M_X)$, keeping only the first term of eq(2.2) or eq(2.3); we will discuss the possible magnitude of these threshold terms later. We also consider only the simplest possibility $k_i = 1$ corresponding to the simplest Grand Unification schemes or to level one Kac Moody string theories.

To ease the presentation we will first discuss the determination of the gauge couplings ignoring, for the moment, the origin of electroweak breaking. Given initial values for M_X, μ, $m_{\frac{1}{2}}$, m_0, A, B and a choice for α_X the renormalisation group equations for the masses may be integrated down in energy from M_X. Of course the evolution of masses stops at the scale equal to the running mass at that scale, corresponding to the physical mass of the state. Instead of a single SUSY threshold we need to include the effect of 11 thresholds namely those for the gluinos, the winos, the left- and the right-handed sleptons , the left- and the right-handed squarks, the left-handed stop and sbottom squark pair and the right-handed stop, the Higgsinos and the Higgs. In doing this we have ignored the small corrections which come from electroweak breaking contributions to the winos and the squarks. This, together with the fact we include only top Yukawa coupling effects, means only the left-handed stop and sbottom are split from the other families. It also means the right-handed stop has a mass different from the other families and the up and down squarks are almost degenerate. The Higgsinos get a common mass determined by μ while the Higgs bosons can be non-degenerate. However, as we will discuss, over most of the acceptable range of parameters the Higgs boson threshold may be taken degenerate to a very good approximation. In Fig.1 we summarise the threshold structure used in this analysis.

The results of the renormalisation group running for representative initial values of parameters are given in Fig.2. It may be seen that the coloured states are systematically heavier than their uncoloured counterparts for the effect of radiative corrections involving gauge couplings is to increase the mass at low

[2] Analyses of R at LEP currently yield anomalously large values of $\alpha_s(M_Z)$ which we shall disregard in this paper.

energies; the states with the smallest gauge interactions are the lightest. Note that the effect of Yukawa couplings is to reduce the mass squared, so that the stop and sbottom squarks are lighter than their flavour partners.

Once one has a mass spectrum the renormalisation group equations for the gauge couplings may be integrated *up* in energy using the experimentally determined values at M_Z as boundary values. The beta functions change at the appropriate mass scales as the threshold for the supersymmetric states is passed and this can be done in detail using the results of the renormalisation group evaluation of the mass spectrum. If the gauge couplings intersect (within experimental errors) M_X and α_X may be determined and compared with the input values used in the first step of determining the mass spectrum. In practice this procedure must be iterated to find a self consistent solution for M_X and α_X, adjusting the value of $\alpha_s(M_Z)$ in order to achieve unification of couplings. The results of this analysis for the case $A_0 = B_0 = 0$ and for various values of $\mid \frac{\mu_0}{m_0} \mid$, are shown in Fig.3 where the subscript 0 denotes that the parameters are evaluated at the unification scale. From Fig.3 it may be seen that a wide range of supersymmetry breaking masses $m_{\frac{1}{2}}$ and m_0 between 2 and 90 TeV give consistency with the predictions for the gauge couplings. The sensitivity to the value taken for μ_0 is such that if it is reduced the mass scale for the remaining supersymmetric states is increased. Comparing with the analysis of Amaldi et al [4] which took $\alpha_s = 0.108 \pm 0.005$ we see that broadly the result of including the non-degenerate spectrum is to increase the effective SUSY scale (the average scale of the SUSY breaking masses) by a factor of 3-10 , the range reflecting the uncertainty of the Higgsino mass.

It is perhaps appropriate to comment about the meaning of this fit, for the three gauge couplings are described in terms of three effective parameters M_X, α_X and the effective supersymmetry mass scale, meaning there will *always* be a fit and apparently no test of unification! However this is not quite fair for the resulting values of the parameters must be *reasonable* if the scheme is to make sense. Thus M_X should be less than the Planck scale but large enough to inhibit proton decay in Grand Unified theories. Also α_X must be positive and within the perturbative domain (although it may be sensible to contemplate non-perturbative unification). Finally the supersymmetry mass scale must be large enough to explain why no supersymmetric states have been found and small enough to avoid the hierarchy problem (as we will see the latter gives a very strong constraint). In the analysis presented above the values of the parameters satisfy these conditions consistent with the hypothesis of minimal unification.

Once an acceptable set of parameters has been obtained from this iterative procedure the resulting supersymmetric spectrum is known. To a good approximation, along the line $m_{\frac{1}{2}} = m_0$, the results of Fig.2 apply , appropriately scaled for different m_0. We will present below detailed spectra of the favoured case once the constraints of electroweak breaking and fine tuning have been satisfied.

3 Electroweak symmetry breaking and the hierarchy problem

The results contained in Fig.3 extend the analysis of [4] to the case where the supersymmetric spectrum is non-degenerate with mass differences given by the radiative corrections. The overall effect is to move up the expectation for the masses of supersymmetric states, somewhat disappointing for the prospects of finding such states. However we have not so far considered electroweak breaking. In the MSSM with soft supersymmetry breaking terms at the unification scale the Higgs masses are also determined. For a viable theory it is necessary that the electroweak symmetry be spontaneously broken and the way that may come about is for the radiative corrections to drive the Higgs mass *squared* negative thus triggering spontaneous symmetry breaking. As noted above the gauge interactions increase the masses (squared) and only the (top) Yukawa interactions can drive the mass squared negative. The effect of the radiative corrections involving this coupling is to reduce the stop and the Higgs masses squared but due to the large positive QCD radiative corrections which affect only the stop it is the Higgs scalar mass squared that is driven negative as desired. Since the effective potential of the Higgs scalar is completely determined in a supersymmetric theory by its gauge and Yukawa couplings its resultant vacuum expectation value is fixed, corresponding to a prediction for M_W. The most significant radiative correction in this respect comes from the top Yukawa coupling so the value of M_W is largely determined by the value of m_t.

In presenting the results of the analysis of electroweak breaking we choose to start with the case $A_0 = B_0 = 0$, which is the favoured case for models derived from the string. At scales below M_X, A evolves to negative values while B initially becomes positive but in general reaches a maximum and may subsequently become negative.

