

KAL 92037
Copy 2 R61
ACCN: 215459
RAL-92-037

Science and Engineering Research Council

Rutherford Appleton Laboratory

Chilton DIDCOT Oxon OX11 0QX

RAL-92-037

***** RAL LIBRARY R61 *****
ACC_No: 215459
Shelf: RAL 92037
R61

LIBRARY, R61
29 JUN 1992
RUTHERFORD APPLETON
LABORATORY

Analysing $\gamma\gamma \rightarrow \pi\pi$ and what it can tell us

D Morgan

June 1992

Science and Engineering Research Council

"The Science and Engineering Research Council does not accept any responsibility for loss or damage arising from the use of information contained in any of its reports or in any communication about its tests or investigations"

Analysing $\gamma\gamma \rightarrow \pi\pi$ and what it can tell us*

D MORGAN

*Rutherford Appleton Laboratory, Chilton, Didcot,
Oxon, OX11 0QX, England.*

Abstract

Among $\gamma\gamma$ reactions, the process $\gamma\gamma \rightarrow \pi\pi$ is particularly favoured for amplitude analysis. The combination of quite detailed data on both charge channels and exceptionally prescriptive theoretical constraints allows a full prediction of the low energy cross-section and a detailed amplitude analysis through the $f_2(1270)$ region. Key issues that may thereby be explored are:

- (i) the coupling to $\gamma\gamma$ of the broad $I = 0$ S -wave resonance;
- (ii) the capacity of precise low energy $\gamma\gamma \rightarrow \pi\pi$ data to refine our knowledge of the $\pi\pi$ $I = 0$ scattering length, a_0^0 .

1 Introduction

Shortly after the last two-photon meeting at Shores in 1988, much more comprehensive data on $\gamma\gamma \rightarrow \pi\pi$ became available. Based on this, Michael Pennington and I carried out a detailed amplitude analysis from threshold to 1.4 GeV¹. For our study, we used MKII's high statistics $\pi^+\pi^-$ data² and the first round of Crystal Ball data on $\pi^0\pi^0$.³ Since then, extra (normalized) data on $\pi^+\pi^-$ from CELLO has been published⁴ and further data on $\pi^0\pi^0$ will be presented at this meeting by Crystal Ball⁵. Given all this extra data, we plan to repeat our 1990 analysis. What will we be seeking to learn and how will we go about learning it?

Foremost in importance among the outputs of such analyses are the resonances widths $\Gamma(R \rightarrow \gamma\gamma)$, especially those for scalar mesons. These should distinguish different types of scalars – glueballs, molecules, regular $(q\bar{q})$ compounds – *provided simple calculations are a reliable guide*. For some time, this was in serious doubt. Since the $(q\bar{q})$ tensors give prominent signals in $\gamma\gamma$ reactions, one would expect to see comparable $(q\bar{q})$ scalar signals. Non-relativistic quark models predict reduced widths in the ratio $\hat{\Gamma}_S/\hat{\Gamma}_D \simeq 15/4$; relativistic variants⁶ bring the predicted ratio down to $\hat{\Gamma}_S/\hat{\Gamma}_D \simeq 2$. For a long time experiment saw no signal and this was advertised as a major problem⁷.

*To appear in Proceedings of Photon-photon '92 (9th International Workshop on photon-photon collisions), San Diego, California, March 22-26, 1992.

Based on a programme of work on $\gamma\gamma$ physics in collaboration with Michael Pennington.

According to our 1990 analysis, there is no problem, at least in the $I = 0$ sector; a large scalar signal above 1 GeV ($\Gamma_{f_0} \geq 5$ keV) is possible, indeed likely. Subsequent confirmation of a sizeable scalar signal have come from an analysis of the CELLO $\pi^+\pi^-$ data by Feindt and Harjes⁸ ($\Gamma_{f_0} = 5\text{-}10$ keV) and from analysis of a preliminary version⁹ of the new Crystal Ball data ($\Gamma_{f_0} = 4.3 \pm 0.8 \pm 0.6$ keV). Re-investigating the allowed range for Γ_{f_0} will be a first priority for any new analysis using data from both $\pi^+\pi^-$ and $\pi^0\pi^0$ final states. (Provided absolute normalizations are reliable, the need to fit both channels is a powerful constraint on fits to $\gamma\gamma \rightarrow \pi\pi$).

