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Abstract

We classify the different neutrino-mass patterns arising in string-inspired GUT and supersymmetric GUT models based on the flipped $SU(5) \otimes U(1)$ and the $SU(4) \otimes O(4)$ gauge groups. Phenomenologically interesting spectra are obtained through the interplay of the two seesaw mechanisms present, with typical neutrino masses $\sim 10^{-3}$ eV in the supersymmetric GUT models and of order $0.1 - 10$ KeV in the ordinary GUTs.

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After the successes of string theory several attempts have been made by model builders to try to make contact with the overly successful $SU(3)_c \otimes SU(2)_L \otimes U(1)$ Standard Model (SM), in particular through those gauge groups which now belong to the classics of grand unification, like $SU(5)$, $SO(10)$ and E_6. One of the main difficulties seemed at first to be the absence of Higgs fields in the adjoint or any higher self-conjugate representation in K=1 string theories (K being the Kac-Moody level), which were needed in usual GUTs in order to spontaneously break the gauge symmetry down to the SM and obtain the doublet-triplet mass splitting in a natural way, thus avoiding a quick proton decay induced by the exchange of light colour-triplet scalars. This problem has been however overcome in the construction of phenomenologically promising models, which require only small Higgs representations, from Calabi-Yau compactifications [1], orbifolds [2] and four dimensional superstrings [3-6]. In particular, starting from $SO(10)$ there are only two possible subgroups, the "flipped" $SU(5) \otimes U(1)$ [3], and the $SU(4) \otimes O(4)$ [4] which is isomorphic to $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, that can lead to the SM without the use of adjoint Higgs representations. In fact, the corresponding gauge symmetries are broken by just two incomplete Higgs multiplets $\mathbf{H}$ and $\bar{\mathbf{H}}$ of the $\mathbf{16}$ and $\bar{\mathbf{16}}$ representations of $SO(10)$, instead of using any of the large Higgs representations, like the $\mathbf{126}$, required in the standard $SO(10)$ models. The low-energy behaviour of these models has been explored in the corresponding minimal GUT versions [3,4] as well as in different versions of the string [3,5,6].

Among the requirements that had to be satisfied was the seesaw suppression of large masses for the three known light neutrinos, whose Dirac masses are equal to the masses of the up-quarks, as in ordinary GUTs. Since in these models the right-handed neutrinos do not get a tree-level mass, so that the ordinary seesaw mechanism is absent, an alternative had to be found. This has been achieved through the introduction of extra singlet states coupled to the right-handed neutrinos, which gave rise to ultra-suppressed and therefore practically unobservable neutrino masses [4]. It has been pointed out later on [7], that large Majorana masses of order 10^8 GeV could be generated radiatively at the two-loop level for the right-handed neutrinos via the Witten mechanism [8]. This would effectively suppress the masses of the left-handed neutrinos down to the range of 10^{-3} eV to 10 KeV, with possible interesting consequences in astrophysics and cosmology. In particular, neutrinos in the mass range 10^{-3} to 10^{-2} eV could provide a solution to the solar neutrino problem via the well known MSW mechanism [9], while masses around $50 - 100$ eV could solve the cosmological dark matter problem. And yet, the eventual exixtence of Dirac states in the 10 keV range could possibly confirm the still speculative 17 keV "Simpson neutrino" [10]. In view of all this, a detailed study

of the neutrino mass spectrum in the string-inspired GUT models is particularly appealing. However, since in the context of supersymmetric theories, the radiatively generated masses cannot be much larger than the electroweak scale without spoiling the SUSY protection of the hierarchy [11], the Witten mechanism is not effective in superstring-derived models, contrary to what was assumed in ref.[7]. Nevertheless, one can obtain an analogous suppression due to the presence of appropriate non-renormalizable terms, which naturally arise in string theories [12]. In the following we explore in detail the possible scenarios for neutrino masses and mixings in $SU(4) \otimes O(4)$ and flipped $SU(5) \otimes U(1)$. We concentrate first on the simple GUT models, where their embedding in $SO(10)$ allows a full prediction of the masses in terms of the up-quark masses and the other parameters of the models. Subsequently we discuss the same models in the context of supersymmetry, where, due to the lack of the uniqueness of the structure of the non-renormalizable terms from string theory, only the relevant mass scales may be obtained.