The neutral Higgs potential is given by

$$V(H_1, H_2) = m_1^2 \mid H_1^0 \mid^2 + m_1^2 \mid H_2^0 \mid^2 + \mu B H_1^0 H_2^0 + \frac{\frac{3}{5}g_1^2 + g_2^2}{8}(\mid H_1^0 \mid^2 - \mid H_2^0 \mid^2)^2 \tag{3.1}$$

The masses of the two Higgs fields are given by

$$\begin{aligned} m_1^2 &= m_{H_1}^2 + \mu^2 \\ m_2^2 &= m_{H_2}^2 + \mu^2 \end{aligned} \tag{3.2}$$

with

$$m_{H_1}^2(M_X) = m_{H_2}^2(M_X) = m_0^2 \tag{3.3}$$

The masses μ, m_{H_1}, m_{H_2} evolve differently; for m_{H_i} we have [19]

$$\frac{dm_{H_i}^2}{d\ln(Q)} = \frac{1}{8\pi^2}(-3M_2^2 g_2^2 - \frac{3}{5}M_1^2 g_1^2 + 3h^2\delta_{i2}(m_{\tilde{t}}^2 + m_{\tilde{t}^c}^2 + m_{H_2}^2 + A^2)) \tag{3.4}$$

where the M_i are the gaugino masses and the tilde denotes the supersymmetric state. Thus the difference between m_1, m_2 is due to the term involving the top Yukawa coupling. This term will drive m_2^2 negative if it is large enough, triggering electroweak breaking. With the usual definitions

$$\begin{aligned} \frac{v_2}{v_1} &= \omega = \cot\theta \equiv \tan\beta \quad (\beta = \frac{\pi}{2} - \theta) \\ m_3^2 &= B\mu \\ \sin 2\theta &= \frac{2B\mu}{m_1^2 + m_2^2} \end{aligned} \tag{3.5}$$

we have

$$M_Z^2 = \frac{(m_1^2 - m_2^2) - (m_1^2 + m_2^2)\cos 2\theta}{\cos 2\theta} \tag{3.6}$$

where the running masses and couplings are evaluated at M_Z.[3] This equation follows from demanding that the Higgs vevs. v_1, v_2 give the correct electroweak symmetry breaking value $v^2 = v_1^2 + v_2^2 = 2M_W^2/g_2^2$. From eq(3.4) we see that

[3] For large top Yukawa couplings there are additional corrections coming from further radiative corrections to the effective potential [21].

tuning h_0 will give the correct value of M_Z. Thus to each point of the solution plane of Fig.3 we may assign a definite h_0 (or equivalently m_t) which gives the correct electroweak breaking scale. Thus we may conveniently present the results of the analysis of electroweak breaking by drawing contours of constant m_t on the $m_0, m_{\frac{1}{2}}$ plots. These are shown by the dotted lines in Fig.3 for the case $m_t = 100, 160$ GeV.

Before doing so we note there are several constraints for acceptable electroweak breaking.

- The potential should be bounded from below

$$\begin{aligned} 2 \mid m_3^2 \mid &< m_1^2 + m_2^2 \\ \text{i.e.} \mid \sin 2\theta \mid &< 1 \\ m_1^2 m_2^2 &< m_3^4 \end{aligned} \tag{3.7}$$

- Colour should be unbroken. For large μ the top Yukawa couplings can drive a squark mass squared negative before the Higgs. Thus we must constrain values of the initial parameters so that all masses squared other than the Higgs are positive.

- The top mass is constrained by the LEP measurements to be 100 GeV$<$ $m_t <$ 160 GeV.

Taking care to check that these conditions are satisfied we have determined the values of m_t needed for acceptable values of electroweak breaking. The results for m_t throughout the $m_{\frac{1}{2}}$, m_0 plane are shown in Fig.3 from which it may be seen that part of the previously allowed region is excluded by the requirement of acceptable radiative electroweak breaking coupled with the LEP bounds on the top mass. The variation with respect to m_0 is obvious for larger m_0 requires larger h to drive m_2^2 negative at the correct scale. The same applies to increasing the value of μ.

3.1 The fine tuning problem

It is remarkable that the intricacies of this radiatively generated mass spectrum are such that an acceptable scale of electroweak breaking results for values for m_t within the range allowed by the precision LEP measurements. However there is a concealed fine tuning problem associated with this solution. The problem is that the value of the Higgs mass and hence of M_W is sensitively dependent on the top Yukawa coupling. For example with the choice of parameters corresponding to Fig.2 if M_W is constrained within 0.4% (experimental uncertainty) then we find that h_0 must be tuned to within about two parts per million, or the top mass must be constrained to about 0.3 MeV! The reason for this extreme sensitivity may be seen from the form of the term in the renormalisation group responsible for the Higgs mass, eq(3.4). As we have discussed the last term proportional to the top Yukawa coupling, h, drives the mass squared negative at some point Q_0 far from M_X. Expanding around this point and, for illustration, keeping only the dominant term involving the stop mass, below Q_0 the Higgs mass squared is given by (here we also neglect the running of μ)

$$m_2^2(Q^2) = \frac{3}{16\pi^2}h^2(m_{\tilde{t}}^2 + m_{\tilde{t}^c}^2)\ln(Q^2/Q_0^2) \tag{3.8}$$

We see that the negative Higgs mass squared which sets the scale for the vacuum expectation value of H_2 and hence M_W is proportional to the stop mass squared and to the top Yukawa coupling squared. Using this equation it is easy to compute the sensitivity to the top mass for, ignoring the dependence of Q_0 on h, we apparently have

$$\frac{\delta m_2^2}{m_2^2} = \frac{\delta m_t^2}{m_t^2} \tag{3.9}$$

Clearly this does not explain the extreme sensitivity found above. The reason for this discrepancy lies in our neglect of the h dependence of Q_0. It is determined by the renormalisation group equations in terms of the initial scale M_X. These equations do not admit an analytic solution but to illustrate the source of the fine tuning problem encountered here we solve for Q_0 ignoring the gauge couplings in eq(3.4) and also the running of the squark and Yukawa coupling. Then the dominant squark mass contribution gives

$$Q_0^2 = M_X^2 \exp\left(-\frac{16\pi^2(m_0^2+\mu^2)}{3h^2(3m_0^2)}\right) \tag{3.10}$$

This shows that the sensitivity of the Higgs mass to the top mass is enhanced. Using eqs(3.10) and (3.8) we find

$$\frac{\delta m_2^2}{m_2^2} \approx \frac{\delta m_t^2}{m_t^2}\left(\frac{\ln(M_X^2/Q^2)}{\ln(Q_0^2/Q^2)}\right) \tag{3.11}$$

This is an enhancement of $(\frac{\ln(M_X^2/Q^2)}{\ln(Q_0^2/Q^2)})$ in the sensitivity of the Higgs mass to the top mass coming from the fact that Q_0 is driven by a large logarithm involving the unification scale M_X. Although eq(3.11) is only a rough approximation it does serve to explain the origin of the extreme sensitivity of the Higgs mass to the top Yukawa coupling which may be found exactly from the full renormalisation group equations.

In ref[22] "reasonable" scales for the supersymmetry thresholds were estimated by demanding that the sensitivity of the electroweak breaking scale to any of the parameters of the standard model should be less than some value, c. In the case we are interested in c is defined through

$$\frac{\delta M_W^2}{M_W^2} = c\frac{\delta h^2}{h^2} \tag{3.12}$$

where the scale is the electroweak breaking scale.