Other issues bearing on any new analysis include:

- (a) two-photon coupling of other $I = 0$ scalars, especially $f_0(975)(S^*)$. Previous measurements have yielded rather small values in keeping with the currently favoured molecule interpretation of $f_0(S^*)$ (cf. T Barnes, these Proceedings¹⁰);
- (b) the likely predominance of helicity $\lambda = 2$ in $f_2(1270) \rightarrow \gamma\gamma$ ¹¹;
- (c) low energy $\pi^0\pi^0$ and $\pi^+\pi^-$ cross-sections and their predictability from the OPE Born term modified by final state interactions (FSI). This has significant bearing on future measurements at the Frascati Φ -factory, DAΦNE¹².

2 Strategy for analysing $\gamma\gamma \rightarrow \pi\pi$

2.1 General scheme

Any amplitude analysis devoid of theory input is heavily under-determined. For quasi-real photons, the D.C.S. is the incoherent sum of $\lambda = 0$ and $\lambda = 2$ helicity contributions (the possibility of distinguishing such components is greatly enhanced when one or both ingoing photons is appreciably off-shell¹³)

$$\frac{d\sigma}{d\Omega} = \frac{\beta}{128\pi^2 s} [|M_{++}|^2 + |M_{+-}|^2] \quad (1)$$

with partial wave expansions for the helicity amplitudes

$$M_{++} = e^2 \sqrt{16\pi} \sum_{J=0(2)\infty} F_{J0}(s) Y_{J0}(\theta, \phi) \quad (2)$$

$$M_{+-} = e^2 \sqrt{16\pi} \sum_{J=0(2)\infty} F_{J2}(s) Y_{J2}(\theta, \phi). \quad (3)$$

Partial wave analysis aims to identify the separate contributions from $J = 0, 2, 4, \dots$ for λ and $I = 0$ and 2, yet the cross-section is the above incoherent sum and, furthermore, angular information is limited. One therefore needs to exploit all available constraints. Fortunately, for the reaction $\gamma\gamma \rightarrow \pi\pi$, we have two powerful circumstances working in our favour:

- (i) Dominance at low energies of the OPE Born term, and,

(ii) known and relatively simple final state interactions (FSI) in conjunction with standard S -matrix constraints.

Via the low energy theorem¹⁴, OPE dominance yields an absolute prediction for the cross-section near threshold; this furnishes a useful ‘pivot’ for our amplitude analysis providing we are fitting absolutely normalised data. Further consequences of OPE dominance are that $I = 2$ amplitudes are comparable in magnitude to $I = 0$ (the OPE Born terms are in the ratio $B^{I=2}/B^{I=0} = 1/\sqrt{2}$) and that relatively high partial waves enter at low energies.

The S -matrix constraints of unitarity and analyticity have important consequences extending up into the resonance region. Unitarity fixes the imaginary part (discontinuity) of the partial wave amplitude (PWA), F , via the relation

$$\text{Im } F = F^* \rho T \quad (4)$$

with T the associated strong interaction PWA and ρ the corresponding phase space factor. (Eq (4) is to be understood as a scalar or matrix equation as appropriate, i.e. if there is one or several channels). This equation is particularly constraining where just one channel is open (thus for $\pi\pi$ below 1 GeV) or where a single Breit-Wigner resonance dominates (as for the $I = 0$ D wave in the $f_2(1270)$ region). It then implies

$$\text{phase } (F) = \text{phase } (T) \quad (\text{the strong interaction phase shift}) \quad (5)$$

A corollary is that representing F as a sum of Born and resonance contributions is WRONG.

The precise formulation of analyticity entails writing a dispersion relation; however we may capture its essence by the schematic statement

$$\text{phase changes} \iff \text{modulus changes.} \quad (6)$$

An important consequence is that resonance poles are universal and that idiosyncratic resonance dynamics cannot occur in particular processes like $\gamma\gamma \rightarrow \pi\pi$.