The particle content of the $SU(4) \otimes O(4)$ and the flipped $SU(5) \otimes U(1)$ model with its transformation properties under the two gauge groups is summarized in Tab.1. In the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ model the fifteen fermions of the SM plus the right-handed neutrino ν^c belong, for each generation, to the $(\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ representations, denoted by $\mathbf{F}$ and $\bar{\mathbf{F}}$, respectively. In the flipped $SU(5) \otimes U(1)$ model they belong to $\mathbf{F} = (\mathbf{10},\mathbf{1})$, $\bar{\mathbf{f}} = (\bar{\mathbf{5}},-\mathbf{3})$, and $\mathbf{l}^c = (\mathbf{1},\mathbf{5})$. Both gauge groups are broken at the grand unification scale $M_G \simeq 10^{16}$ GeV by the vacuum expectation values of two Higgs fields, $\mathbf{H}$ and $\bar{\mathbf{H}}$, which, as already mentioned, belong to incomplete spinorial representations of $SO(10)$. The masses of the ordinary fermions in the $SU(4) \otimes O(4)$ model are induced by the two vacuum expectation values of the bidoublet Higgs field $\mathbf{h} = (\mathbf{1},\mathbf{2},\mathbf{2})$. The field $\mathbf{h}$ can be fitted with the multiplet $\mathbf{D} = (\mathbf{6},\mathbf{1},\mathbf{1})$ in the fundamental $\mathbf{10}$ representation of $SO(10)$. The $\mathbf{D}$ multiplet combines with the colour triplets in $\mathbf{H}$ and $\bar{\mathbf{H}}$ forming massive states at the GUT scale, thus avoiding a fast proton decay. This is also the case in the flipped $SU(5) \otimes U(1)$ model, where the colour-triplet $\mathbf{D}_3$ is part of $\mathbf{h} = (\mathbf{5},-\mathbf{2})$, which contains the SM Higgs field. Here $\mathbf{h}$ and its complex conjugate form a $\mathbf{10}$ representation of $SO(10)$. In addition, in both models there is a set of fields ϕ_m, singlets under the two gauge groups, which arise naturally in string theories, and mix with the right-handed neutrinos, providing the novel seesaw scenario mentioned in the introduction. A minimal set would consist of n_g such singlets (n_g being the number of generations). Their mass is generated by an extra singlet scalar field (also denoted by ϕ), which develops a vacuum expectation value (vev) at the electroweak scale [4]. In the supersymmetric case there will be instead $n_g + 1$ superfields, of which one has the scalar component which develops the above vev.

For the supersymmetric $SU(4)\otimes O(4)$ model, we can now write the most general superpotential, satisfying the discrete symmetry $\bar{\mathbf{H}} \to -\bar{\mathbf{H}}$, which is essential for forbidding a tree-level Majorana mass at the GUT-scale for the ordinary left-handed neutrinos [7]:

$$\begin{aligned}\mathcal{W}_{(4)} &= \lambda_1^{ij} F_i \bar{F}_j h + \lambda_2^{im} \bar{F}_i H \phi_m + \lambda_3 H H D + \lambda_4 \bar{H} \bar{H} D + \lambda_5^m h h \phi_m \\ &+ \lambda_6^{mnq} \phi_m \phi_n \phi_q + \lambda_7^{ij} F_i F_j D + \lambda_8^{ij} \bar{F}_i \bar{F}_j D + \lambda_9^n D D \phi_n \,. \end{aligned} \tag{1}$$

Correspondingly, in the flipped $SU(5)\otimes U(1)$ model one has [3]:

$$\begin{aligned}\mathcal{W}_{(5)} &= \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i l_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\ &+ \lambda_6^{im} F_i \bar{H} \phi_m + \lambda_7^m h \bar{h} \phi_m + \lambda_8^{mnq} \phi_m \phi_n \phi_q \,. \end{aligned} \tag{2}$$

In the non-supersymmetric GUT models, some of the terms in eqs.(1,2) become Yukawa terms and the others correspond to cubic-scalar interactions. Notice that in this case three of the singlets have to be fermion fields while one has to be a scalar field.

