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Abstract

We show how heavy quark effective theory, including $\frac{1}{M_Q}$ corrections, may be matched onto dynamical quark models by making a specific choice of K_μ, m and v_μ in the $p_\mu = mv_\mu + K_\mu$ expansion. We note that Wigner rotations of heavy quark spins arise at $O(p^2/m^2)$ in non-relativistic models but at $O(\Lambda_{QCD}/M_Q)$ or $O(velocity - transfer)$ in HQET and so are necessary for a consistent treatment.

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There is currently considerable experimental and theoretical interest in hadrons that contain a single heavy quark. On the experimental side, flavour changing weak decays such as $B \rightarrow D^* e\nu$ provide essential input to completing the elements in the CKM matrix as part of a larger strategy of unravelling the source of CP violation. The problem for theory is to extract these CKM elements at **quark** level from experiments that involve **hadrons**. Historically this has relied on model calculations where a heavy quark is bound in a heavy hadron: these have the virtue of being applied to b, c, s or even u, d flavours but suffer from model dependent assumptions and, in the case of u, d flavours, considerable dispute as to reliability and self-consistency. Recently there has been interest in an approach (heavy quark effective theory, HQET) which exploits newly discovered symmetries of QCD that apply to Green functions of heavy quarks that are nearly on-shell [1,2]. In HQET the states depend on the velocity of the heavy quark and involve an expansion around infinite quark mass with v held finite.

In practice $m_Q \neq \infty$. This is perhaps a reasonable approximation for b, but for c and s one needs to understand corrections of $O(\Lambda_{QCD}/m_Q)$. However, there is some arbitrariness in how one defines the variables. The states in HQET depend upon the velocity v of the heavy quark which is conserved in the absence of external interactions (the "velocity superselection rule" [2]). Ref 3 has noted some ambiguities in procedure concerning the relation between the field of velocity v_μ and a heavy-light meson that contains this heavy quark. For example, quoting from ref 3 "Is the meson's momentum mv_μ or does it differ by some amount K_μ of order of the hadronic scale Λ? Is m the mass of the meson or the quark?" We propose a physically motivated choice that enables immediate contact with the formalism for describing composite systems e.g. heavy quark in a heavy hadron. This highlights that the **light** quark(s) in a heavy-light system play a nontrivial dynamical role and are not "spin-inert"[‡] even though, at first sight, only the heavy quark appears to be involved directly in flavor changing decays such as $B \rightarrow D^* l\nu$. Furthermore we shall see that this spin activity survives even in the leading order when $m_Q \rightarrow \infty$.

First we review the HQET expansion to $O(1/M_Q)$.

A heavy quark Q is interacting with light degrees of freedom whose four momenta are of order Λ_{QCD} and much smaller than the heavy quark mass, m_Q. In HQET one writes

$$p_Q^\mu = m_Q v^\mu + K^\mu \tag{1}$$

where K^μ is a residual momentum, small compared to m_Q. The heavy quark

[‡]See e.g. the comments on p 348 and 356 of ref 16

spinor is written [2]

$$Q = e^{-im_Q v\cdot x}[h_v^{(Q)} + \chi_v^{(Q)}] \tag{2}$$

where h survives as $m_Q v \to \infty$ and χ is an $O(K/m_Q)$ correction.

The decomposition is defined explicitly by [2]

$$\not{v} h_v^{(Q)} = h_v^{(Q)},\ \not{v}\chi_v^{(Q)} = -\chi_v^{(Q)} \tag{3}$$

and it is straightforward to verify that the equations of motion then impose the constraints, to $O(1/m_Q^2)$

$$\frac{\not{K}}{2m_Q} h_v = \chi_v \quad ; \quad v\cdot K = 0 \tag{4}$$

This is a perfectly legitimate and actively researched procedure when applied to the Lagrangian for a single heavy quark. However, many practical applications deal specifically with a heavy quark (Q) within a many-quark system, in which case there are kinematic correlations among the Q and other constituents that are manifested in processes involving recoil, e.g. $H_1(Q_i) \to H_2(Q_j) l\nu$ or in Compton scattering. The requirements of gauge invariance, current conservation and Lorentz invariance **constrain** the definition of dynamical variables such that, for example, the low energy theorems of Compton scattering are satisfied in an arbitrary frame for a many quark system [4,6,8].