To see the effect of the above two principles consider the simple case where the strong interaction final state comprises a narrow, background free, (Breit-Wigner) resonance. The $\gamma\gamma$ amplitude then takes the simple form

$$F_i = \frac{\Gamma_{\gamma\gamma}^R \Gamma_i^R / 2}{E_R - E - i\Gamma_{\text{tot}}/2} \quad (7)$$

and the sole task of $\gamma\gamma$ studies is to measure the two-photon width, $\Gamma_{\gamma\gamma}^R$. For realistic situations, governed by more complicated formulae than (7), extracting $\Gamma_{\gamma\gamma}^R$ is still the prime aim.

2.2 Note on the Strong Interaction Input

A prerequisite of the type of amplitude analysis to be described is adequate knowledge of the relevant strong interactions, here $I = 0$ and $2 \pi\pi \rightarrow \pi\pi$ and $I = 0 \pi\pi \rightarrow K\bar{K}$ etc. Most of the needed information is available and uncontraversial. Thus:

- (i) $\pi\pi$ is essentially elastic up to $K\bar{K}$ threshold (~ 1 GeV).
- (ii) Aside from a few specific details, $\pi\pi$ phase shifts are well known up to 1.4 GeV and beyond.
- (iii) Of $I = 0$ resonances and their associated parameters, $f_2(1270)$ is well determined. Similarly, for $f_0(975)$, although there are still details to settle, the major features are known. *The only serious question that confronts us is the nature and specification of the first broad $I = J = 0$ resonance:*

The latest (1990) Particle Data Tables¹⁵ lists $f_0(1400)$ with width $\Gamma=150-400$ MeV. I suggest that the data¹⁶ indicates a much lighter and broader object than PDG proposes. I base this on:

- (i) the behaviour of the phase shift, δ_0^0 , which, when one removes the rapid excursion associated with $f_0(975)$, shows a slow, steady ascent through 90° (see figure).
- (ii) Resonance pole fits¹⁷ to a comprehensive set of reactions leading to $\pi\pi$ and $K\bar{K}$ final states indicates such an object [$f_0(\epsilon)$ - mass ~ 900 , width ~ 700 MeV].
- (iii) General duality notions require that the 'effective resonance position' should equal the weighted mean of the cross-section peak.

On this basis, I advocate something like $f_0(1000)$ with width, $\Gamma, \sim 700$ MeV. It is this object along with $f_2(1270)$ and $f_0(975)$ whose $\gamma\gamma$ excitations we seek to extract.

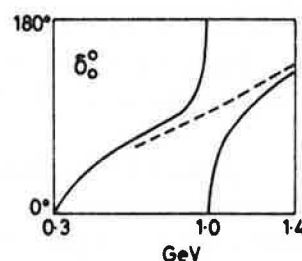


Fig 1. Form of phase-shift, δ_0^0 ¹⁶ (modulo 180°) (full line) and with rapid phase excursion from $f_0(S^-)$ removed (dashed line).

2.3 How the constraints of §2.1 restrict $\gamma\gamma \rightarrow \pi\pi$ amplitudes in various domains of energy

The above constraints operate at various levels of prescriptiveness depending on energy. Low energy $\gamma\gamma \rightarrow \pi\pi$ is essentially fixed. Among features thereby comprehended is the very different behaviour of $\gamma\gamma$ production of the respective charge channels, $\pi^+\pi^-$ and $\pi^0\pi^0$. The $\pi^+\pi^-$ process has a large low energy peak dominated by OPE whilst $\pi^0\pi^0$ has a small cross-section arising from the difference between final state interactions in the $I = 0$ and $2 S$ wave channels. Low energy $\gamma\gamma \rightarrow \pi^0\pi^0$ thus affords a rather delicate test of our understanding of these interactions within the dispersive

approach.

The reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ at low energy has also been singled out in studies of chiral perturbation theory (χ PT)¹⁸ as affording exceptionally assumption-free ('gold-plated') predictions. As we shall see, χ PT is appreciably less successful in explaining existing data than the dispersion method.