As can be seen from the form of eqs.(1,2), after the spontaneous breaking of the symmetry, the neutrino mass matrix, in addition to the ordinary Dirac mass terms, also contains mixing terms between the right-handed neutrinos and the singlets, as well as mass terms for the latter. In particular, in both models, the Dirac neutrino masses are equal to the up-quark masses:

$$m_{\nu D}^{ij} = M_u^{ij} = \lambda_1^{ij} < h^0 > \,. \tag{3}$$

On the other hand, the $\nu^c - \phi$ mixing comes from the $\lambda_2^{im} \bar{F}_i < H > \phi_m$ term in $\mathcal{W}_{(4)}$, or correspondingly from the $\lambda_6^{im} F_i < \bar{H} > \phi_m$ in $\mathcal{W}_{(5)}$, and are therefore proportional to M_G. The masses of the singlets M_X^{ij} arise from the $\lambda^{mnq} < \phi_m > \phi_n \phi_q$ terms, and are $\propto 10^2$ GeV. As already discussed in the introduction, in the non-supersymmetric models, there are also Majorana masses M_R^{ij} for the right-handed neutrinos of order 10^8 GeV, which arise radiatively at the two-loop level through the so-called Witten mechanism [8]. Although in the supersymmetric models these contributions can be at most of the order of the electroweak scale, a large Majorana mass for ν^c of order 10^{14} GeV may arise from the presence of non-renormalizable terms such as $\bar{F}_i H H \bar{F}_j / M_S$ for $SU(4)\otimes O(4)$ and $F_i \bar{H} \bar{H} F_j / M_S$ for $SU(5)\otimes U(1)$, M_S being the string unification scale $\sim 10^{18}$ GeV [13].

The structure of the one-generation neutrino mass matrix in the $(\nu_L,\, \nu_L^c,\, \phi)$ basis is therefore, for both the flipped $SU(5)\otimes U(1)$ and the $SU(4)\otimes O(4)$ models, of the following form:

$$\mathcal{M} = \begin{pmatrix} 0 & m_u & 0 \\ m_u & M_R & M_G \\ 0 & M_G & M_X \end{pmatrix} . \tag{4}$$

As has been discussed in ref.[7], there are two different see-saw suppression mechanisms present. One of them is due to the implementation of a massive singlet which couples to the right-handed neutrino and leads to a ultra-light neutrino mass proportional to $m_u^2 X/M_G^2$ (X being the characteristic mass scale for the singlets) and, in the absence of M_R, to two states proportional to M_G. The other type of seesaw mechanism, which more closely resembles the standard one, is effective only if the corresponding M_G entries are absent. This leads to a light neutrino mass proportional to m_u^2/M_R, a heavy neutrino state proportional to M_R, and the singlet. This means that in the non-supersymmetric case the mass of the light neutrinos ranges from 10^{-5} eV for the first generation to 10 keV for the third one, while in the supersymmetric case their mass is shifted down by six orders of magnitude. Therefore, in the multi-generation case, in order to have some of the ordinary neutrinos with a mass in the eV range, the submatrix M_G should be singular. Interestingly, this is quite often the case in the string-derived models, as a consequence of extra discrete symmetries.