No fine tuning would correspond to $c \approx 1$ but, more conservatively, a value of $c = 10$ was chosen as a measure of a reasonable theory *i.e.h* must be tuned to finer than $\frac{1}{10}$ the W mass *uncertainty*. As we have just discussed the requirement that the electroweak breaking scale be generated from the unification scale introduces more sensitivity to the top Yukawa coupling than is found in the MSSM without unification. From eqs(3.8),(3.10) we expect

$$c \approx \frac{m_1^2}{M_Z^2} \tag{3.13}$$

Because $m_1^2 = m_0^2 + \mu_0^2 + km_{\frac{1}{2}}^2$, $k \approx \frac{1}{2}$ we see that c increases rapidly with all three parameters $m_0, m_{\frac{1}{2}}, \mu_0$. Thus constraining $c \leq 10$ dramatically reduces the allowed region in Fig.3. Of course eq(3.13) is only approximate and in the analysis presented below we compute c using the full renormalisation group equations.

3.2 Fully constrained analysis

In this section we present the results following from applying the constraints of section 3.1 and the fine tuning constraint $c \leq 10$ with $\alpha_s \leq 0.118$. From Fig.4 and imposing the exact constraint replacing eq(3.13) we find the fits are restricted to the region $\mid \frac{\mu_0}{m_0} \mid \approx 1$, $\alpha_s \approx 0.118$ in the $m_0, m_{\frac{1}{2}}$ plane. The full SUSY spectra for each of the "extremes " labelled by Z and X in Fig.4 are listed in Table 1, and the evolution of the masses from M_X shown in Fig.5. This includes the (small) contributions to the masses coming from the spontaneous breaking of electoweak symmetry [18]. The neutralino and chargino masses are obtained by diagonalising the full mass matrices. The spectra are very strongly constrained and lie not far from the present experimental limits with all SUSY masses less than 1 TeV and sleptons much less, of order 200 GeV.[4] Note that the fine tuning constraint is largely independent of the details of the unification at M_X and may be expected to give similar limits on the SUSY spectrum in any unification scheme having a very large M_X.

We can also determine the mass of the Higgs bosons from the results of this analysis. This requires minimisation of the full Higgs potential and determination of the masses at the minimum. In order to determine the Higgs spectrum it is necessary to determine β defined in eq(3.5). This is particularly sensitive to the parameters B_0 and A_0 and so we extend the analysis to discuss the possibility of a non-zero value for them. The sensitivity to B_0 may be seen by considering the values of β found as one tracks the solution going from Z to X in Fig.4. For vanishing B_0, there is a point at which $\sin 2\theta$ is zero ie $\tan\beta = \infty$. Since we constrain the bottom Yukawa coupling to be less than the top Yukawa coupling (the radiative breaking scenario breaks down otherwise[19]) we should allow only $\tan\beta \leq \frac{m_t}{m_b}$. Imposing this constraint removes a range of m_t between 130 and 155 GeV. If nonzero B_0 is allowed a non-zero initial value of θ is generated since $\sin 2\theta = \frac{\mu_0 B_0}{m_0^2+\mu_0^2}$. If $|\sin 2\theta| < 1$ at all scales we need $|B_0| \leq \frac{m_0}{2}$. We find that allowing B_0 in this range allows any value of $\tan\beta \geq 3$ as shown in Fig.6. The one remaining parameter is A_0. For completeness, we consider the effects of allowing A_0, as well as B_0, to be non-zero. The results are shown in Fig.7. For reasonable values of A_0, B_0, $\tan\beta$ remains $\geq 2-3$.

For the parameter choice of Fig.5 at the extremes Z and X the resulting Higgs spectrum is given in Table 1. As noted above, however, these masses are sensitive to the value of $\tan\beta$ and change for non-zero A_0, B_0. As has recently been noted [21] there are also large radiative corrections to the quartic terms in the effective potential for large values of the top mass, but these have been evaluated and are easily included.

Finally we consider the predictions for the light quark masses that may result from Grand Unification. In $SU(5)$ the prediction $m_b = m_\tau$ applies at

[4]Note that for each solution (with $B_0 = 0$) there are two solutions with opposite signs for μ_0. We take only the one which evolves to a positive $\sin 2\theta$ at M_Z.

the unification scale and must be radiatively corrected at low scales. We have calculated these corrections using the full 2 loop evolution of h_b, h_τ for the non-SUSY [23] and SUSY [24] ranges. The results are shown in Fig.4. While it is the constraint of keeping $c \leq 10$ which strongly forces the relatively low values of $m_0, m_{\frac{1}{2}}$ we note, from Fig.4, that it is this region which is also favoured by the value of m_b.

4 Discussion and Conclusions

In this paper we have confronted the predictions of supersymmetric unified theories with experiment. Assuming the (SU(5)) relation amongst the couplings and the minimal particle content up to the unification scale there is a remarkable agreement provided the masses of the new supersymmetric states is low. With the preferred method of supersymmetry breaking in a hidden sector the supersymmetric spectrum is determined in terms of five parameters, m_0, $m_{\frac{1}{2}}$, μ, A and B. Using this and the full renormalisation group equations for both the couplings and the masses we found agreement with the unification predictions for the gauge couplings for a range of supersymmetric masses between 10^2 GeV and 10^5 GeV, the effect of incorporating the non-degenerate spectrum being to increase somewhat the allowed mass of the supersymmetric states for a given value of $\alpha_s(M_Z)$.

The analysis was extended to include the prediction for the electroweak breaking scale which follows because the radiative corrections included in the renormalisation group analysis drive the Higgs mass squared negative at some scale below the unification scale. The prediction is sensitive to the top quark mass and we find it again in remarkable agreement with experiment for a top quark mass in the range needed for consistency with the precision LEP experiments. However if the SUSY masses are near their upper value allowed by the coupling unification the top quark mass must be fine tuned to a very high degree *i.e.* the natural scale of W mass lies close to the supersymmetric mass. This fine tuning may be reduced only if the SUSY mass scale is low and asking that it be less than one part in ten ($c < 10$) forces $m_0, m_{\frac{1}{2}} = O(10^2 \text{ GeV})$ (*c.f.* Fig.4). In this case unifiaction of the couplings is only possible for $\alpha_s(M_Z)$ at its upper allowed value ≈ 0.118 as determined by jet analyses at LEP. It is also noteworthy that the $SU(5)$ mass relation for the bottom quark mass is in best agreement with the upsilon mass for the low SUSY scale although the dependence on the scale is quite weak.