3 Detailed prediction of low energy $\gamma\gamma \rightarrow \pi\pi$ from dispersion relations¹⁹

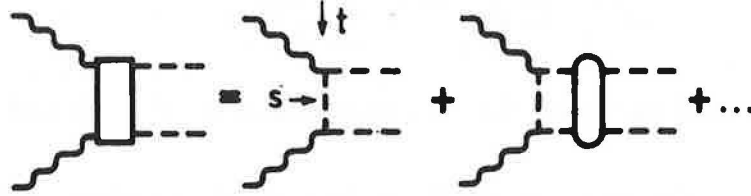


Fig 2. $\gamma\gamma \rightarrow \pi\pi$: One pion exchange (OPE) and modification from final state interactions (s and t are the usual Mandelstam variables).

The idea is to construct $\gamma\gamma \rightarrow \pi\pi$ partial wave amplitudes from a dispersion relation embodying the above principles. The key steps are as follows: Because OPE dominates t -channel exchange for small t , the near left hand cut in the s -channel is known

$$\text{Im}F^{+-} = \text{Im}B_{\pi}^{+-} \quad (s < 0) \quad (8)$$

(Throughout, F^{+-} and F^{00} signify the $\gamma\gamma$ S -wave amplitudes leading to the physical channels $\pi^+\pi^-$ and $\pi^0\pi^0$ whilst $F^I (I = 0, 2)$ denote the corresponding isospin eigenstates. B_{π}^{+-} is the one pion exchange (OPE) S -wave Born amplitude for $\gamma\gamma \rightarrow \pi^+\pi^-$. We also include effects from ρ and ω exchange – for more details, see ref [19].)

Above the s -channel threshold, the phase of F^I is fixed by unitarity

$$\text{phase}(F^I) = \delta_0^I \equiv \Phi_0^I(\text{known}) \quad (s > 4m_{\pi}^2). \quad (9)$$

From the low energy theorem¹⁴,

$$F^{+-}(s) \rightarrow B^{+-}(s), \quad F^{00}(s) \rightarrow 0 \quad (s \rightarrow 0) \quad (10)$$

Finally, PCAC²⁰ imposes Adler zeros for the physical $\gamma\gamma \rightarrow \pi\pi$ S -wave amplitudes close to $s = 0$ (again see [19] for details)

$$\begin{aligned} F^{00}(s = s_0) &= 0, \quad s_0 = 0(m_{\pi}^2) \\ (F^{+-} - B_{\pi}^{+-}) &= 0(s^2) \text{ at } s = 0 \end{aligned} \quad (11)$$

From these requirements, one can predict the low-energy cross-section by first using the Omnès construction

$$\Omega^I(s) = \exp\left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds' \Phi^I(s')}{s'(s'-s)}\right] \quad (12)$$

and then writing a twice subtracted dispersion relation

$$F^I(s) = B^I(s) + (c^I + d_s^I)\Omega^I(s) - \frac{s^2\Omega^I(s)}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds' B^I(s') \text{Im}[\Omega^I(s')^{-1}]}{s'^2(s'-s)} \quad (13)$$

with constants fixed to secure the low-energy theorem ($c^I = 0$) and the Adler zero condition.

Computing the Omnès functions, $\Omega^I(s)$, from the known phase shifts and inserting

$$B = B_\pi + B_\rho + B_\omega \quad (14)$$

one is able to predict the low energy $\gamma\gamma \rightarrow \pi\pi$ cross-section, in particular that for $\gamma\gamma \rightarrow \pi^0\pi^0$, for a range of assumptions on the input phases at low and medium energies. (For calculating the FSI modification to OPE, sensitivity to the high energy behaviour of the phases, ϕ^I , is small. Furthermore, the correlation between the $I = 0$ and 2 $\pi\pi$ scattering lengths from analyticity and crossing (as embodied in the Roy equations²¹) is assumed. See [19] for details).

Results are shown in Fig 3 and compared to the corresponding measurements from Crystal Ball³ and to the prediction from χ PT¹⁸. The dispersion prediction is seen to do appreciably better. Reasons for this are firstly that χ PT fails to do full justice to the non-linear effects of unitarity; and, secondly, that χ PT has the Weinberg value²² of 0.15 for the $I = 0$ S -wave scattering length, a_0^0 , whilst the data seemingly prefers $a_0^0 \simeq 0.2$ as in our central prediction.

Given much more precise data there is evident scope (Fig 4) for constraining the $I = 0$ $\pi\pi$ scattering length quite tightly. The Φ -factory, DAΦNE¹², under construction at Frascati, should provide such data.