We shall consider now the three generation case, where the entries of the matrix in eq.(4) become 3×3 sub-matrices. Since the embedding of the two models in $SO(10)$ in the non- supersymmetric case leads to definite predictions for the neutrino spectrum, we shall consider this case first. We start with the quark mass matrices, which we assume to be of Fritzsch-type [14], a structure which may arise in models with only nearest-neighbour generation interactions and where the scale is fixed by the heaviest (third generation) fermion mass. This *Ansatz*, which has proved successful in understanding the entries of the Kobayashi-Maskawa mixing matrix in terms of the quark masses, corresponds to the following choice for M_u:

$$M_u = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \tag{5}$$

where, due to the observed mass hierarchy of quark masses, $A \simeq \sqrt{m_u m_c}$, $B \simeq \sqrt{m_c m_t}$ and $C \simeq m_t \simeq 10^2$ GeV.

The structure of the radiative mass matrix M_R is fixed by the Yukawa coupling in the diagram of Fig.1, that is λ_8^{ij} in eq.(1) for the $SU(4) \otimes O(4)$ model, and λ_1^{ij} in eq.(2) for $SU(5) \otimes U(1)$. In this last case, λ_1^{ij} is also the Yukawa coupling which fixes the down-quark mass matrix, $M_d = \lambda_1 < h^0 >$, so that in this case M_R is fully determined. On the other hand, in the $SU(4) \otimes O(4)$ model, λ_8^{ij} is not fixed by the charged fermion masses, and therefore the predictibility for the neutrino masses is in general lost. However, the embedding of this model in $SO(10)$ reduces the number of independent Yukawa couplings in the superpotential, and in particular

λ_8^{ij} becomes equal to $\lambda_1^{ij} = M_u^{ij}/ < h^0 >= M_d^{ij}/ < \bar{h}^0 >$ in $\mathcal{W}_{(4)}$. Therefore, when both models are embedded in $SO(10)$, the radiative mass matrices become identically proportional to $M_u/ < h^0 >$:

$$M_R = \begin{pmatrix} 0 & aR & 0 \\ aR & 0 & bR \\ 0 & bR & cR \end{pmatrix}, \tag{6}$$

where the radiative scale, obtained from the diagram in Fig.1, is given [7] by:

$$R \simeq \varepsilon \left(\frac{\alpha_G}{4\pi}\right)^2 \frac{3}{16} M_G, \tag{7}$$

α_G being the GUT coupling constant ($\simeq 0.02$), and ε is the mixing between D_3 and $\bar{d}_H^c$, assumed to be of order 0.1. The parameters a, b, c in eq.(6), in the case where the two models are imbedded in $SO(10)$, are equal to the mass parameters[1] A, B, C of eq.(5) divided by $< h^0 >\simeq< \bar{h}^0 >$, which, for simplicity, we have assumed to be equal to $m_t \simeq 10^2$ GeV.

The mass matrix M_X for the extra singlets[2], whose structure is arbitrary in both models, has been assumed to be proportional to the identity matrix, at a scale of 10^2 GeV. Notice that as long as M_X is not singular, its particular structure does not affect substantially the neutrino mass spectrum. On the other hand, the structure of the matrix M_G is crucial and leads to phenomenologically different mass patterns. As already pointed out, the singularity of the matrix M_G is essential for obtaining at least some neutrino masses in the interesting range $10^{-3} - 10^4$ eV. In particular, a perturbative study of the secular equation[3] of the full 9×9 mass matrix has shown that the general features of the neutrino spectrum are mainly determined by the rank of M_G. Labeling the entries of M_G by $M_{ij}\,(i,j = 1,2,3)$ one can classify the various cases of the rank-two and rank-one classes. This classification can be carried out simultaneously for both the $SU(4)\otimes O(4)$ and the flipped $SU(5)\otimes U(1)$ models, because, though the two GUT models are not related to each other, the neutrino mass matrices become identical in the $SO(10)$-embedded versions, due to the equality of M_R and the freedom in choosing M_X and M_G.

