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Scalar Mesons in ϕ Radiative Decay:

their implications for spectroscopy and for studies of CP-violation at ϕ factories

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Abstract

Existing predictions for the branching ratio for $\phi \to K\bar{K}\gamma$ *via* $\phi \to S\gamma$ (where S denotes one of the scalar mesons $f_0(975)$ and $a_0(980)$) vary by several orders of magnitude. Given the importance of these processes for both hadron spectroscopy and CP-violation studies at ϕ factories (where $\phi \to K^0\bar{K}^0\gamma$ poses a possible background problem), this state of affairs is very undesirable. We show that the variety of predictions is due in part to errors and in part to differences in modelling. The latter variation leads us to argue that the radiative decays of these scalar states are interesting in their own right and may offer unique insights into the nature of the scalar mesons. As a byproduct we find that the branching ratio for $\phi \to K^0\bar{K}^0\gamma$ is $\lesssim 0(10^{-7})$ and will pose no significant background to proposed studies of CP-violation.

1 Introduction

There are predictions in the existing literature for the branching ratio for $\phi \to K\bar{K}\gamma$ *via* $\phi \to S\gamma$ (where S denotes one of the scalar mesons S^* (now called $f_0(975)$) or δ (now called $a_0(980)$)) that vary by several orders of magnitude [1-5]. Clearly not all of these predictions can be correct! Given the importance of these processes for both hadron spectroscopy and CP-violation studies, this state of affairs is clearly undesirable. Moreover, in view of the impending ϕ factory, DAΦNE [6], and other developing programmes [7], there is an urgent need to clarify the theoretical situation.

The scalar mesons (*i.e.*, mesons with $J^{PC_n} = 0^{++}$) have been a persistent problem in hadron spectroscopy.* We shall show in this paper that the radiative decays of the ϕ meson to these states can discriminate among various models of their structure. In addition to the spectroscopic issues surrounding the scalar mesons, there is a significant concern that the decay $\phi \to K^0\bar{K}^0\gamma$ poses a possible background problem to tests of CP-violation at future ϕ factories: the radiated photon forces the $K^0\bar{K}^0$ system to be in a C-even state, as opposed to the C-odd decay $\phi \to K^0\bar{K}^0$. Looking for CP-violating decays in $\phi \to K^0\bar{K}^0$ has been proposed as a good way to measure ε'/ε [10], but because this means looking for a small effect, any appreciable rate for $\phi \to K^0\bar{K}^0\gamma$ (namely, a branching ratio $\phi \to K^0\bar{K}^0\gamma \gtrsim 10^{-6}$) will limit the precision of such an experiment. Estimates [4] of the *non-resonant* $\phi \to K^0\bar{K}^0\gamma$ rate give, in the absence of any resonant contribution, a branching ratio of the order of 10^{-9}, far too small to pose a problem. The uncertainty in the theoretical estimates, and the potential experimental ramifications, arise due to the presence of the scalar mesons $f_0(975)$ and $a_0(980)$, which are strongly coupled to the $K\bar{K}$ system. Estimated rates for the *resonant* decay chain $\phi \to S + \gamma$, followed by the decay $S \to K^0\bar{K}^0$, vary by three orders of magnitude, from a branching ratio of the order of 10^{-6} down to 10^{-9}. These variations in fact reflect the uncertainties in the literature for the expected branching ratio for $\phi \to S\gamma$ which vary from 10^{-3} to 10^{-6} [11] . Here we concentrate on this resonant process.

We shall show that the variability of the predictions for $\phi \to S\gamma$ is due in part to errors and in part to differences in modelling. On the basis of this model dependence, we argue that the study of these scalar states in $\phi \to S\gamma$ may offer unique insights into the nature of the scalar mesons. These insights should help

*For an historical perspective see Ref. [8]; for a more recent study see Ref. [9].

lead in the future to a better understanding of not only quarkonium but also glueball spectroscopy. As a byproduct we predict that the branching ratio for $\phi \rightarrow K^0\bar{K}^0\gamma$ is $\lesssim 0(10^{-7})$ (*i.e.*, the branching ratio for $\phi \rightarrow S\gamma$ is $\lesssim 0(10^{-4})$) and will pose *no* significant background to studies of CP-violation at DAΦNE.

2 Probing the nature of the scalar mesons below 1 GeV

The scalar mesons are spectroscopically interesting for several reasons. One is that, while agreeing on little else, it is an essentially universal prediction of theory (lattices, bags, flux tube models, QCD sum rules, ...) that the lowest-lying glueball has scalar quantum numbers and a mass in the 1.0 - 1.5 GeV mass range. Clarifying the presently confused nature of the known 0^{++} mesons may be pivotal in the quest to identify this glueball. Another is the possibility that the two best known [12] scalar mesons, the $f_0(975)$ and the $a_0(980)$, are $qq\bar{q}\bar{q}$ states. The original proposal [13] for this interpretation, based on the bag model, also predicted many other states which have not been seen (although this shortcoming is now understood to some degree [14]). The $qq\bar{q}\bar{q}$ interpretation of these two states was later revived in a different guise within the quark potential model as the "$K\bar{K}$ molecule" interpretation [15]. Since providing a test of this particular interpretation is one of the main results to be presented here, we first briefly elaborate on these two models of multiquark states.

In the naive bag model the $qq\bar{q}\bar{q}$ states consist of four quarks confined in a single spherical bag interacting via one gluon exchange. It is obvious that such a construction will lead to a rich spectroscopy of states. Although it is not clear how to treat or interpret the problem of the stability of this spectrum under fission into two bags [14], it is very interesting that the dynamics of this model predicts that the lowest-lying such states will (in the SU(3) limit) form an apparently ordinary ("cryptoexotic") nonet of scalar mesons. It is, moreover, probable that a better understanding of bag stability could solve both the problem of unwanted extra predicted states and also a problem with the a_0 itself: in the naive model it can "fall apart" into $\pi\eta$ so that it is difficult to understand its narrow width, given the presently accepted pseudoscalar meson mixing angle (see footnote 22 in the first of Refs. [13]). In the absence of an understanding of how to overcome these difficulties, we will not consider the bag picture further in this paper†.

†See, however, Refs. [16] for a possible way out of the $a_0 \rightarrow \pi\eta$ problem.

In the potential model treatment [15] it is found that the low-lying $qq\bar{q}\bar{q}$ sector is most conveniently viewed as consisting of weakly interacting ordinary mesons: the resulting spectrum is normally a (distorted) two particle continuum. Within the ground state u,d,s meson-meson systems, the one plausible exception to this rule is found in the $K\bar{K}$ sector (*i.e.*, the $K\bar{K}$ channel and those other channels strongly coupled to it): the $L = 0$ (*i.e.*, $J^{PC_n} = 0^{++}$) spectrum seems to have sufficient attraction to produce weakly bound states in both $I = 0$ and $I = 1$. There are a number of phenomenological advantages to the identification of these two states with the $f_0(975)$ and $a_0(980)$. Among them are:

1) It is immediately obvious why the $f_0(975)$ and $a_0(980)$ are found just below $K\bar{K}$ threshold: they bear much the same relationship to it that the deuteron bears to np threshold.

2) The problem of the f_0 and a_0 widths is solved. If these states were $^3P_0$ quarkonia with flavours corresponding to ω and ρ (as suggested by their degeneracy), then $\Gamma(f_0 \to \pi\pi)/\Gamma(a_0 \to \pi\eta)$ would be about 4 in contrast to the observed value of about $\frac{1}{2}$. At least as serious is the problem in the quarkonium picture with the absolute widths of these states: models [17-19] predict, for example,

$$\Gamma(f_0 \to \pi\pi) \simeq (3-6)\Gamma(b_1 \to (\omega\pi)_S) \quad (2.1)$$

$$\simeq 500 - 1000 MeV \quad (2.2)$$

versus the observed partial width of 25 MeV. We have already noted the problem in the bag model $qq\bar{q}\bar{q}$ interpretation with $a_0 \to \pi\eta$. In the $K\bar{K}$ molecule picture the narrow observed widths are a natural consequence of weak binding: $(K\bar{K})_{I=0} \to \pi\pi$ and $(K\bar{K})_{I=1} \to \pi\eta$ occur slowly because the $K\bar{K}$ wavefunction is diffuse.