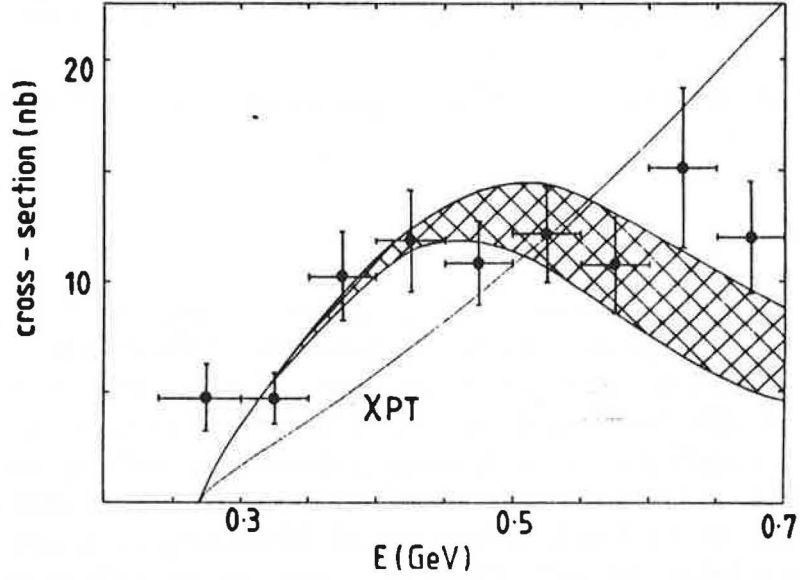


Fig 3. Low energy $\gamma\gamma \rightarrow \pi^0\pi^0$ cross-section. Crystal Ball results³ compared to representative dispersive predictions with $a_0^0=0.2$ and to lowest order chiral perturbation theory (χPT)¹⁸ (see ref [19] for details).

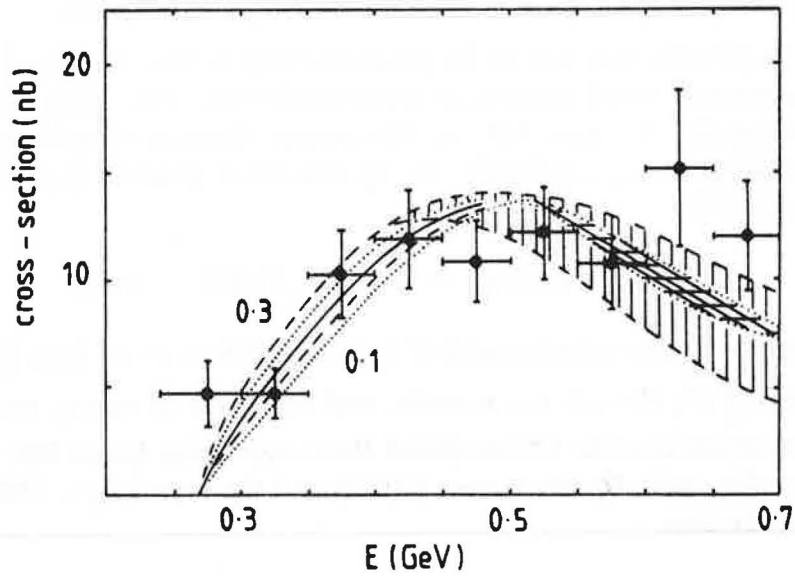


Fig 4. Low energy $\gamma\gamma \rightarrow \pi^0\pi^0$ cross-section (as in Fig 3). Crystal Ball results³ compared to dispersive predictions with a_0^0 varying in steps $a_0^0 = 0.1(0.05)0.3$ (see ref [19] for details).

4 Extending amplitude analysis into the resonance region

As one moves up in energy, unitarity is no longer quite so prescriptive but it still fixes the resonance content and phases.