This is our point of departure.

We suggest that it may be useful to impose these constraints at the outset and thereby identify a **particular** choice of v, m and K from the more general set of possibilities consistent with Eq. 1. Our aim is to choose the variables so that the χ, h spinors of the HQET refer respectively to the motion of the heavy quark relative to the heavy hadron and the overall motion of the system with v_μ identified as the hadron's velocity; the K_μ will be a function of the relative momentum of the heavy quark in the hadron. Our prescription, which is consistent with the general precepts of HQET, leads to immediate and consistent insertion of the heavy quark in a heavy hadron, hence isolating an "effective heavy quark theory".

At first sight this appears trivial. It is tempting to exploit the familiar separation of overall, $\vec{P}$, and relative, $\vec{K}$, momentum variables for a non-relativistic composite system

$$\left.\begin{array}{rcl} \vec{p}_Q & = & \frac{m_Q}{M}\vec{P} + \vec{K} \\ E_Q & = & \frac{m_Q}{M}E + \frac{\vec{K}\cdot\vec{P}}{M} \end{array}\right\} \tag{5}$$

and to compare with eq.(1) where M is the hadron's mass and $v_\mu = (E/M; \vec{P}/M)$ is its four-velocity. However, this is wrong. The reason why, and the correct solution, can be seen if we first recall where Eq. 5 comes from. If $p_\mu = (\omega, \vec{k})$ in the hadron $\vec{P} = 0$ frame, (where $k = O(\Lambda_{QCD})$) then after a Lorentz boost to a frame with $\vec{P} \neq 0$, the variables are

$$\left.\begin{array}{lcl} \vec{p}_Q & = & \frac{\omega_Q \vec{P}}{M} + \vec{k} + \frac{\vec{k}\cdot\vec{P}}{M(E+M)}\vec{P} \\ E_Q & = & \frac{E}{M}\omega_Q + \frac{\vec{k}\cdot\vec{P}}{M} \\ \sigma(\vec{v}) & = & \vec{\sigma} + \frac{(\vec{k}\times\vec{P})\times\vec{\sigma}}{2M(E+M)} \end{array}\right\}. \tag{6}$$

If $m, M \to \infty$ with $k/M, P/M \to 0$ then eq (5) obtains; furthermore in this limit $\sigma(\vec{v}) \equiv \vec{\sigma}$ and the spin is trivial. However, in HQET, $m_Q \to \infty$ with v **held finite.** As $v^\mu = P^\mu/M$ is the hadron's velocity, then eq (6) becomes (to $O(\Lambda^2/m_Q^2)$)

$$\left.\begin{array}{lcl} \vec{p}_Q & = & m_Q\vec{v} + \vec{k} + \frac{\vec{k}\cdot\vec{v}}{1+v_0}\vec{v} \\ E_Q & = & m_Q v_0 + \vec{k}\cdot\vec{v} \\ \sigma(\vec{v}) & = & \vec{\sigma} + \frac{(\vec{k}\times\vec{v})\times\vec{\sigma}}{2m_Q(1+v_0)} \end{array}\right\} \tag{7}$$

The equations for $(\vec{p}_Q, E_Q)$ have the form of eq (1); notice that $\vec{\sigma}(\vec{v})$ is no longer trivial and that v is understood to be the **hadron's** velocity which,as is clear from eq.7, **differs** from the heavy quark velocity in general. As we shall see later, this is important when matching with explicit composite wavefunctions.

If m_Q in eq (1) is identified as the quark mass (technically, its energy $\omega = \sqrt{m^2 + \vec{k}^2}$ in the hadron's rest frame) then the residual momentum K^μ has the frame dependent form (from eq 7)

$$K^\mu \equiv k^\mu + \frac{\vec{v}\cdot\vec{k}}{1+v_0}v^\mu \tag{8}$$

Notice that the constraint equation $K \cdot v = 0$ implies that $k \cdot v = -k_0$ or, equivalently, that

$$\frac{\vec{v}\cdot\vec{k}}{1+v_0} = k_0. \tag{9}$$

Alternatively, if one wishes to identify K^μ as the residual momentum k^μ of the heavy quark in the hadron's rest frame, then it is the mass parameter that must be regarded as frame dependent in order to realise eqs(7)

$$p_Q^\mu = m_Q^{eff} v^\mu + k^\mu \quad ; \quad m_Q^{eff} = \omega_Q + \frac{\vec{v}\cdot\vec{k}}{1+v_0}. \tag{10}$$

Now we shall study the decomposition of the heavy quark spinor, eqs(2-4), in terms of these variables.