3) Both the f_0 and a_0 seem to bear a special relationship to $s\bar{s}$ pairs: their $K\bar{K}$ "couplings" are very large and they are observed in channels which would violate the Okubo-Zweig-Iizuka (OZI) rule [20] for an ω, ρ -like pair of states [21].

4) The $\gamma\gamma$ couplings of the f_0 and a_0 are about an order of magnitude smaller than expected for $^3P_0$ quarkonia [22], but consistent with the expectations for $K\bar{K}$ molecules [23].

Although these observations argue against the viability of the $^3P_0$ quarkonium interpretation of the $f_0(975)$ (and probably also the $a_0(980)$), they are insufficient to rule it out completely. (Moreover, a unitarized variant of the quark model [24], in which the scalar mesons are strongly mixed with the meson-meson continuum, avoids several of these problems. In addition to this conservative alternative, the recent analysis of Ref. [9] has raised the possibility that the $f_0(975)$ is really a combination of two effects, one of which is a candidate for a scalar glueball.)

The main purpose of this paper is to point out a simple (and to us unexpected) experimental test which sharply distinguishes among these alternative explanations. We show that the rates for $\phi \to f_0(975)\gamma \to \pi\pi\gamma$ and $\phi \to a_0(980)\gamma \to \pi\eta\gamma$ in the quarkonium, glueball, and $K\bar{K}$ molecule interpretations differ significantly; furthermore, the ratio of branching ratios

$$\frac{\phi \to a_0(980)\gamma}{\phi \to f_0(975)\gamma}$$

also may prove to be an important datum in that it can have a model-dependent value anywhere from zero to infinity (see Table 2)!

In the quarkonium interpretation, $\phi \to f_0(975)\gamma$ and $\phi \to a_0(980)\gamma$ are simple electric dipole transitions quite similar in character to several other measured electric multipole transitions, including not only the light quark transitions $a_2(1320) \to \pi\gamma$, $K^*(1420) \to K\gamma$, $a_1(1275) \to \pi\gamma$, and $b_1(1235) \to \pi\gamma$, but also such decays as $\chi_{c0} \to \psi\gamma$ and $\chi_{b0} \to \Upsilon\gamma$. From the comparison between theory and experiment given in Ref. [17], we expect that the quark model predictions for these processes given in Table 1 are reliable to within a factor of two. Thus if the f_0 is an $s\bar{s}$ quarkonium, the branching ratio for $\phi \to S\gamma$ would typically be of the order of 10^{-5}.

If the $f_0(975)$ is a glueball (in Ref. [9] there is a glueball component of the "S^* effect", dubbed the $S_1(991)$, which couples to $\pi\pi$ and is responsible for the resonant behaviour seen in $\pi\pi$ phase shift analyses; the other component, dubbed the $S_2(998)$, is practically uncoupled to $\pi\pi$) then one would naturally expect $\phi \to f_0(975)\gamma \to \pi\pi\gamma$ to be even smaller than in the quarkonium interpretation since the decay would be OZI-violating. The remarks made above on the strong decay widths of the quarkonium states would suggest that quarkonium - glueball mixing, through which we presume the OZI-violation would proceed, must be small for the f_0 (975) to remain narrow. Thus we can crudely estimate the glueball - quarkonium mixing angle to be less than $[\Gamma(f_0 \to \pi\pi)/\Gamma(^3P_0 \to \pi\pi)]^{\frac{1}{2}}$ so that if f_0 (975) is a glueball

$$\Gamma(\phi \to f_0\,(\text{glueball})\,\gamma) \;\leq\; \frac{\Gamma(f_0 \to \pi\pi)}{\Gamma(^3P_0 \to \pi\pi)}\Gamma(\phi \to f_0\,(\text{quarkonium})\,\gamma) \quad (2.3)$$

$$\leq\; \frac{1}{20}\Gamma(\phi \to f_0\,(\text{quarkonium})\,\gamma) \quad (2.4)$$

Thus if f_0 (975) is a glueball, this branching ratio should be more than an order of magnitude smaller than it would be to a ϕ-like quarkonium.

Table 1: ϕ photodecays to quarkonia

quarkonium	formula	ϕ branching ratio
$f_0 = \sqrt{\frac{1}{2}}(u\bar{u} + d\bar{d})\ {}^3P_0$	$0^{a)}$	$\lesssim 10^{-6}$
$f_0 = s\bar{s}\ {}^3P_0$	$\frac{4\alpha \lvert d_{f_0\phi}\rvert^2 \omega^3}{243}$	$\simeq 1 \times 10^{-5}$
$a_0 = \sqrt{\frac{1}{2}}(u\bar{u} - d\bar{d})\ {}^3P_0$	$0^{b)}$	$\lesssim 10^{-6}$

a) proceeds through $\omega - \phi$ and $f_0 - f_0'$ mixing
b) proceeds through $\omega - \phi$ mixing only

If the f_0 is a quarkonium consisting only of nonstrange flavours, with a_0 its isovector quarkonium partner, these states will be OZI decoupled in the ϕ radiative decay. The OZI-violating production rate via a $K\bar{K}$ loop, *viz.* $\phi \to \gamma K\bar{K} \to \gamma a_0$, may be calculated. This calculation reveals some interesting points of principle which shed light on the role of finite hadron size in such loop calculations; this calculation will be discussed in the next section.

Interesting questions arise in the case of $qq\bar{q}\bar{q}$ or $K\bar{K}$ bound states ("molecules"). The quark contents of these two systems are identical but their dynamical structures differ radically. The situation here has its analog in the case of the deuteron which contains six quarks but is not a "true" six-quark bound state. The essential feature is whether the multiquark system is confined within a hadronic system with a radius of order $(\Lambda_{QCD})^{-1}$ or is two identifiable colour singlets spread over a region significantly greater than this (with radius of order $(\mu E)^{\frac{1}{2}}$ associated with the interhadron binding energy E for a system of reduced mass μ). In the former case the branching ratio may be as large as 10^{-4} (see Ref. [5] and section 4); the branching ratio for a diffuse $K\bar{K}$ molecular system can be much smaller, as discussed below.

The ratio of branching ratios is also interesting. The ratio of $\Gamma(\phi \to \gamma a_0)/\Gamma(\phi \to \gamma f_0)$ is approximately zero if they are quarkonia (the f_0 being $s\bar{s}$ and the a_0 being OZI decoupled), it is approximately unity if they are $K\bar{K}$ systems, while for $q^2\bar{q}^2$ the ratio is sensitively dependent on the internal structure of the states. This sensitivity in $qq\bar{q}\bar{q}$ arises because $\phi \to S\gamma$ is an $E1$ transition whose matrix element, being proportional to $\Sigma e_i \vec{r}_i$, probes the electric charges of the constituents weighted by their vector distance from the overall centre of mass of the system. Thus, although the absolute transition rate for $S = qq\bar{q}\bar{q}$ depends on unknown dynamics, the ratio of a_0 to f_0 production will be sensitive to the internal spatial structure of the scalar mesons through the relative phases in $I = 0$ and 1 wavefunctions and the relative spatial distributions of quarks and antiquarks.