The minimal unification of the gauge interactions also determines the unification scale which we find to be $M_X \approx 10^{16}$ GeV, sufficient in Grand Unified supersymmetric theories to inhibit nucleon decay below experimental limits. From the most optimistic point of view this may be seen as evidence (albeit circumstantial) for unification. Is there a realistic (string) model realising this simple scheme? In Grand Unification the minimal unification assumptions which lead to the successful predictions discussed above can be realised if the MSSM is embedded in a GUT such as SU(5) which breaks directly to $SU(3) \otimes SU(2) \otimes U(1)$ at the unification scale M_X and all the resulting massive states are nearly degenerate. There is no constraint in Grand Unification which prohibits such a pattern of unification and it provides a nice example of the minimal unification discussed here. In string theories, however, there are much stricter constraints for a given four dimensional string theory has definite multiplet structure and, moreover, the unification scale is determined in terms of the Planck scale (*c.f.* eq(2.4)). The simplest realisation of minimal string unification is for the gauge group after compactification to be just $SU(3) \otimes SU(2) \otimes U(1)$ with the minimal particle content *i.e.* there is no need for any additional heavy states. While no example of such a string theory has yet been constructed it seems likely that something quite close to it can be found. However the prediction for M_X is difficult to satisfy for a survey [26] of all orbifold models shows that typically M_X is increased from the string unification scale $M_{SU} \approx 1.03g10^{18}$ GeV. Only for very special assignments of the matter fields to (twisted) multiplets can one obtain values of the b_i' in eq(2.3) which lead to a *reduction* in the unification scale. The alternative is to give up the minimal unification assumption and to add states beyond the minimal set. For example in flipped $SU(5)$ it is necessary to evolve the $SU(5) \otimes U(1)$ couplings from M_{SU} to M_X before using the $SU(3) \otimes SU(2) \otimes U(1)$ evolution. Unfortunately this spoils the agreement with the low-energy couplings in the original version of the theory but in non- minimal versions the effect of states with masses of order 10^{13} GeV can lead to acceptable results [8]. This example nicely illustrates both the additional predictive power of the string theory and the uncertainties introduced once non- minimal theories are introduced.

In summary we have found a remarkable agreement between the precision measurements of standard model parameters and the minimal unification predictions of the supersymmetric standard model. Perhaps the main conclusion to be drawn from this analysis is that we are very close to the threshold for producing the new supersymmetric states. It is worth stressing that the low supersymmetric threshold determined here follows from demanding the absence of fine tuning and does not depend sensitively on the details of the unification, only that it should occur at a high scale. Thus the contours in Fig.4 giving the upper bound on the supersymmetric masses should be viewed as a general constraint on a unified version of the MSSM.

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Table 1

Masses of the supersymmetric states for the two solutions (Z and X of Fig.4) with $\alpha_s(M_z) = 0.118$, (Z) with $m_t = 160$ GeV, (X) with $m_t = 100$ GeV. Here $|\mu_0/m_0|$=1 and $B_0 = A_0 = 0$. The soft supersymmetry breaking contributions have been corrected by including electroweak symmetry breaking terms. The masses (which are in GeV units) of the two solutions represent the range of values of our predictions. The light Higgs does not have the quartic corrections included.

	Parameters	
$m_{1/2}$	140	230
m_0	190	120
μ_0	190	−120
m_t	160	100
$\tan\beta$	21	5
	Gauginos	
$\tilde{\gamma}$	57	83
$\tilde{Z}$; $\tilde{W}$	99; 99	120; 112
$\tilde{g}$	354	559
	Sleptons	
$\tilde{\ell}_L$	220	206
$\tilde{\ell}_R$	195	146
	Squarks	
$\tilde{u}_L, \tilde{c}_L$; $\tilde{d}_L, \tilde{s}_L$	365; 373	511; 517
$\tilde{u}^c_L, \tilde{c}^c_R$	359	495
$\tilde{d}^c_R, \tilde{s}^c_R, \tilde{b}_R$	358	491
$\tilde{t}_L$; $\tilde{b}_L$	325; 335	491; 497
$\tilde{t}^c_R$	273	452
	Higgs, Higgsinos	
H^0	91, 264	84,221
$H^\pm$; A^0	276; 264	232; 218
$\tilde{H}^0$	205, 225	139, 226
$\tilde{H}^\pm$	229	227

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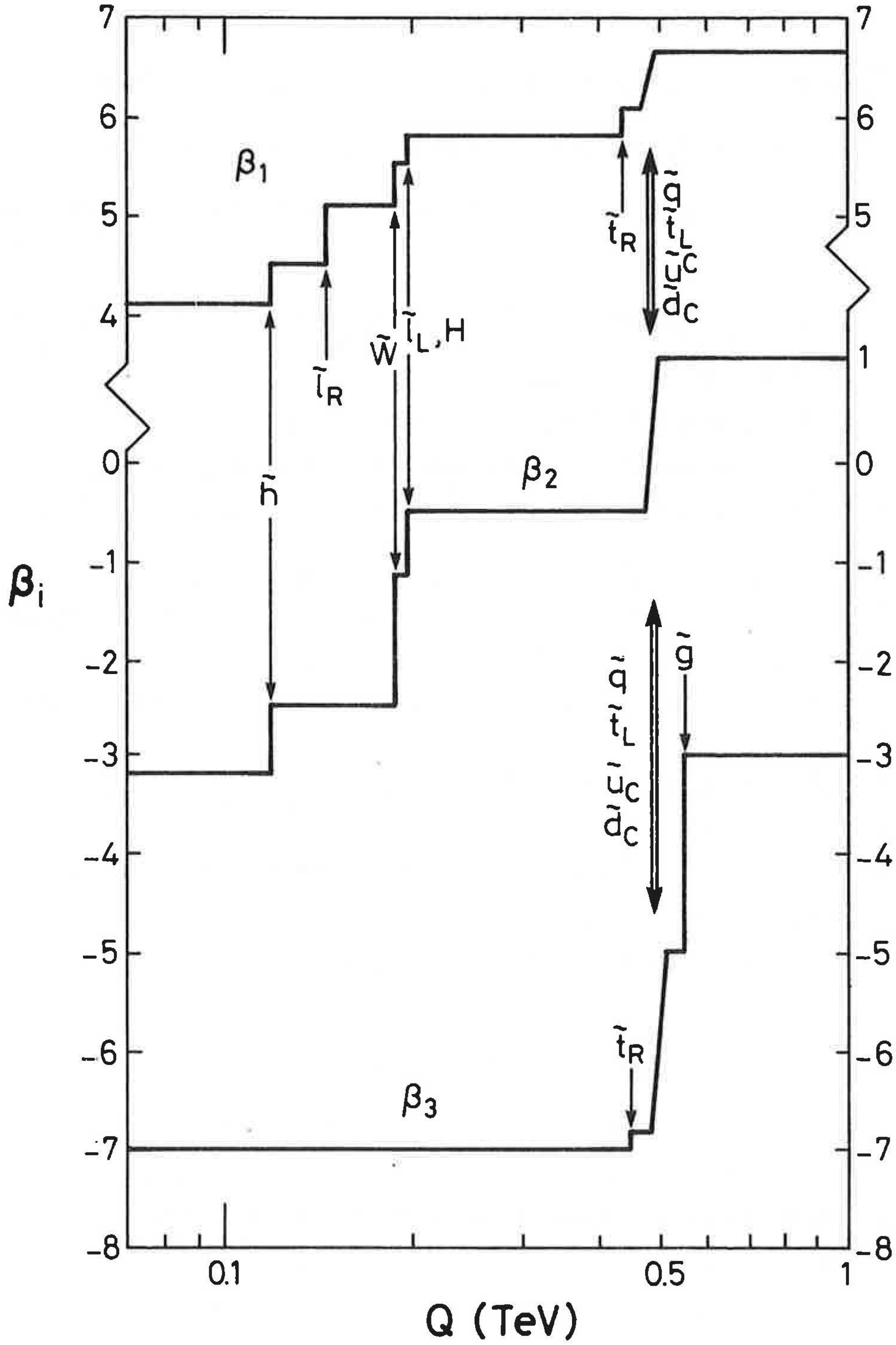