As shown in Tab.2, we have obtained three distinct sets of eigenvalues for the rank-two class containing two ultra-light mass eigenvalues which up to some ratio

[1]In the "non-embedded case", while these expressions for the parameters a, b, c still hold for $SU(4) \otimes O(4)$, in the $SU(5) \otimes U(1)$ model they become proportional to the corresponding entries which characterize the down-quark mass matrix.

[2]For simplicity, and without any loss of generality, we assume that only three singlets mix with the right-handed neutrinos.

[3]In getting the approximate expressions for the mass eigenvalues given in Tabs. 2,3, we have followed a procedure analogous to the one used in ref.[15].

of up-quark masses are proportional to $m_u^2 X/M_G^2$, four eigenvalues of order M_G, and one proportional to X. In the cases (II) and (III), the remaining two Majorana states combine to form a single pseudo-Dirac state with a mass proportional to $\sqrt{m_u m_c} \simeq 100$ MeV (II) and to $\sqrt{m_t m_c} \simeq 10$ GeV (III), respectively. Since the experimental upper bounds on the three standard neutrinos are ≤ 35 MeV the spectra (II) and (III) are not realistic. In the rank-two class there remains only the spectrum of case (I) as a good candidate, which, in addition to the almost massless ν_e and ν_μ, contains a 10 keV heavy τ- neutrino and a state with a mass proportional to R. Such a heavy τ-neutrino could be compatible with our present understanding of cosmology only if it decays with a lifetime shorter than $\simeq 10^{13}$ sec, which, as the discussion on "the 17 keV neutrino" has revealed [16], cannot be achieved within the framework of the Standard Model or any theory which is identical to it at low energies, like the present version of the models we have been considering here. Therefore, the presence of a Nambu-Goldstone boson into which this neutrino could decay seems to be required. This could be achieved by breaking some of the extra $U(1)$ global symmetries of the string at some intermediate mass scale, or by introducing a horizontal symmetry à la Peccei-Quinn [17].

We turn now to the rank-one class, which also contains three distinct cases, shown in Tab.3. Obviously, now there is only one ultra-light state, the ν_e, whose mass is proportional to $m_u^2 X/M_G^2$ (up to some ratio of quark masses), and so there are two eigenvalues of order M_G and two of order X. For the same reasons stated above, the spectrum (V) has to be rejected. In the spectrum (IV), $\nu_{\mu L}$, which mixes with $\nu_{\tau L}$ at the ten percent level ($\sin\theta \simeq \sqrt{m_2/m_3} \simeq \sqrt{m_c/m_t}$), has a mass in the 100 eV range, and could therefore be a potential candidate for solving the dark-matter problem, while $\nu_{\tau L}$, having a mass of order 10 keV, has to decay. On the other hand, we have found in the spectrum (VI) two light states of a few keV, which are linear combinations of ν_μ and ν_τ, and form a pseudo-Dirac state. It is conceivable that the radiative mass scale can be pushed up by an order of magnitude, *e.g.* by a corresponding increase of the mixing parameter ε (leading then to a maximal mixing between, say, D_3 and $\bar{d}_H^c$), in which case these neutrinos would be allowed to be stable and become dark-matter candidates. It should be pointed out that, due to the superlightness of the electron neutrino, the $\nu_\mu \pm \nu_\tau$ states do not mix with ν_e at the 10% level, as would be necessary in order to be identifiable with the 17 keV Simpson neutrino [10]. So, the absence of such a neutrino state can be attributed to the presence of singlets and the coupling of at least one of them to the neutrino sector, a common feature of superstring-derived versions of GUT models.

These results indicate that in absence of appropriate decay modes for the tau neutrino and in some cases also for the muon neutrino, the masses obtained in these

two ordinary GUT models would give a far too large contribution to the energy density of the Universe and are therefore in conflict with its present age. The only viable neutrino spectrum, apart from the trivial one, where all three neutrinos are practically massless (M_G non-singular), would then correspond to case (VI), where at least one singlet field couples to the right-handed tau neutrino, if the mixing parameter ε is chosen of order one. The eventual presence of more singlets, does not add any new cases to the six we have discussed here plus the original one with all three neutrinos being practically massless.