For example, suppose that the state's constituents are distributed about the centre of mass with the structure $(q\bar{s})(\bar{q}s)$, where q denotes u or d, and (ab) represents a spherically symmetric cluster. Then

$$\left\{ \begin{array}{c} f_0 \\ a_0 \end{array} \right\} = \frac{1}{\sqrt{2}}[(u\bar{s})(\bar{u}s) \pm (d\bar{s})(\bar{d}s)] \tag{2.5}$$

and the $E1$ matrix element will be

$$M \sim [(e_u + e_{\bar{s}}) \pm (e_d + e_{\bar{s}})] = e_{K^+} \pm e_{K^0}$$

and hence the ratio $\Gamma(\phi \to \gamma f_0)/\Gamma(\phi \to \gamma a_0)$ will be unity. The quarks are distributed *as if* in a $K\bar{K}$ molecular system (which is a specific example of this configuration) and only the absolute branching ratio will distinguish $q^2\bar{q}^2$ from $K\bar{K}$.

If the distribution is $(q\bar{q})(s\bar{s})$ then the matrix element

$$M \sim [(e_q + e_{\bar{q}}) - (e_s + e_{\bar{s}})] = 0 \quad .$$

Here the quark distributions mimic $\pi^0\eta$ (in the a_0) or $\eta\eta$ (in the f_0). In this case the absolute branching ratios will be suppressed. Most interesting is the case where $S = D\bar{D}$, where D denotes a diquark, *i.e.* where

$$\left\{ \begin{array}{c} f_0 \\ a_0 \end{array} \right\} = \frac{1}{\sqrt{2}}[(us)(\bar{u}\bar{s}) \pm (ds)(\bar{d}\bar{s})] \tag{2.6}$$

in which case

$$M \sim [(e_u + e_s) \pm (e_d + e_s)]$$

so that

$$\frac{\Gamma(\phi \to \gamma a_0)}{\Gamma(\phi \to \gamma f_0)} = (\frac{1+2}{1-2})^2 = 9 \quad .$$

The absolute rate in this case depends on an unknown overlap between $K\bar{K}$ and the diquark structure; nonetheless the dominance of a_0 over f_0 would be rather distinctive. For convenience these possibilities are summarised in Table 2.

3 The $K\bar{K}$ Loop Contribution to $\phi \to S\gamma$

The ϕ and the S (where $S = a_0$ or f_0) each couple strongly to $K\bar{K}$, with the couplings g_ϕ and g for ϕK^+K^- and SK^+K^- being related to the widths by

$$\Gamma(\phi \to K^+K^-) = \frac{g_\phi^2}{48\pi m_\phi^2}(m_\phi^2 - 4m_{K^+}^2)^{3/2} \tag{3.1}$$

and

$$\Gamma(S \to K^+K^-) = \frac{g^2}{16\pi m_S^2}(m_S^2 - 4m_{K^+}^2)^{1/2} \tag{3.2}$$

for kinematical conditions where the decay is allowed. Hence, independent of the dynamical nature of the S, there is an amplitude $M(\phi \to S\gamma)$ for the decay $\phi \to S\gamma$ to proceed through the charged K loop (fig. 1), $\phi \to K^+K^- \to S(\ell) + \gamma$

Table 2: some qualitative implications of $\frac{\Gamma(\phi\to a_0\gamma)}{\Gamma(\phi\to f_0\gamma)}$

scalar meson constitution	$\frac{\Gamma(\phi\to a_0\gamma)}{\Gamma(\phi\to f_0\gamma)}$	absolute branching ratios	comments
$K\bar{K}$ molecule	$1^{(a)}$	$a_0 \simeq f_0 \simeq 4\times 10^{-5}$	$K\bar{K}$ dominates loop diagrams
$q^2\bar{q}^2$: $K\bar{K}$-like "bag" $D\bar{D}$-like "bag" $(n\bar{n})(s\bar{s})$-like "bag"	$1^{(b)}$ $9^{(c)}$ –	rates probably $< 10^{-6}$, see section 5	see section 5
$(q\bar{q})^3P_0$: $f_0 = n\bar{n}$ $f_0 = s\bar{s}$	– $\simeq 0$	see Table 1	see Table 1
f_0 glueball, a_0 quarkonium	–	$\leq 10^{-6}$	see text

a) neglecting I=0, 1 mixing effects.
b) if $\frac{<r_A>}{<r_q>} \equiv x$, then this ratio is $(\frac{3}{1+2x})^2$.
c) if $\frac{<r_A>}{<r_q>} \equiv x$, then this ratio is $(\frac{3}{1-2x})^2$.

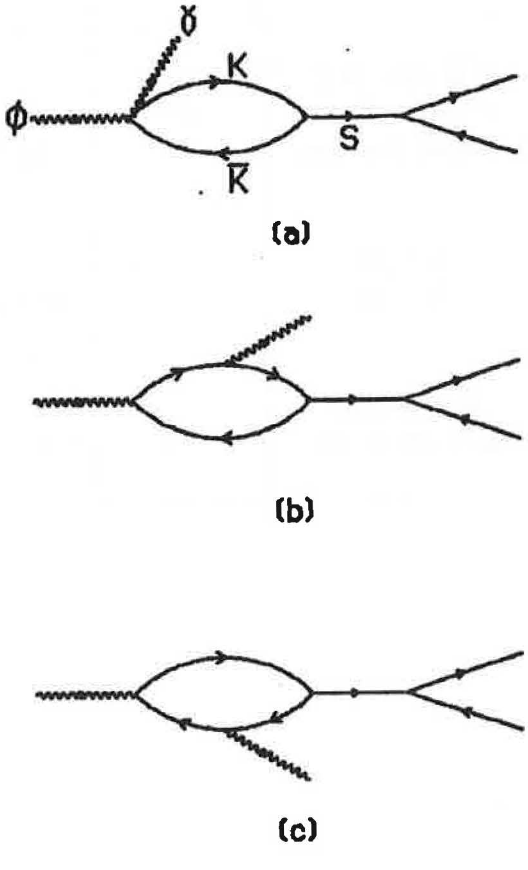


Figure 1. The contact (a) and loop radiation (b,c) contributions.

where the $K^{\pm}$ are real or virtual and S is the scalar meson with four momentum ℓ. The amplitude describing the decay can be written

$$M(\phi(p,\epsilon_\phi) \to S(\ell)+\gamma(q,\epsilon_\gamma)) = \frac{eg_\phi g}{2\pi^2 i m_K^2} I(a,b)[(p\cdot q)(\epsilon_\gamma\cdot\epsilon_\phi)-(p\cdot\epsilon_\gamma)(q\cdot\epsilon_\phi)] \quad (3.3)$$

where ϵ_γ and ϵ_ϕ (q and p) denote γ and ϕ polarisations (momenta).

The quantities a, b are defined as $a = \frac{m_\phi^2}{m_K^2}, b = \frac{\ell^2}{m_K^2}$ so that $a - b = \frac{2p.q}{m_K^2}$ is proportional to the photon energy, and $I(a,b)$ which arises from the loop integral is

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\{f(\frac{1}{b}) - f(\frac{1}{a})\} + \frac{a}{(a-b)^2}\{g(\frac{1}{b}) - g(\frac{1}{a})\} \quad (3.4)$$

where

$$\begin{aligned} f(x) &= \begin{cases} -(\arcsin(\frac{1}{2\sqrt{x}}))^2 & x > \frac{1}{4} \\ \frac{1}{4}[\ln(\frac{\eta_+}{\eta_-}) - i\pi]^2 & x < \frac{1}{4} \end{cases} \\ g(x) &= \begin{cases} (4x-1)^{1/2}\arcsin(\frac{1}{2\sqrt{x}}) & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{1/2}[\ln(\frac{\eta_+}{\eta_-}) - i\pi] & x < \frac{1}{4} \end{cases} \\ \eta_\pm &= \frac{1}{2x}(1 \pm (1-4x)^{1/2}) \end{aligned} \quad (3.5)$$

Note that ℓ^2 may in general be virtual, though we shall here concentrate on the real resonance production where $\ell^2 = m_S^2$ with $m_S \simeq 975$ or 980 MeV.

Even though Refs. [1-4] use essentially the same values for the couplings and other parameters, they obtain different results. Our results confirm those of Ref. [1] apart from a minor numerical error. Ref. [5] claims that the value of the loop calculation depends on the dynamical nature of the S. Since the coupling $S \to K\bar{K}$ is input from data it is somewhat surprising that the result can discriminate amongst models of the S. We confirm the numerical result of Ref. [5] and discuss its physical significance below.