We choose to define h_v to be independent of m_Q and hence

$$h_v = \sqrt{\frac{1+v_0}{2}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{v}}{1+v_0} \end{pmatrix} \tag{11}$$

With K^μ defined at eq(8) and taking care to include, consistently, the $O(\vec{v}/m)$ spin-rotation (eq 7c) and the constraint at eq(9), the $O(k/m_Q)$ correction to the heavy quark spinor is

$$\chi_v \equiv \frac{\not{K}}{2m_Q} h_v = \sqrt{\frac{1+v_0}{2}} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{v}}{1+v_0} \\ 1 \end{pmatrix} \frac{\vec{\sigma}\cdot\vec{k}}{2m_Q}. \tag{12}$$

where the $\vec{\sigma}$ operator acts on spinor states as defined in the hadron $\vec{v} = 0$ frame.

Note the ordering of $\vec{\sigma}\cdot\vec{v}\vec{\sigma}\cdot\vec{k}$ that arises in the HQET expansion, eq(12). We shall now show that this matches the spinor representation of a composite system if the latter is constructed in the system's rest frame and then boosted to arbitrary velocity.

In the hadron rest frame we write formally

$$H(\vec{v} = 0) = Q(\vec{v} = 0; \vec{k}) \otimes S(\vec{v} = 0; -\vec{k}) \tag{13}$$

where $Q(\vec{k}), S(-\vec{k})$ refer to the heavy quark and spectator system respectively with equal and opposite three momenta ("relative momenta"). To $O(k^2/m^2)$ the spinor of the heavy quark is

$$Q(\vec{v} = 0; \vec{k}) = \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{k}}{2m_Q} \end{pmatrix} \tag{14}$$

(we are neglecting binding energy at present).Upon boosting to a frame with velocity $\vec{v}$, this becomes

$$Q(\vec{v}; \vec{k}) = \sqrt{\frac{1+v_0}{2}} (1 + \frac{\vec{\alpha}.\vec{v}}{1+v_0}) Q(\vec{v} = 0; \vec{k}) \tag{15}$$

$$= \sqrt{\frac{1+v_0}{2}} \begin{pmatrix} 1 + \frac{\vec{\sigma}\cdot\vec{v}\vec{\sigma}\cdot\vec{k}}{2m_Q(1+v_0)} \\ \frac{\vec{\sigma}\cdot\vec{v}}{1+v_0} + \frac{\vec{\sigma}\cdot\vec{k}}{2m_Q} \end{pmatrix} \tag{16}$$

If the spectator is a single (anti)quark, the $S(\vec{v}; -\vec{k})$ follows from eq.16 by replacing $\vec{k}$ by $-\vec{k}$ and $2m_Q$ by $2m_q$ (upper component) or $E_q + m_q$ (lower component). (For a multiquark system the $S(\vec{v}, -\vec{k})$ is a direct product of spinors with appropriate momenta relative to the centre of momentum of the system). This way of writing the spinors for a boosted composite system is essentially that developed by Brodsky and Primack (see section 4 of ref 4) and establishes the identity with the HQET decomposition at eq(11,12). Note the $\vec{\sigma} \cdot \vec{v}\vec{\sigma} \cdot \vec{k}$ term in the upper component of eqs (12, 16): this has frequently been overlooked in the literature and will be seminal in what follows.

The careful separation of the quark momentum within the hadron from the overall motion of the system is crucial when considering interactions that transfer momentum (velocity) (such as spin dependent gluon exchange energy shifts in spectroscopy (ref 5) or the interactions with external currents, as in electromagnetic [4,6,7,8] or weak interactions). The use of the spinor eq(16) and consistent accounting of the spectator wavefunction $S(\vec{v}, -\vec{k})$ in the boosted version of eq(13) most effectively leads to the correct interaction Hamiltonian.