Fig.1 A typical example of the threshold structure used in this analysis. (Here m_0=0.12 TeV, $m_{\frac{1}{2}}$=0.23 TeV, μ_0=0.12 TeV)

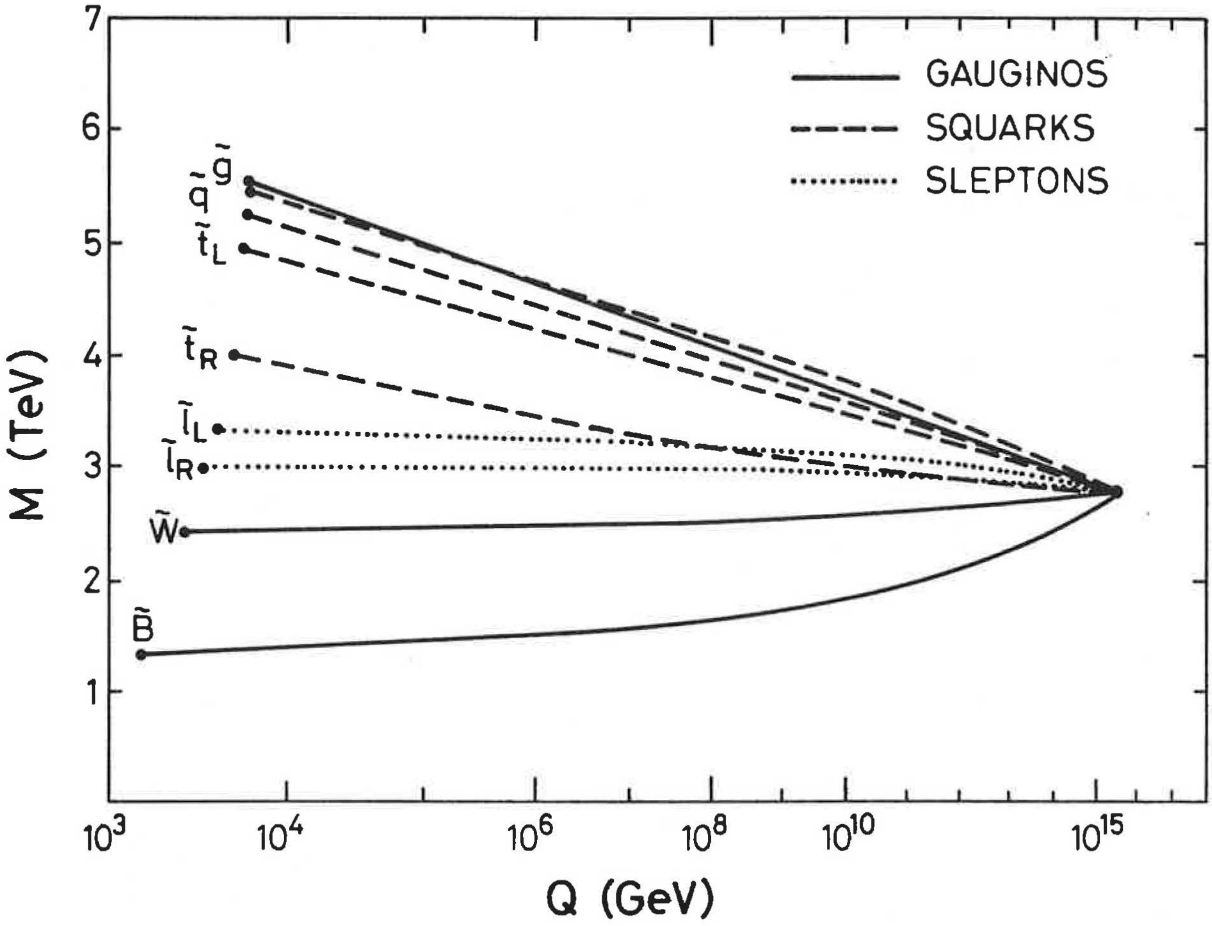

Fig.2 Evolution of the masses of gauginos, squarks and sleptons for the particular case $m_0 = m_{\frac{1}{2}} = \mu_0$=2.8 TeV, for which unification of the couplings occurs if $\alpha_s(M_Z) = 0.110$