The resonant contributions to the $\phi \to K^0\bar{K}^0\gamma$ branching fraction give a differential decay width

$$\frac{d\Gamma}{dk^2} = \frac{|I(a,b)|^2 g_\varphi^2 g^2}{4m_K^4\pi^4}\chi \quad (3.6)$$

where χ is given by

$$\chi = \frac{\alpha}{128\pi^2 m_\varphi^3}\frac{\frac{1}{3}(m_\varphi^2 - \ell^2)^3(1-\frac{4m_K^2}{\ell^2})^{1/2}}{(\ell^2 - m_S^2)^2 + m_S^2\Gamma_S^2} \quad (3.7)$$

Here ℓ^2 is the invariant mass squared of the final $K^0\bar{K}^0$ system, and hence the resonance.

The limitations and problems in the existing literature concerning attempts to calculate the above are discussed in Ref. [11]. Here we shall briefly review the loop calculation in order to assess the existing literature and to highlight the novel features of the case where the S is a $K\bar{K}$ bound state with a finite size.

Calculation of the integral $I(a,b)$

Upon making the ϕ and K interactions gauge invariant, one finds for charged kaons

$$H_{int} = (eA_\mu + g_\phi \phi_\mu) j^\mu - 2eg_\phi A^\mu \phi_\mu K^\dagger K \tag{3.8}$$

where A^μ, ϕ_μ and K are the photon, phi and charged kaon fields, $j^\mu = iK^\dagger(\vec{\partial}^\mu - \overleftarrow{\partial}^\mu)K$. If the coupling of the kaons to the scalar meson is assumed to be simply the *point-like* one SK^+K^-, then gauge invariance generates no extra diagram and the resulting diagrams are in figs. (1). Immediately one notes a problem: the contact diagram fig. (1a) diverges. The trick has been to calculate the finite sum of figs. (1b) and (1c) and then, by appealing to gauge invariance, to extract the correct finite part of fig. (1a). This is done either by

a) (Refs. [1-3]) Fig. (1a) contributes to $A^\nu \phi^\mu g_{\mu\nu}$ whereas figs. (1b) and (1c) contribute both to this and to $p_\nu q_\mu A^\nu \phi^\mu$. Therefore one need calculate only the latter diagrams, since the finite coefficient of the $p_\nu q_\mu$ term determines the result by gauge invariance.

b) (Ref. [5]) These authors compute the imaginary part of the amplitude (which arises only from figs. (1b) and (1c)) and write a *subtracted* dispersion relation, with the subtraction constrained by gauge invariance. This is also sufficient to determine the amplitude.

In section 4 we shall consider the case where the scalar meson is an extended object, in particular a $K\bar{K}$ bound state. The $SK\bar{K}$ vertex in this case involves a momentum-dependent form factor $f(k)$, where k is the kaon, or loop, momentum which will be scaled in $f(k)$ by k_0, the mean momentum in the bound state wavefunction or, in effect, the inverse size of the system. In the limit where $R \to 0$ (or $k_0 \to \infty$) we recover the formal results of approaches (a,b) above, as we must, but our approach offers new insight into the physical processes at work. In particular, in this more physical case there is a further diagram (fig. (2d))

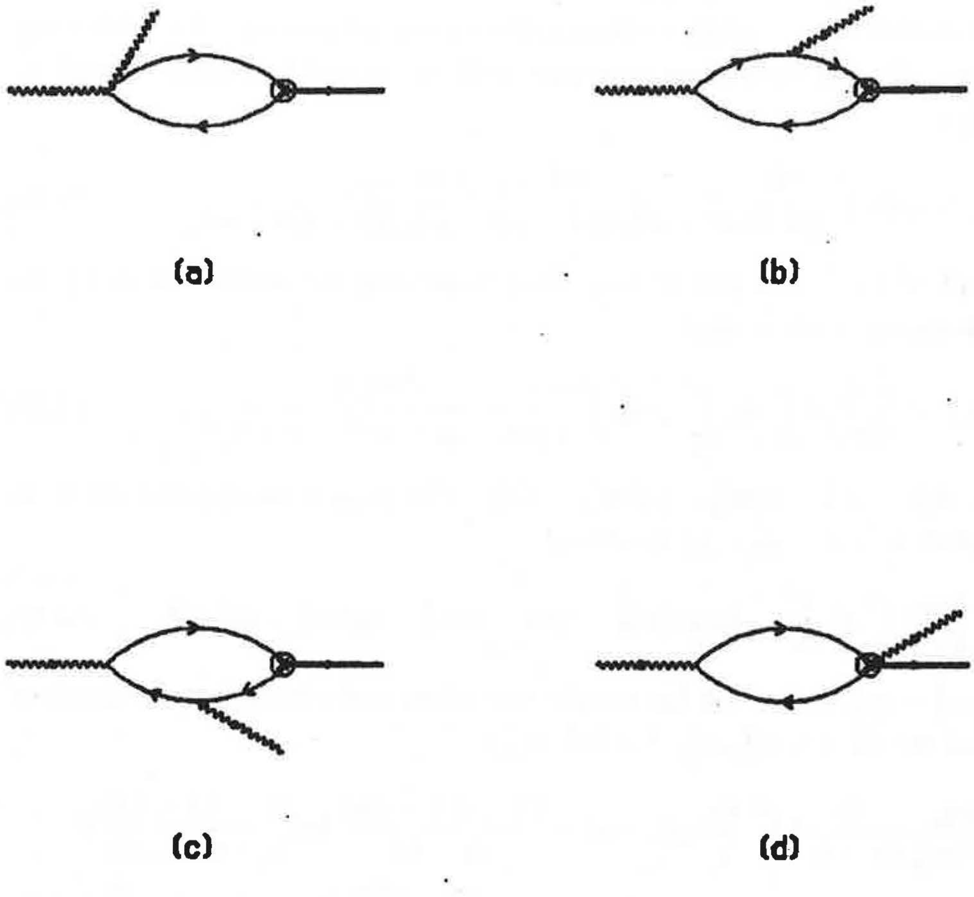


Figure 2. As fig. 1 but with an extended scalar meson. Note the new diagram (d).

proportional to $f'(k)$ since minimal substitution yields

$$f(|\vec{k} - e\vec{A}|) - f(|\vec{k}|) = -e\vec{A} \cdot \hat{k}\frac{\partial f}{\partial k} \tag{3.9}$$

As we shall see, this exactly cancels the contribution from the seagull diagram fig. (2a) in the limit where $q_\gamma \to 0$, and gives an expression for the finite amplitude which is explicitly in the form of a difference $M(q) - M(q = 0)$. This makes contact with the subtracted dispersion relation approach of Ref. [5].