As an example consider a "meson" consisting of a heavy quark, charge e_Q, mass m_Q, and a quasi-free electrically neutral spectator fermion with mass m_q. The corresponding electromagnetic interaction for this system is [4,6,8]

$$H_I = \left\{ e_Q \frac{\vec{p}_Q \cdot \vec{A}_Q}{m_Q} - \frac{e_Q g}{2m_Q} \vec{\sigma}_Q \cdot \vec{B}_Q - \frac{e_Q}{2m_Q}(2g-1)\vec{\sigma}_Q \cdot \left(\vec{E}_Q \times \frac{\vec{p}_Q}{2m_Q} \right) \right\} + \frac{1}{4M_T} \left(\frac{\vec{\sigma}_q}{m_q} - \frac{\vec{\sigma}_Q}{m_Q} \right) \cdot (e_Q \vec{E}_Q \times \vec{p}_q) \tag{17}$$

where $\vec{A}_Q$, $\vec{B}_Q$ and $\vec{E}_Q$ are the electromagnetic fields at the position of the heavy quark. We allow the possibility that $g \neq 1$ from the QCD effects. The second line in Eq. 17 shows the crucial spectator-dependent or "nonadditive terms" explicitly that are known to be essential in satisfying the low energy theorems of Compton scattering[4,8]. They arise because

(i) When the spinor is written in the manner of eqs.(11,12) or (16) it manifests correlations between the relative (k) and overall(v) motion - specifically the $\vec{\sigma} \cdot \vec{v}\vec{\sigma} \cdot \vec{k}$ upper component. This term in the spinor of the "active" quark (the one interacting with the external current) generates the $\vec{\sigma}_Q \vec{E}_Q$ term in the second line of eq17.

(ii)The momentum of the spectator(s) is conserved in a nontrivial manner. There is a transfer from relative(k) to overall(v) momentum of the spectator(s) induced by the recoil of the system; this induces a contribution from the spectators to the Wigner rotation of the hadron's overall spin. For a single light quark spectator, as in Eq17, this spin rotation is given by $\bar{S}(\vec{v}', -\vec{k}')S(\vec{v}, -\vec{k})$ (where $\vec{k} - \vec{k}' = (M - m_Q)(\vec{v} - \vec{v}')$). This generates the $\vec{\sigma}_q \vec{E}_Q$ term in the second line of eq17 and calls into question the traditional assumption that the spectators are "spin- inert".[§]

The result is that the vector current $J_\mu = (J_0, \vec{J})$ for the heavy quark m_Q in terms of the velocity v is (to $0((v-v')^2\ ;\ \vec{v}^3)$

$$\vec{J} = \frac{1}{2}(\vec{v}+\vec{v}')-\frac{igM}{2m_Q}\vec{\sigma}_Q\times(\vec{v}'-\vec{v})-\frac{i}{8}(v_0'-v_0)\left(\frac{2gM}{m_Q}\vec{\sigma}_Q-\vec{\sigma}_T\right)\times(\vec{v}'+\vec{v})+\vec{J}(\vec{k}) \quad (18)$$

and

$$J_0 = \frac{1}{2}(v_0+v_0')-\frac{i}{8}\left(\frac{2gM}{m_Q}\vec{\sigma}_Q-\vec{\sigma}_T\right)\cdot[(\vec{v}'+\vec{v})\times(\vec{v}'-\vec{v})]+J_0(k) \quad (19)$$

where

$$\vec{J}(\vec{k}) = \frac{\vec{k}'+\vec{k}}{2m_Q}-\frac{i}{4}(v_0'-v_0)\left\{\left[\frac{M}{m_Q}(2g-1)-1\right]\vec{\sigma}_Q\times\frac{\vec{k}'+\vec{k}}{2m_Q}+\vec{\sigma}_q\times\frac{\vec{k}'+\vec{k}}{2m_q}\right\}, \quad (20)$$

$$J_0(k) = \frac{k_0'+k_0}{2m_Q}-\frac{i}{4}\left\{\left[\frac{M}{m_Q}(2g-1)-1\right]\vec{\sigma}_Q\times\frac{\vec{k}'+\vec{k}}{2m_Q}+\vec{\sigma}_q\times\frac{\vec{k}'+\vec{k}}{2m_q}\right\}\cdot(\vec{v}'-\vec{v}) \quad (21)$$