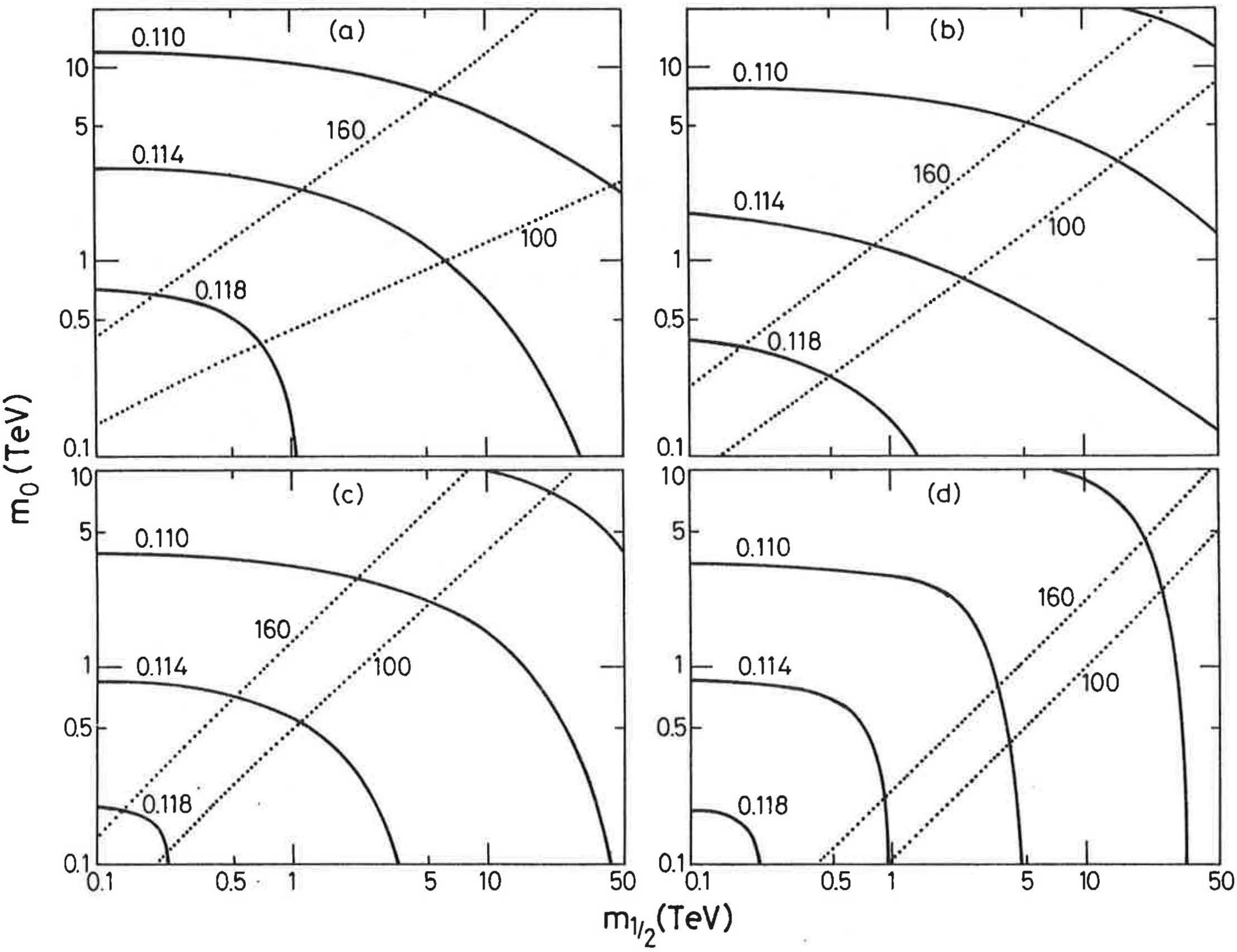

Fig.3 Allowed values of the parameters $m_0, m_{\frac{1}{2}}$ for different values of μ_0; $\mid \frac{\mu_0}{m_0} \mid = 0.2, 0.4, 1.0, 5.0$ in Figs (a), (b), (c), (d) respectively. The solid lines indicate the values of $\alpha_s(M_Z)$ necessary to achieve unification of the couplings at $Q = M_X$. The dotted lines show the values of m_t, 100 and 160 GeV, resulting from tuning the top Yukawa coupling to get the correct E.W. symmetry breaking.