First let us briefly summarise the calculation of the Feynman amplitude in the standard point-like field theory approach, as it has caused some problems in Refs. [2,3]. If we denote $M_{\mu\nu} = [p_\nu q_\mu - (p.q)g_{\mu\nu}]H(m_\phi, m_S, q)$ (see eq. (3.3)), then the tensor for fig. (3) may be written (compare with eqs. 8 and 6 of Refs. [2] and [3], respectively)

$$M_{\mu\nu} = egg_\phi \int \frac{d^4k}{(2\pi)^4} \frac{(2k-p)_\mu (2k-q)_\nu}{(k^2 - m_K^2)[(k-q)^2 - m_K^2][(k-p)^2 - m_K^2]} \tag{3.10}$$

We will read off the coefficient of $p_\nu q_\mu$ after combining the denominators by the standard Feynman trick so that

$$M_{\mu\nu} = \frac{egg_\phi}{(2\pi)^4} 8 \int_0^1 dz \int_0^{1-z} dy \int_{-\infty}^{\infty} \frac{d^4k k_\mu k_\nu}{[(k - qy - pz)^2 - c + i\epsilon]^3} \tag{3.11}$$

where $c \equiv m_K^2 - z(1-z)m_\phi^2 - zy(m_S^2 - m_\phi^2)$. The $p_\nu q_\mu$ term appears when we make the shift $k \to k + qy + pz$ to obtain

$$H = \frac{egg_\phi}{4\pi^2 i} \int_0^1 dz \int_0^{1-z} dy \; yz[m_K^2 - z(1-z)m_\phi^2 - zy(m_S^2 - m_\phi^2)]^{-1}. \tag{3.12}$$

Note that $m_S^2 < m_\phi^2$ and so one has to take care when performing the y integration. One obtains (recall $a = m_\phi^2/m_K^2, b = m_S^2/m_K^2$)

$$H = \frac{egg_\phi}{4\pi^2 i m_K^2} \frac{1}{(a-b)} \{ \int_0^1 \frac{dz}{z} [z(1-z) - \frac{(1 - z(1-z)a)}{(a-b)} \ln(\frac{1 - z(1-z)b}{1 - z(1-z)a})] \\ \frac{-i\pi}{(a-b)} \int_{1/\eta_+}^{1/\eta_-} (1 - z(1-z)a)\frac{dz}{z} \} \tag{3.13}$$

where $\eta_\pm \equiv \frac{a}{2}(1 \pm \rho)$ with $\rho \equiv \sqrt{1 - 4/a}$. (In performing the integrals, one must take care to note that $a > 4$ whereas $b < 4$ (which causes $\rho_a^2 > 0, \rho_b^2 < 0$)). Our calculation has so far only taken into account the diagram where the K^+ emits the γ; the contribution for the K^- is identical, so the total amplitude is double that of eq. (3.13) and therefore in quantitative agreement with eqs. (3) and (4) of Ref. [1]. Straightforward algebra confirms that this agrees with eqs. (9-11) of Ref. [5].

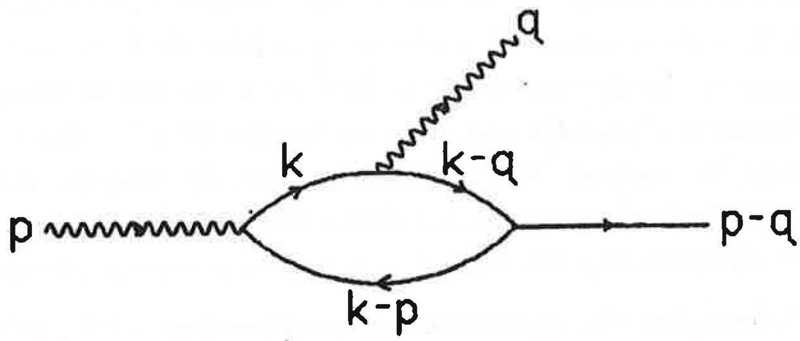


Figure 3. Momentum routing.

Numerical evaluation, using $m(f_0) = 975$ MeV and $g^2/4\pi = 0.6$ GeV^2 leads to

$$\Gamma(\phi \to f_0\gamma) = 6 \times 10^{-4} MeV \tag{3.14}$$

somewhat at variance with the value of 8.5×10^{-4} MeV quoted in Ref. [1] ‡. Ref. [5] does not directly quote a rate for $\phi \to f_0\gamma$. Instead, it quotes values for $\phi \to \gamma f_0 \to \gamma\pi\pi$ (for example) and claim that this depends upon the $q\bar{q}$ or $q^2\bar{q}^2$ structure of the f_0. However, the differences in rate (which vary by an order of magnitude between $q\bar{q}$ and $q^2\bar{q}^2$ models) *arise because different magnitudes for the $fK\bar{K}$ couplings have been used in the two cases.* In the $q^2\bar{q}^2$ model a value for $g^2(fK\bar{K})$ was used identical to ours and, if one assumes a unit branching ratio for $f_0 \to \pi\pi$, the rate is consistent with our eq. (3.14) (Ref. [5] has integrated over the resonance). In the case of the a_0, Ref. [5] notes that in the $q^2\bar{q}^2$ model the relation between $g^2(a_0K\bar{K})$ and $g^2(a_0\pi\eta)$ implies $\Gamma(a_0 \to \pi\eta) \simeq 275$ MeV. In the $q\bar{q}$ model, in contrast, Ref. [5] uses as *input* the experimental value of $\Gamma(a_0 \to \pi\eta) \simeq 55$ MeV which implies a reduced value for $g^2(a_0\pi\eta)$ and, therefore, for $g^2(a_0\bar{K}K)$: the predicted rate for $\phi \to \gamma a_0 \to \gamma\pi\eta$ is correspondingly reduced.

Thus we believe that the apparent structure-dependence of the reaction $\phi \to S\gamma$ claimed in Ref. [5] is suspect. The calculation has assumed a point-like scalar field which couples to point-like kaons with a strength that can be extracted from experiment. The computation of a rate for $\phi \to K\bar{K} \to \gamma S$ will depend upon this strength and cannot of itself discriminate among models for the internal structure of the S.

We shall now consider the production of an extended scalar meson [11] which is treated as a $K\bar{K}$ system (based on the picture developed in Refs. [15]).

4 $K\bar{K}$ loop production of an extended scalar meson

Suppose that K^+ and K^- with three momenta $\pm\vec{k}$ produce an extended scalar meson in its rest frame. The interaction Hamiltonian $H = g\phi(|\vec{k}|)SK^+K^-$ is in general a function of momentum. Now make the replacement $\vec{k} \to \vec{k} - e\vec{A}$, and expand $\phi(|\vec{k} - e\vec{A}|)$ to leading order in e; one then finds a new electromagnetic contribution

$$H_{K^+K^-f_0\gamma} = -eg\phi'(k)\hat{k} \cdot \vec{A} \ . \tag{4.1}$$

‡However, J. Pestieau, private communication, confirms our value.

The finite range of the interaction, which is controlled by $\phi(k)$, implies that the currents flow over a finite distance during the $K\bar{K} \to S$ transition: this current is the "interaction current". The above current given by minimal substitution is not unique, in the sense that the transverse part $\vec{\epsilon}_\gamma \cdot \vec{j}$ cannot be determined by the requirement of gauge invariance alone. However, it should describe the process under consideration accurately since the radiated photon is soft: the details of the interaction current are not important in the soft photon regime [25]. The effect of this form factor is readily seen in time ordered perturbation theory. (In this section we will work in the non-relativistic approximation. This suffices both to make our point of principle and to provide numerically accurate estimates for nonrelativistic $K\bar{K}$ bound states such as the f_0 and a_0 in the Ref. [15] picture. In general there are further time orderings whose sum gives the relativistic theory; see below.)

There are four contributions: ($H_{1,4}$ correspond to figs. (2a) and (2d), while $H_{2,3}$ correspond to figs. (2b) and (2c), where the γ is emitted from the K^+ or K^- leg). We write these (for momentum routing see fig. (3))

$$H_{2,3} = 2egg_\phi \int d^3k \frac{\phi(k) 2\vec{\epsilon}_\gamma.\vec{k}(\vec{k}.\vec{\epsilon}_\phi \pm \frac{1}{2}\vec{q}.\vec{\epsilon}_\phi)}{D(E)D_1 D(q\pm)} \tag{4.2}$$

$$H_1 = 2egg_\phi \int d^3k \frac{\phi(k)\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi}{D_1} \tag{4.3}$$

$$H_4 = 2egg_\phi \int d^3k \frac{\phi'(k)\vec{\epsilon}_\gamma.\hat{k}\vec{\epsilon}_\phi.\vec{k}}{D(0)} \tag{4.4}$$

where

$$D_1 \equiv m_\phi - q - D(E)$$

$$D(q^\pm) \equiv m_\phi - 2E^\pm \tag{4.5}$$

$$D(0) \equiv m_\phi - 2E(k) \tag{4.6}$$

$$D(E) \equiv E^+ + E^- \tag{4.7}$$

and where $E^\pm = E(k \pm q/2)$ with $E(P)$ the energy of a kaon with momentum P. Note that H_1 is the (form-factor-modified) contact diagram and H_4 is the new contribution arising from the extended $SK\bar{K}$ vertex.