are functions of the relative momentum $\vec{k}$, and $\vec{\sigma}_T = \vec{\sigma}_Q+\vec{\sigma}_q$. (For a flavour changing vector current between initial(i) and final(f) states, the $\frac{M}{m_Q}$ in leading order becomes the average, namely, $\frac{1}{2}(\frac{M_i}{m_{Q_i}}+\frac{M_f}{m_{Q_f}})$; there are also terms proportional to (m_i-m_f) at order $1/M$ which we shall not discuss in this paper).

The nonadditive terms in Eq. 17 are crucial in generating the correct overall Wigner rotation proportional to $\vec{\sigma}_T$ in Eqs.18,19; they summarise the kinematic spin rotation associated with a boost from the quark to hadron overall c.m. frame. The spectator system does not simply act as a spin-neutral system (contrast e.g. ref. 9 and 16). Thus even though a flavor changing weak decay at first sight involves only the heavy active quark, we see that at non-zero velocity transfer the **light quark spectators do not trivially factor out in the effective Lagrangian of the HQET and can play a dynamical role even when** $M_Q \to \infty$.

The careful accounting of Wigner rotations is important when studying polarisation [10] at large momentum transfer. Examples of immediate relevance include the polarisation [10] of vector mesons D^*, K^* in semi-leptonic decays such as

§A covariant representation of states (e.g. refs 16,17) incorporates the spectator spin-rotation correctly at leading order but it is not immediately transparent how the required spin-rotation arises in that formalism. The utility of our detailed approach is that it shows how the change in overall velocity arises as a trade-off with relative momentum and enables contact to be made between the covariant approach, generalised to O(1/M), and explicit models.

$B \to D^* e\nu, D \to K^* e\nu$ and of the Λ_c, Λ_s in their baryonic analogues [11]: indeed, the importance of such care in maintaining covariance in $B \to D^* e\nu$ has already been noticed in an explicit model [12]. Note, in particular, that the $\vec{\sigma}_T$ in eq.18,19 does not contribute to the transition $B \to D^*$; if this had been incorrectly written as $\vec{\sigma}_Q$ then the term in parenthesis in eq18,19 would have its strength underestimated by (roughly) a factor of two. We find that the relationships among form factors for $\Lambda_b \to \Lambda_c e\nu$ at $O(1/m)$ (ref 13,14) are unaffected in the approximation that $\omega = m_Q$, essentially because the light quark spectator system has no net spin.

The explicit appearance of light quark dynamics might appear to frustrate some of the hopes that HQET makes clean cut statements about processes involving heavy quark hadrons. However, the form of Eqs.18 to 21 and the physical interpretation of the light-quark contributions, suggest that HQET can remain effective if judicious choice of frame is made. For flavor changing transitions involving heavy quarks, the Wigner rotation subtelties may be bypassed in leading order by choosing to work in the particular frame where $\vec{v} = -\vec{v}'$ (this frame [15] would correspond to the Breit frame in the case $m_i = m_j$). The terms linear in $\vec{k}/m_q$ integrate to zero for transitions involving S-state hadrons, but can give non-trivial contributions elsewhere (e.g.ref18). In studies of hadron spectroscopy for a two-body system one need only work in the $\vec{v} = 0$ frame for these concerns to become academic[8]; however for baryons, where the $v_{qq} = 0$ frame differs in general from the $\vec{v}_{Qqq} = 0$ frame, these problems are rather central, in particular for the P-wave and other excited states[5]. In transitions where the light quarks are active, such as $H_1 \to H_2 + (\pi or \rho)$, there can be analogous spin rotations of the spectator heavy quark which need to be taken into account.

In this note we have ignored interactions between the heavy quark and other constituents; at $O(1/m_Q)$ these can generate dynamical spin couplings between heavy and light quarks in addition to the "kinematic" ones discussed here. The incorporation of scalar or vector binding potential follows the procedure developed in refs 7,8 (for the case of vector currents where $m_Q^i = m_Q^f$); application of these ideas to flavour changing transitions, involving vector and axial currents, will be described in detail elsewhere.

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