4.1 Method

To extend our amplitude analysis into the resonance region, we have first to decide what partial waves to include and how to parameterize them. The scheme adopted in ref [1] was as follows: For partial waves with J greater than 2, we simply used the unmodified OPE Born amplitudes. For $I = 2$ S and D waves, we used the FSI modified Born amplitudes. In parameterizing the remaining waves, we have to cater for possible resonance contributions (all within the unitarity framework of eq (4)). For the $I = 0$ D wave we used an $f_2(1270)$ dominated form merging into the FSI modified Born expression below 600 MeV. There are in principle two helicity couplings $\lambda = 0$ and 2. The analysis of ref [1] allowed a free admixture of both, however there are growing indications that the $\lambda = 0$ component is suppressed:

- (a) Fits to CELLO results on $\gamma\gamma \rightarrow a_2 \rightarrow 3\pi^{23}$ gives $\sigma(\lambda = 0/\lambda = 2) \lesssim 0.06^{24}$.
- (b) Quark model calculations including relativistic effects⁶ indicate a large suppression. This suggests that in future (until the matter can be tested experimentally¹³) some or most fits should be made with $\lambda = 0$ de-coupling imposed.

The final amplitude that has to be parameterized is that for the $I = 0$ S - wave, in which the presently most interesting dynamical issues arise. The existence of two independent channels, $\pi\pi$ and $K\bar{K}$, in the energy domain of interest leads us to express the corresponding amplitude, F , by the most general form that unitarity prescribes, namely¹⁹:

$$F^{I=J=0} = \alpha_\pi T(\pi\pi \rightarrow \pi\pi) + \alpha_K T(K\bar{K} \rightarrow \pi\pi). \quad (15)$$

Aside from specific factors which enable F to have different zeros from \tilde{T} and to allow for zeros of $\det(\tilde{T})^{19}$, the α 's are smooth, real functions of energy required among other things to reproduce the FSI modified Born expression below 600 MeV. Such a form allows coupling of both the narrow $f_0(975)$ and the broad $f_0(\sim 1000)$ resonances if the data so indicates.

4.2 Results from ref [1] and discussion

We used the above scheme to make joint fits to the MKII $\pi^+\pi^-^2$ and Crystal Ball $\pi^0\pi^0^3$ data for different proportions of the $I = 0$ partial waves $S : D_0 : D_2$ in the f_2

region. There was a marked preference for solutions with a sizeable S -wave component. As an example, Figs 5 and 6 show the fits to the data of [2] and [3] from Solution A of ref [1] with the corresponding $I = 0$ S and D partial wave cross-sections displayed in Fig 7. This solution has $\Gamma_{f_0} \sim 5.5$ keV and $\Gamma_{f_2} \sim 2.2$ keV (cf. the current (1990) PDG average of $\Gamma_{f_2} = 2.76 \pm 0.14$ keV¹⁵). A range of possibilities is allowed for S/D but a suitably weighted sum of Γ_{f_2} and Γ_{f_0} is well determined

$$\Gamma_{f_2} + \frac{1}{4}\Gamma_{f_0} = 3.6 \pm 0.3 \text{ keV}.$$

The simultaneous requirement to fit both $\pi^+\pi^-$ and $\pi^0\pi^0$ was a key feature for constraining S/D in our analysis. It will be interesting to see how inclusion of the new Crystal Ball⁵ and the CELLO⁴ results affect our conclusions. Separate analyses of these additional data sets have already appeared, in each case finding an appreciable S -wave signal. From analysis of the CELLO $\pi^+\pi^-$ data, Feindt and Harjes⁸ find $\Gamma(f_0(1100) \rightarrow \gamma\gamma) = 5$ to 10 keV; an analysis of Crystal Ball's preliminary new $\pi^0\pi^0$ data yielded $\Gamma(f_0(1250) \rightarrow \gamma\gamma) = 4.3 \pm 0.8 \pm 0.6$ keV⁹.

Once it is established that $(q\bar{q})$ scalars are produced significantly in $\gamma\gamma$ reactions, one can use two-photon processes to explore the credibility of other $(q\bar{q})$ candidates, notably $a_0(1300)$ and $f_0(1520)$ ¹⁵, by analysing appropriate final states. Already, a rather small upper limit has been derived by Feindt²⁴ for $a_0(1300) \rightarrow \gamma\gamma$ ($\Gamma(a_0 \rightarrow \gamma\gamma) \cdot B(a_0 \rightarrow \eta\pi) < 0.44$ keV 95% CL).