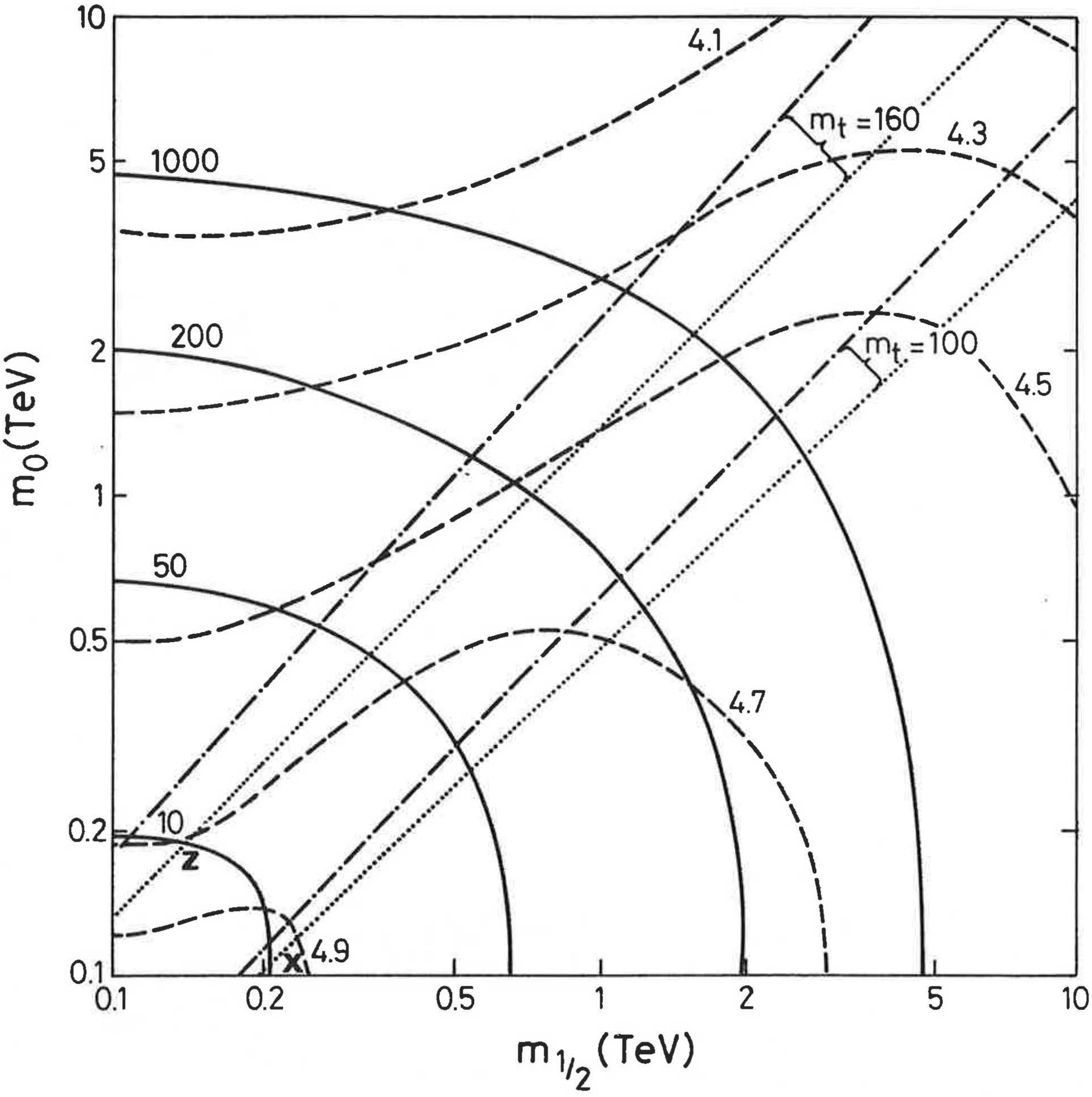

Fig.4 $m_0, m_{\frac{1}{2}}$ plot for $| \frac{\mu_0}{m_0} = 1 |$, $A_0 = 0$ in the low mass region, 100-300 GeV. The diagonal band is the area constrained by $100 < m_t < 160$ GeV for $B_0 = 0$; the diagonal dashed lines indicate how the band is modified for $| B_0 |= m_0$. Other contours indicate values of the fine tuning constant c (evaluated exactly) and m_b (the bottom quark mass) using the full two-loop expressions for $\frac{m_b}{m_\tau}$. The contour c=10 here virtually coincides with the $\alpha_s = 0.118$ contour of Fig (3c).

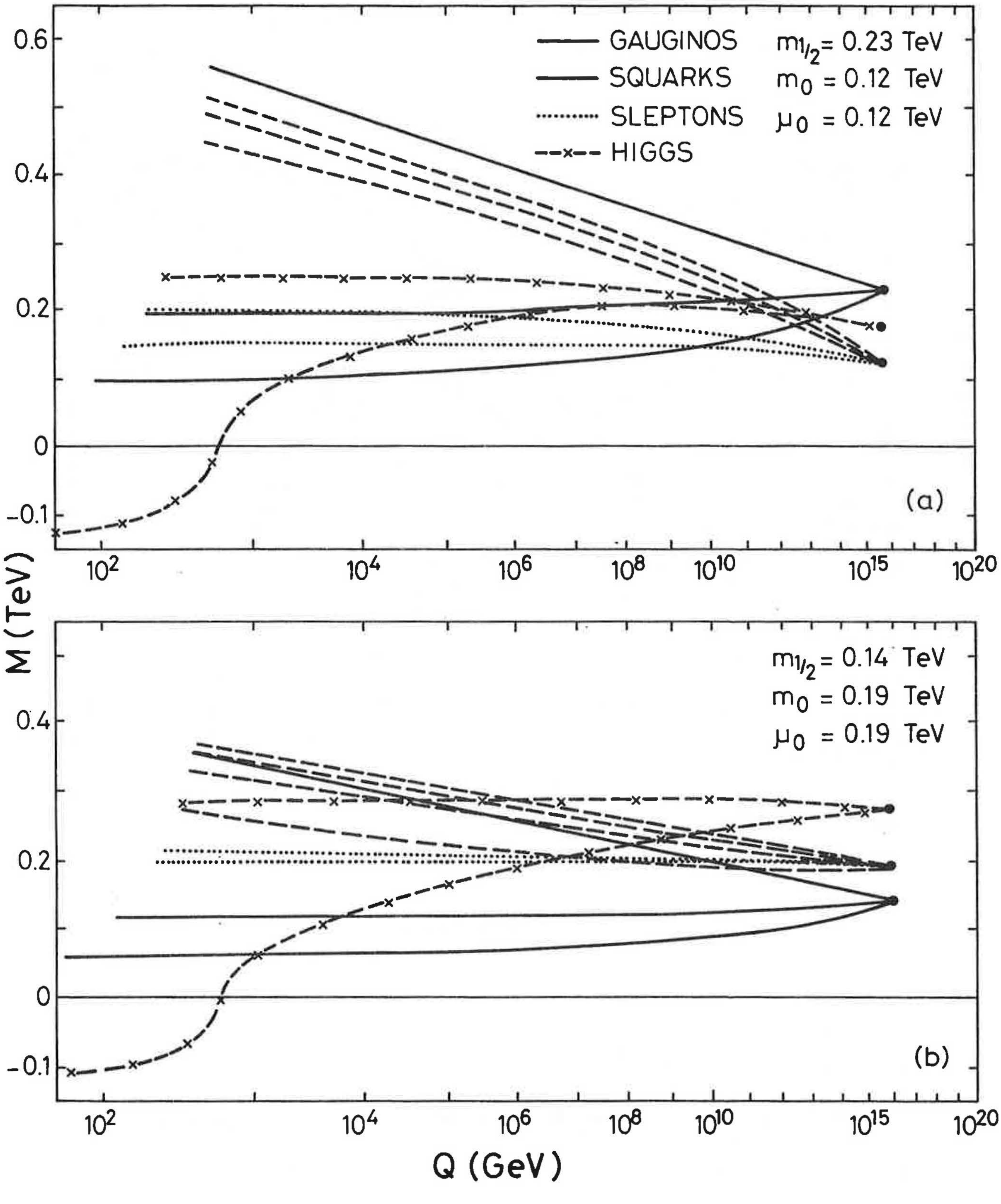

Fig.5 Evolution of the SUSY spectra from $Q = M_X$ for the solutions Z, X in Fig(4). The evolved masses are listed in Table 1.