After some manipulations their sum can be written

$$H = 2egg_\phi\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi \int d^3k [\frac{\phi(k)}{D_1}\{1 + \frac{\vec{k}^2 - (\vec{k}.\hat{q})^2}{D(E)}(\frac{1}{D(q^+)} + \frac{1}{D(q^-)})\} + \frac{\phi'(k)|\vec{k}|}{3D(0)}] \quad . \tag{4.8}$$

If $\lim_{k^2\to\infty}(k^2\phi(k)) \to 0$ [§] we may integrate the final term in eq. (4.8) by parts

[§] Actually, when $k \to \infty$ the relativistic expressions of the next subsection are needed. These show that $\phi(k)$ need only vanish logarithmically to obtain convergence.

and obtain for it

$$H_4 = 2egg_\phi \vec{\epsilon}_\gamma . \vec{\epsilon}_\phi \int d^3k \frac{\phi(k)}{D(0)} \{-1 - \frac{\vec{k}^2 - (\vec{k}.\hat{q})^2}{E(k)D(0)}\} \tag{4.9}$$

This is identical to the $\vec{q} \to 0$ limit of $H_1 + H_2 + H_3$, and hence we see explicitly that the $g_{\mu\nu}$ term (*i.e.*, the term proportional to $\vec{\epsilon}_\gamma.\vec{\epsilon}_\phi$ as calculated above) is effectively subtracted at $\vec{q} = 0$ due to the partial integration of the $\phi'(k)$ contribution, H_4.

If one has a model for $\phi(k)$ one can perform the integrals numerically. For the $K\bar{K}$ molecule, the wavefunction

$$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r} \tag{4.10}$$

is a solution of the Schrodinger equation

$$\{-\frac{1}{m_K} \frac{d^2}{dr^2} + v(r)\}u(r) = Eu(r). \tag{4.11}$$

One may approximate (see Ref. [23])

$$v(r) = -440(MeV)\exp(-\frac{1}{2}(\frac{r}{r_0})^2) \tag{4.12}$$

with $r_0 = 0.57$ fm. Equation (4.11) may be solved numerically, giving $E = -10$ MeV and a $\psi(r)$ which for analytic purposes may, as we shall see, be well-approximated by

$$\psi(r) = (\frac{\mu^3}{\pi})^{1/2} \exp(-\mu r); \quad \mu \equiv \frac{\sqrt{3}}{2R_{K\bar{K}}} \tag{4.13}$$

where $R_{K\bar{K}} \simeq 1.2fm$ (thus $\psi(0) = 3 \times 10^{-2} GeV^{3/2}$; see also Ref. [23]). The momentum space wave function that is used in our computation (see fig. (4)) is thus taken to have

$$\frac{\phi(k)}{\phi(0)} = \frac{\mu^4}{(k^2 + \mu^2)^2} \quad . \tag{4.14}$$

The rate for $\Gamma(\phi \to S\gamma)$ is shown as a function of $R_{K\bar{K}}$ in fig. (5). The nonrelativistic approximation eqs. (4.2-4.9) is valid for $R_{K\bar{K}} \gtrsim 0.3fm$ which is applicable to the $K\bar{K}$ molecule: for $R_{K\bar{K}} \to 0$ the fully relativistic formalism is required and has been included in the curve displayed in fig. 5. As $R_{K\bar{K}} \to 0$ and $\phi(k) \to 1$ we recover the numerical result of the point-like field theory, whereas for the specific $K\bar{K}$ molecule wavefunction above one predicts a branching ratio of some 4×10^{-5} (width $\simeq 10^{-4} MeV$). This is only $\frac{1}{5}$ of the point-like field theory result but is larger than that expected for the production rate of an $s\bar{s}$ scalar meson (see Tables 1 and 2).

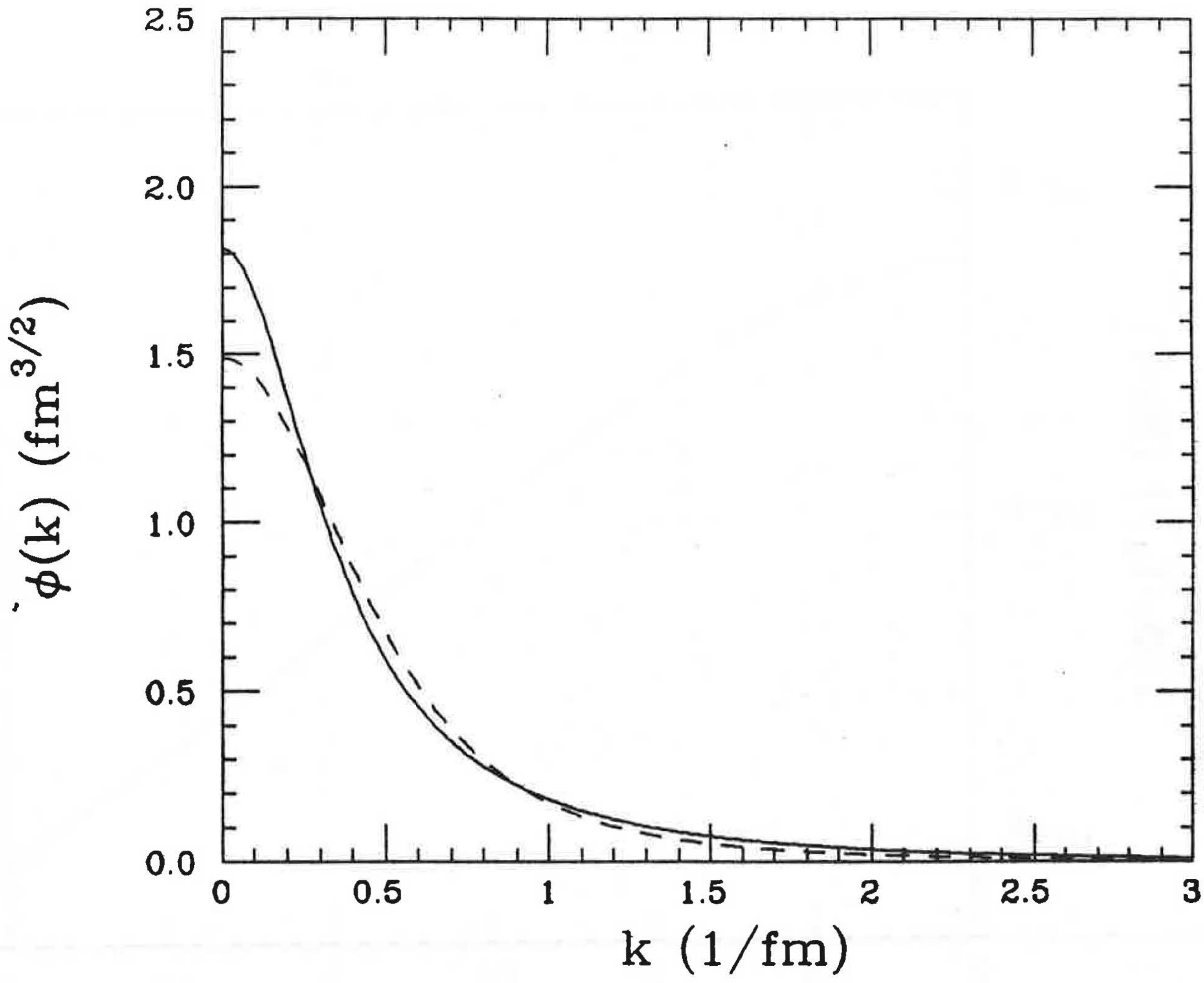


Figure 4. Comparison between the exact momentum space wavefunction $\phi(k)$ (solid) and the approximation of eq.(4.14); k is the relative momentum of the K and $\bar{K}$.