Let us now turn to the supersymmetric version of the two models. In this case, the structure of the non-renormalizable terms in generation space (and therefore of the matrix M_R) is not uniquely fixed by the string theory. However, as long as the matrix M_R is not singular, the classification of the possible neutrino mass spectra, given in Tabs. 2,3, is basically the same as in the non-supersymmetric case, apart from a reduction by six orders of magnitude of the mass eigenvalues which scale as $1/R$. Notice that those cases which were excluded by the experimental constraints on neutrino masses (II, III, V), are still ruled out. However, the neutrinos that previously had masses in the 1-10 keV range, for which a new decay mechanism was needed in order to satisfy the cosmological constraints, now have masses in the range 10^{-3} to 10^{-2} eV, and might therefore provide a solution to the solar neutrino problem via the MSW mechanism [9].

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Table Captions

Table 1. The particle content of two superstring- inspired GUT models. See also refs.[3,4].

Table 2. The ordinary neutrino mass spectrum of the two $SO(10)$-embedded GUT models, in the three cases where the mass matrix M_G^{ij} is singular and has rank two. Case (I) corresponds to the entries $(i,j) = (3,1),(3,2),(3,3)$ being zero, while case (II) corresponds to $(i,j) = (1,1),(1,2),(1,3)$ being zero, and case (III) to $(i,j) = (2,1),(2,2),(2,3)$ being zero. Four of the extra neutrinos have a mass at the GUT scale and one decouples at the electroweak scale. In case (I) one neutrino has a radiative mass of $R \simeq 10^9$ GeV.

Table 3. The ordinary neutrino mass spectrum of the two $SO(10)$-embedded GUT models, in the three cases where the mass matrix M_G^{ij} is singular and has rank one. Case (IV) corresponds to at least one of the entries $(i,j) = (1,1),(1,2),(1,3)$ being different from zero, while the cases (V) and (VI) correspond respectively to at least one non-zero entry of $(i,j) = (2,1),(2,2),(2,3)$ and $(i,j) = (3,1),(3,2),(3,3)$. In all the three cases, in addition to these neutrino states there are two heavy states with a mass at the GUT scale, and two with a mass at the electroweak scale. In case (IV) there are also two mass eigenstates, one of order $R\,m_c/m_t$ and the other is of order R. In case (V) there is only one state proportional to $\simeq R$, while in case (VI) there are two mass-degenerate states of order $R\sqrt{m_u m_c}/m_t$.

Figure Captions

Figure 1. The diagram which induces a Majorana mass for the R-H neutrinos at the two loop level in the non-supersymmetric $SU(4) \otimes O(4)$ model. The broken lines stand for scalar bosons, while the dotted lines stand for the gauge vector bosons. The solid line, as usual, indicates fermions. An analogous diagram holds in the case of the flipped $SU(5) \otimes U(1)$ model, as shown in ref.[7].

References

[1] B.R. Greene, K.H. Kirklin, P.J. Miron and G.G. Ross, Nucl. Phys. **B278** (1986) 667;**B292** (1987) 606.

[2] L.E. Ibanez, J.E. Kim, H.P. Nilles and F. Quevedo, Phys. Lett. **191B** (1987) 3;L.E. Ibanez, J. Mas, H.P. Nilles and F. Quevedo, Nucl. Phys. **B301** (1988) 157;D. Bailin, A. Love and S. Thomas, Phys. Lett. **194B** (1987) 385;Nucl. Phys. **B298** (1988) 75;J.A. Casas, E. Katechou and C. Munoz, Nucl. Phys. **B317** (1989) 171.

[3] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. **194B** (1987) 231;**205B** (1988) 450; **208B** (1988) 209; **231B** (1989) 65.

[4] I. Antoniadis and G.K. Leontaris, Phys. Lett. **216B** (1989) 333.

[5] I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. **245B** (1990) 161.

[6] G.K. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. **243B** (1990) 220.