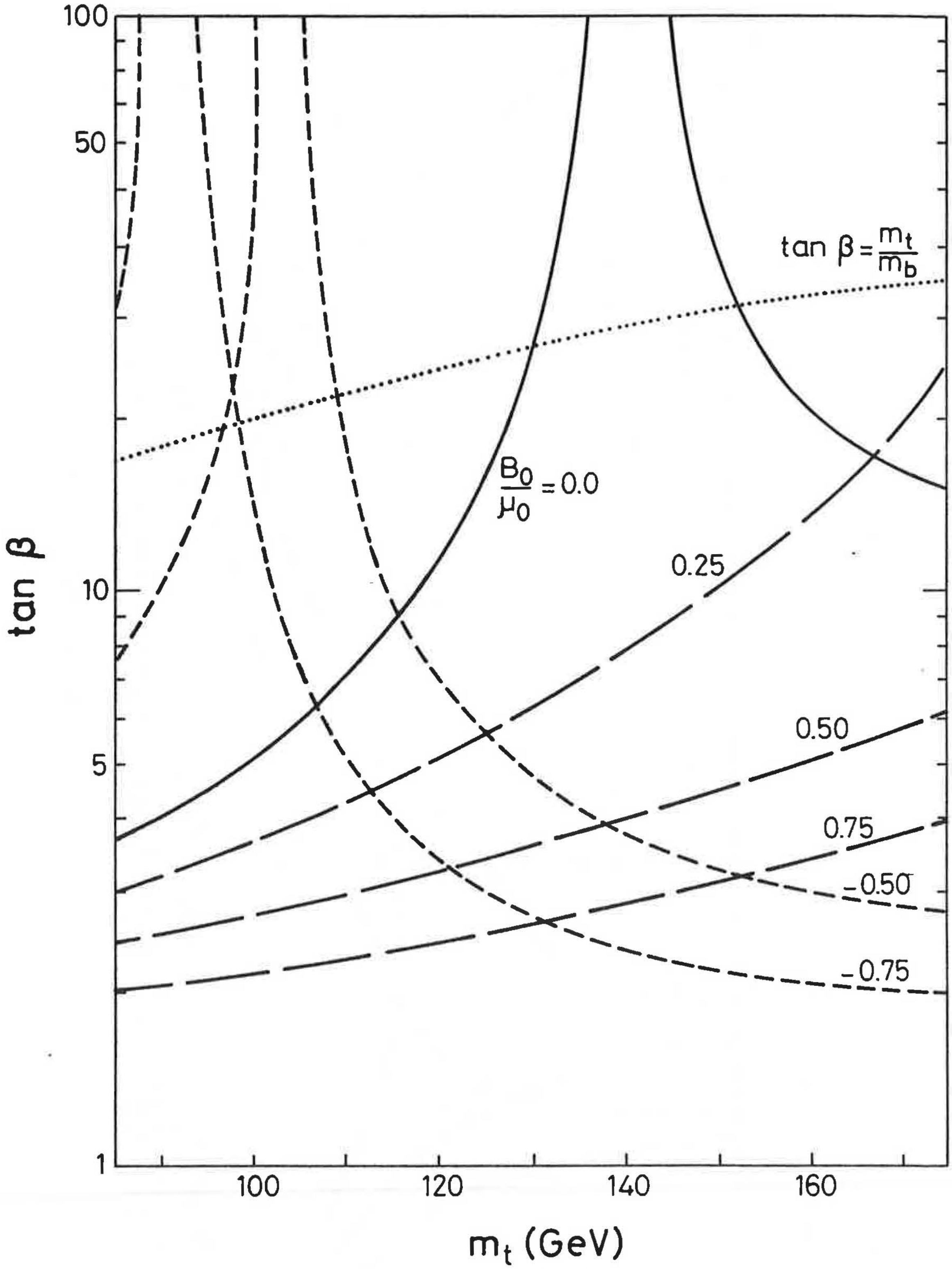

Fig.6 $\tan\beta$, m_t plot for the solutions along the contour $\alpha_s = 0.118$ in Fig(4). The value of B_0 is allowed to depart from zero and the particular value of $\frac{B_0}{m_0}$ is marked on each curve. Thus for $| B_0 | < \frac{1}{2} | m_0 |$, $\tan\beta \geq 3$.

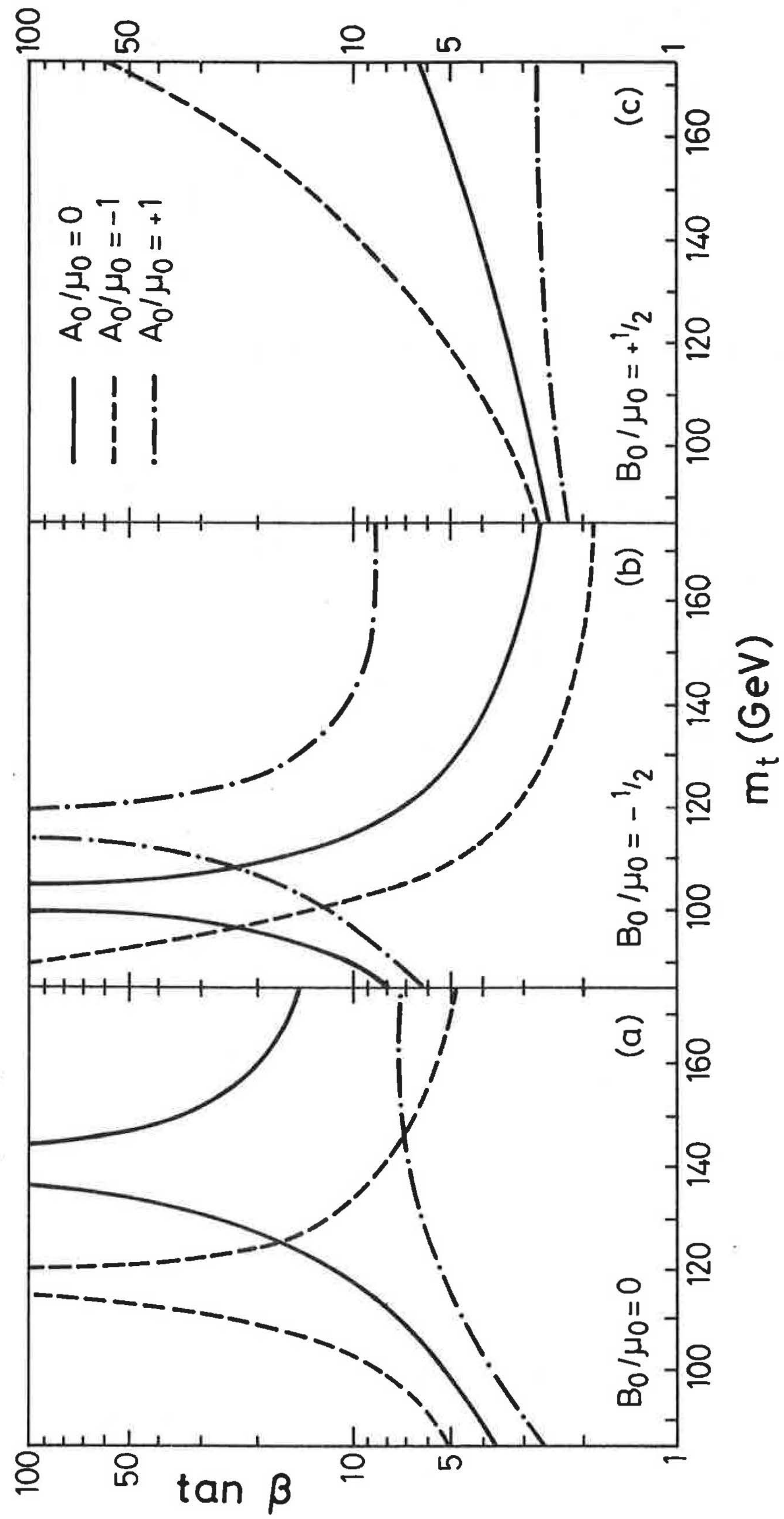

Fig.7 $\tan\beta$, m_t plot for various values of A_0. (a)$B_0 = m_0$ (b)$B_0 = -\frac{1}{2}m_0$ (c) $B_0 = \frac{1}{2}m_0$ and the curves in each case correspond to $\frac{A_0}{m_0} = -1, 0, +1$.