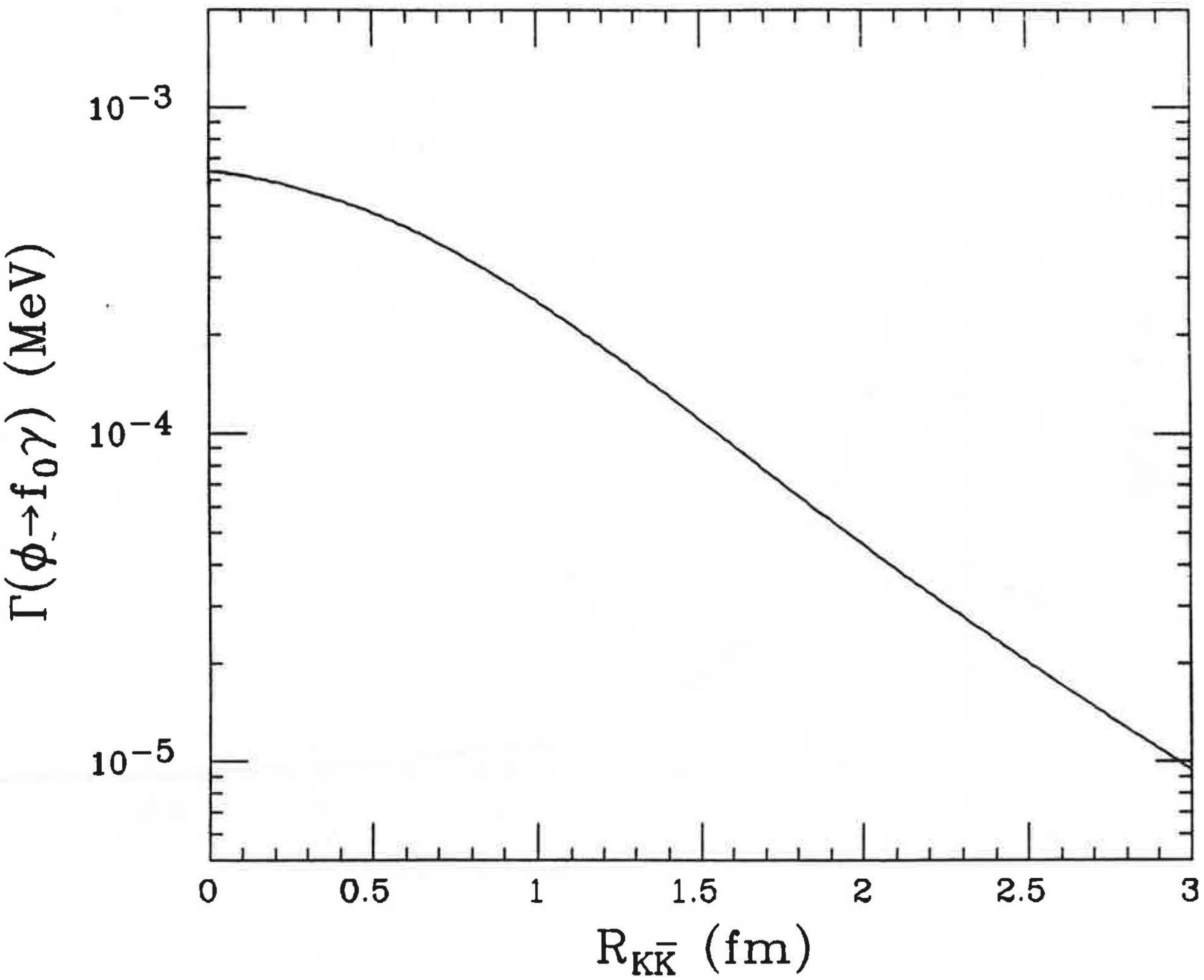


Figure 5. $\Gamma(\phi \rightarrow S\gamma)$ **versus** $R_{K\bar{K}}$.

Connection with Relativistic Field Theory

The nonrelativistic formalism is sufficient for describing the radiation from a $K\bar{K}$ molecule. However, it does not have the proper limit as $R_{K\bar{K}} \to 0$; in this limit relativistic $K\bar{K}$ pairs are important in the loop integral. In this section we show how the relativistic formalism can be obtained from time-ordered perturbation theory and make contact with the relativistic field theory formalism of section 3. The matrix elements for the time-orderings of fig. (6) are

$$M_1^\mu = +ieggg_\phi \int \frac{d^3k}{(2\pi)^3}\, \phi(|\vec{k}|)\, 2\varepsilon_\phi^\mu\, [\, -\frac{1}{2E_+ 2E_-(E_S + E_+ + E_-)} + \frac{1}{2E_+ 2E_-(m_\phi - q - E_+ - E_-)}\,] \qquad (4.15)$$

where the first (second) term corresponds to fig. (6a) (fig. (6b)) and $E_\pm$ is defined by $E_\pm = E(k \pm q/2)$. Using $E_S = m_\phi - q$,

$$-\frac{1}{m_\phi - q + E_+ + E_-} = +\frac{2E_+}{(m_\phi - q + E_-)^2 - E_+^2} - \frac{1}{m_\phi - q + E_- - E_+} \qquad (4.16)$$

and

$$\frac{1}{m_\phi - q - E_+ - E_-} = +\frac{2E_-}{(m_\phi - q - E_+)^2 - E_-^2} + \frac{1}{m_\phi - q - E_+ + E_-}, \qquad (4.17)$$

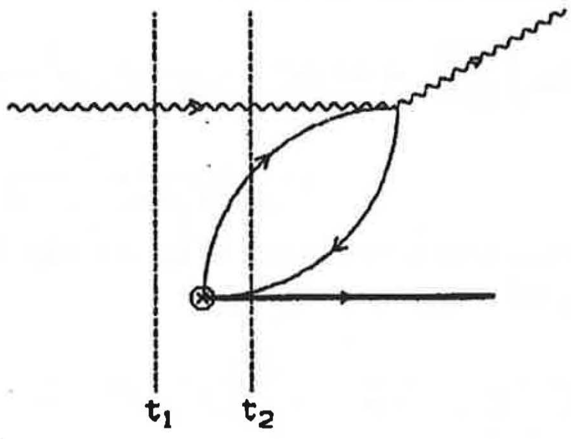
we obtain

$$M_1^\mu = +ieggg_\phi \int \frac{d^3k}{(2\pi)^3}\, \phi(|\vec{k}|)\, 2\varepsilon_\phi^\mu\, [\, \frac{1}{2E_-[(m_\phi - q + E_-)^2 - E_+^2]} + \frac{1}{2E_+[(m_\phi - q - E_+)^2 - E_-^2]}\,]. \qquad (4.18)$$

Analogously, M_2^μ, M_3^μ, and M_4^μ are

$$M_2^\mu = M_3^\mu = +ieggg_\phi \int \frac{d^3k}{(2\pi)^3}\, \phi(|\vec{k}|)\; 2\varepsilon_\phi^\mu\, [\vec{k}^2 - (\vec{k}\cdot\hat{q})^2]$$

$$\times\, [\, \frac{1}{2E_+[(q + E_+)^2 - E_-^2][(m_\phi + E_+)^2 - E_+^2]} + \frac{1}{2E_-[(q - E_-)^2 - E_+^2][(m_\phi - q + E_-)^2 - E_+^2]} + \frac{1}{2E_+[(m_\phi - E_+)^2 - E_+^2][(m_\phi - q - E_+)^2 - E_-^2]}\,] \qquad (4.19)$$



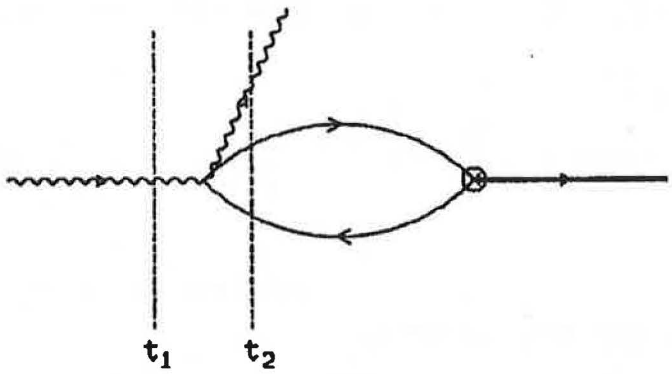


Figure 6. The two time orderings of fig. 2(a).

$$M_4^\mu = +ieggg_\phi \int \frac{d^3k}{(2\pi)^3}\, \phi'(|\vec{k}|)\, \frac{|\vec{k}|}{3}\; 2\varepsilon_\phi^\mu \left[\frac{1}{2E_0[(m_\phi + E_0)^2 - E_0^2]} + \frac{1}{2E_0[(m_\phi - E_0)^2 - E_0^2]} \right] \tag{4.20}$$

where E_0 is defined by $E_0 = E(k)$.