[7] G.K. Leontaris and J.D. Vergados, Phys. Lett. **258B** (1991) 111.

[8] E. Witten, Phys. Lett. **91B** (1980) 81.

[9] S. Mikheyev and A. Smirnov, Sov. J. Nucl. Phys. **42** (1986) 1441;L. Wolfenstein, Phys. Rev. **D17** (1978) 2369;**D20** (1979) 2634.

[10] For a review of 17 keV neutrino experiments see, *e.g.* D.R.O. Morrison, talk given at the "Lepton-Photon Symposium", Geneva, July 1991.

[11] L. Ibanez, Phys. Lett. **117B** (1982) 403;Nucl. Phys. **B218** (1983) 514.

[12] S. Kalara, J.L. Lopez and D.V. Nanopoulos, Phys. Lett. **245B** (1990) 421;J. Rizos and K. Tamvakis, Phys. Lett. **262B** (1991) 227.

[13] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, CERN preprint, CERN-TH 6004/91.

[14] H. Fritzsch, Phys. Lett. **73B** (1978) 317;**85B** (1979) 81; Nucl. Phys. **B155** (1979) 189.

[15] E. Papageorgiu and S. Ranfone, Nucl. Phys. **B369** (1992) 99.

[16] See, for example: G. Gelmini, S. Nussinov and R.D. Peccei, UCLA preprint, UCLA-91-TEP-15, (1991).

[17] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38** (1977) 1140;Phys. Rev. **D16** (1977) 1791.

Table 1

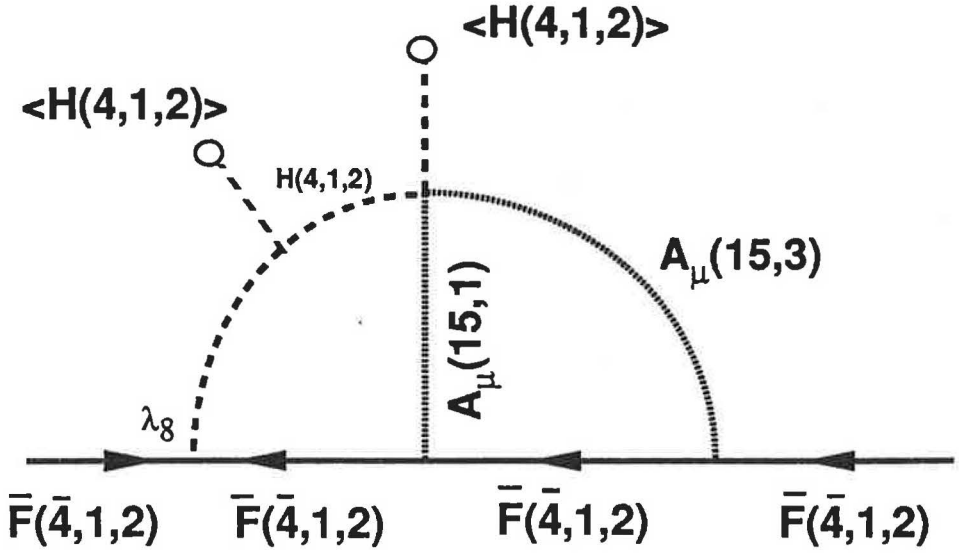
$SU(4) \otimes SU(2)_L \otimes SU(2)_R$	$SU(5)_{fl} \otimes U(1)$
$F = (4,2,1) = (u, d, \nu, e)$	$F = (10,1) = (u, d, d^c, \nu^c)$
$\bar{F} = (\bar{4},1,2) = (u^c, d^c, \nu^c, e^c)$	$\bar{f} = (\bar{5},-3) = (u^c, \nu, e)$
	$l^c = (1,5) = (e^c)$
$H = (4,1,2) = (\bar{u}^c_H, \bar{d}^c_H, \bar{\nu}^c_H, \bar{e}^c_H)$	$H = (10,1) = (u_H, d_H, d^c_H, \nu^c_H)$
$\bar{H} = (\bar{4},1,2) = (u^c_H, d^c_H, \nu^c_H, e^c_H)$	$\bar{H} = (\bar{10},-1) = (\bar{u}_H, \bar{d}_H, \bar{d}^c_H \bar{\nu}^c_H)$
$h = (1,2,2) = (h^0, h^-, \bar{h}^+, \bar{h}^0)$	$h = (5,-2) = (D_3, h^-, h^0)$
$D = (6,1,1) = (D_3, \bar{D}_3)$	$\bar{h} = (\bar{5},2) = (\bar{D}_3, h^+, \bar{h}^0)$
$\phi_m = (1,1,1)\,,\ m = 1,...,4$	$\phi_m = (1,0)$

Table 2

	Masses	Neutrino Spectrum
(I)	$m_1 = m_u^2 X/M_G^2 \simeq 10^{-26}$ eV	$\nu_1 \simeq \nu_{eL}$
	$m_2 = m_c^2 X/M_G^2 \simeq 10^{-21}$ eV	$\nu_2 \simeq \nu_{\mu L}$
	$m_3 = m_t^2/R \simeq 20$ keV	$\nu_3 \simeq \nu_{\tau L}$
(II)	$m_1 = m_u m_c X/M_G^2 \simeq 10^{-23}$ eV	$\nu_1 \simeq \nu_{\tau L}$
	$m_2 = m_t^2 X/M_G^2 \simeq 10^{-17}$ eV	$\nu_2 \simeq \nu_{eL}$
	$m_{3,4} = \sqrt{m_u m_c} \simeq 80$ MeV	$\nu_{3,4} \simeq \frac{1}{\sqrt{2}}(\nu_{\mu L} \pm \nu_{eL}^c)$
(III)	$m_1 = m_u^2 X/M_G^2 \simeq 10^{-26}$ eV	$\nu_1 \simeq \nu_{\mu L}$
	$m_2 = m_t m_c X/M_G^2 \simeq 10^{-19}$ eV	$\nu_2 \simeq \nu_{eL}$
	$m_{3,4} = \sqrt{m_t m_c} \simeq 10$ GeV	$\nu_{3,4} \simeq \frac{1}{\sqrt{2}}\left(\nu_{\tau L} \pm \nu_{\mu}^c\right)$

Table 3

	Masses	Neutrino Spectrum
(IV)	$m_1 = m_u^2 X/M_G^2 \simeq 10^{-26}$ eV	$\nu_1 \simeq \nu_{eL}$
	$m_2 = m_c m_t/R \simeq 0.3$ keV	$\nu_2 \simeq cos\theta\, \nu_{\mu L} - sin\theta\, \nu_{\tau L}$
	$m_3 = m_t^2/R \simeq 20$ keV	$\nu_3 \simeq cos\theta\, \nu_{\tau L} + sin\theta\, \nu_{\mu L}$
(V)	$m_1 = m_u m_c X/M_G^2 \simeq 10^{-23}$ eV	$\nu_1 \simeq \nu_{eL}$
	$m_2 = m_t^2/R \simeq 20$ keV	$\nu_2 \simeq \nu_{\tau L}$
	$m_{3,4} = \sqrt{m_u m_c} \simeq 80$ MeV	$\nu_{3,4} \simeq \frac{1}{\sqrt{2}}(\nu_{\mu L} \pm \nu_{eL}^c)$
(VI)	$m_1 = m_u m_t X/M_G^2 \simeq 10^{-21}$ eV	$\nu_1 \simeq \nu_{eL}$
	$m_{2,3} = \sqrt{m_t m_c} m_t/R = 2$ keV	$\nu_{2,3} \simeq \frac{1}{\sqrt{2}}(\nu_{\tau L} \pm \nu_{\mu L})$


<H(4,1,2)>
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H(4,1,2)
$A_\mu(15,3)$
$A_\mu(15,1)$
λ_8
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