In this way, we obtain "relativistic" expressions for the radiative ϕ meson decays. Matrix elements for the process a–d in fig. (2) may thus be written

$$M_1^\mu = -egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{2\varepsilon_\phi^\mu}{D(k-q/2)D(k+q/2-p)} \tag{4.21}$$

$$M_2^\mu = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k+q-p)\,(2k)^\mu}{D(k+q/2)D(k-q/2)D(k+q/2-p)} \tag{4.22}$$

$$M_3^\mu = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k-q+p)\,(2k)^\mu}{D(k+q/2)D(k-q/2)D(k-q/2+p)} \tag{4.23}$$

$$M_4^\mu = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi'(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k-p)\,\hat{k}^\mu}{D(k)D(k-p)} \tag{4.24}$$

where $D(k)$ is defined by

$$D(k) = k^2 - m_K^2 \tag{4.25}$$

and $\hat{k} = (0, \vec{k}/|\vec{k}|)$. In the particular case where $\phi(|\vec{k}|) = 1$ and $\phi'(|\vec{k}|) = 0$, these reproduce the familiar field theory expressions of Refs. [1-5] and section 3. It is interesting to note the role that $\phi'(|\vec{k}|)$ plays in regularising the infinite integral.

Define the matrix elements $\tilde{M}_j$ $(j = 1-4)$ by $\tilde{M}_j = \varepsilon_\gamma \cdot M_j/[ie\varepsilon_\gamma \cdot \varepsilon_\phi]$ and the decay width is then calculated by

$$\Gamma(\phi \to S\gamma) = \frac{\alpha q}{3m_\phi^2}\, |\tilde{M}|^2 \quad , \quad \tilde{M} = \tilde{M}_1 + \tilde{M}_2 + \tilde{M}_3 + \tilde{M}_4 \tag{4.26}$$

which reproduces the expressions in Refs. [1-5] and provides a check on our formalism. Eqs. (4.21-4.24), when evaluated numerically, give the decay widths shown in Fig. 5. In the limit $R_{K\bar{K}} \to 0$ our numerical results agree with eq. (3.14) which was obtained by using the point-like field theory.

5 A Comment on the OZI Rule

The calculations presented in this paper may have a bearing on one of the least understood characteristics of the low energy strong interactions: the Okubo-Zweig-Iizuka (OZI) rule [20]. If the a_0 were a $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ state, its production in $\phi \rightarrow a_0\gamma$ would vanish in "lowest order" in the quark model, with the $K\bar{K}$ loop contribution presumed to provide a small correction since such processes are OZI-violating (*e.g.*, $\omega - \phi$ mixing could also occur *via* such loops). We have seen that in the point-like approximation $\phi \rightarrow a_0\gamma$ would proceed with a branching ratio of order 10^{-4} via this loop process, as would $f_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. If $f_0 = s\bar{s}$, a similar rate would be obtained from the $K\bar{K}$ loop, but now there would be a direct term which is supposed to be dominant. It is, however, easy to discover that this direct process would only produce a branching ratio of the order of 10^{-5} (see Table 1).

Our calculation provides some insight into this conundrum. If the $K\bar{K}$ system is diffuse, $R_{K\bar{K}} \gtrsim 2fm$, then the loop calculation gives a branching ratio $< 10^{-5}$ (see fig. (5)) and the empirical OZI rule is good. Physically, the rate is suppressed due to the poor spatial overlap between the $K\bar{K}$ system and the ϕ. The point-like field theory does not allow for this: superficially the loops have a large magnitude. The essential observation is that the point-like calculation does not take into account the confinement scale, even though it is clear from our results that the dynamics can depend on it rather critically.

Now consider a ϕ and assume that S is an $(s\bar{s})$ scalar meson, confined in $\Lambda_{QCD}^{-1} \simeq 1\ fm$ and connected by an intermediate state with quark composition $q\bar{q}s\bar{s}$. If this multiquark system is confined in a length scale $\lesssim \Lambda_{QCD}^{-1} \simeq 1fm$ (*i.e.*, it is a "genuine" $q^2\bar{q}^2$ state and separate identifiable kaons are not present), then the point-like field theory calculations, which contain no intrinsic length, are superficially at least roughly applicable. The $\phi \rightarrow \gamma S$ branching ratio *via* the $K\bar{K}$ part of this compact system is then elevated above the 10^{-5} barrier. However, if a pure $K\bar{K}$ intermediate state forms, then it must occupy $> 2\Lambda_{QCD}^{-1}$. The amplitude for the ϕ or a $S(s\bar{s})$ to fluctuate to this scale of size would be small and it is this supression that is at the root of the OZI rule in this process.

We see from this reasoning that the contribution of diagrams which correspond at the quark level to $q\bar{q}s\bar{s}$ loops really contain two distinct contributions at the hadronic level. These are first of all the diffuse contributions which can arise from hadronic loops corresponding to nearby thresholds, in this case from $K\bar{K}$. Then there are "short distance" contributions where approximating the $q\bar{q}s\bar{s}$ system as a $K\bar{K}$ system is potentially very misleading: a realistic calculation of such contributions would at least have to include a very large set of hadronic loops. A step in this direction has recently been taken in Refs. [26]. These authors have considered the loop contributions to, *e.g.*, $\omega - \phi$ mixing in the $^3P_0$ quark pair creation model, and found that there is a systematic tendency for the sum of all hadronic loops to cancel. In fact, they show that (in their model) the incompleteness of the cancellation of OZI-violating hadronic loops is precisely due to nearby thresholds.

6 Conclusions

There is still much thought needed on the correct modelling of the $K\bar{K}$ or $q^2\bar{q}^2$ scalar meson and the resulting rate for $\phi \to S\gamma$: the present paper merely makes a start by clarifying the present literature, making the first predictions for the production of a $K\bar{K}$ molecule, and pointing out the utility of the ratio of branching ratios as a filter. However, these results in turn raise questions that merit further study. For example, there are interesting interference effects possible between the $a_0(I = 1)$ and $f_0(I = 0)$ states which have not been considered. These two nearly degenerate states lie so near to the $K\bar{K}$ thresholds that the mass difference between neutral and charged kaons is not negligible: for example, their widths straddle the K^+K^- threshold but only barely cross the $K^0\bar{K}^0$ threshold (at least in the case of the relatively narrow f_0).

Although there is clearly much to be done, it is already clear that there may be unique opportunities for probing dynamics in $\phi \to S\gamma$ and investigating the nature of the scalar mesons below 1 GeV. Moreover, we can already conclude that the branching ratio of $\phi \to S\gamma$ will be between 10^{-4} and 10^{-5} depending on the dynamical nature of these scalars and so will generate nugatory¶ background to studies of CP-violation at DAΦNE or other ϕ-factories.

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¶**nūg' atory**, a. Trifling, worthless, futile; inoperative, not valid. [f. L *nugatorius (nuggari* trifle f. prec., -ORY)] [28]

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