Returning to analysis of $\gamma\gamma \rightarrow \pi\pi$ reactions, the other $R \rightarrow \gamma\gamma$ width one can measure is that for $f_0(975)$ (S^*) for which our analysis¹ gave

$$\Gamma_{f_0(S^*)} = 0.63 \pm 0.14 \text{ keV}$$

Strong interaction properties of S^* are assumed in extracting this signal. Such a value is more in line with the molecule than the $(q\bar{q})$ picture unless S^* is pure $(s\bar{s})$. The size of the $f_0(S^*) \rightarrow \gamma\gamma$ signal needs confirming.

What would really help to clarify the status of $f_0(975)$ (and $a_0(980)$) would be better statistics on the reactions $\gamma\gamma \rightarrow K\bar{K}$ (K^+K^- and $K^0\bar{K}^0$). To establish a measurable non-zero signal down towards threshold in both channels would tie things down greatly. (Since the respective charge final states $|K^+K^- \rangle$ and $|K^0\bar{K}^0 \rangle$ are a (constructive/destructive) superposition of $I = 0$ and 1 combined fitting of both systems would be required). Progress in understanding $a_0 \rightarrow \gamma\gamma$ is particularly desirable since, via the $\pi\eta$ mode, this is the only $\gamma\gamma \rightarrow$ scalar coupling straightforwardly visible in the data.

Acknowledgement

It is a pleasure to thank the organizers of photon-photon '92 for providing a very stimulating meeting.

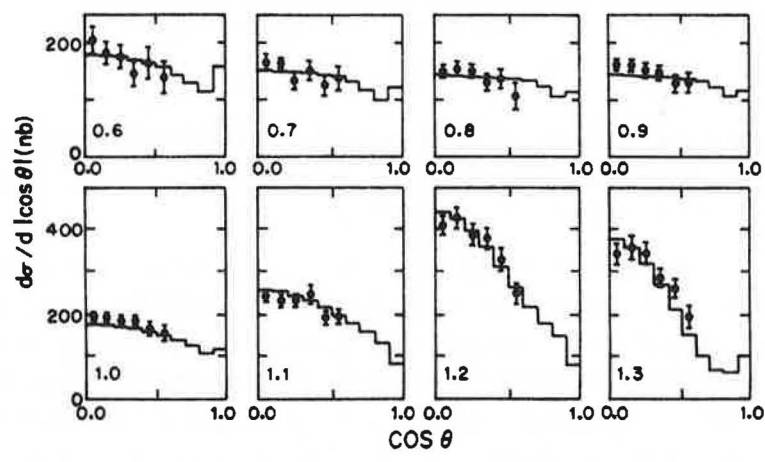
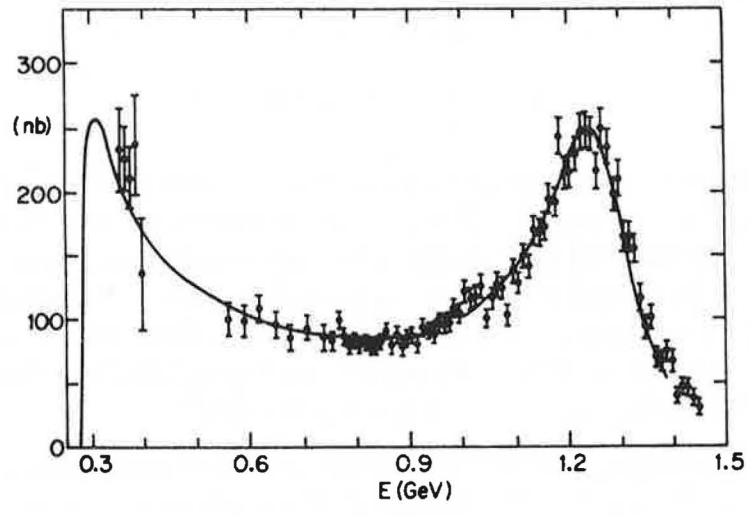


Fig 5. Amplitude analysis of $\gamma\gamma \rightarrow \pi\pi$ (ref [1]): Integrated cross-section and angular distribution from Fit A compared to $\gamma\gamma \rightarrow \pi^+\pi^-$ data from MKII².

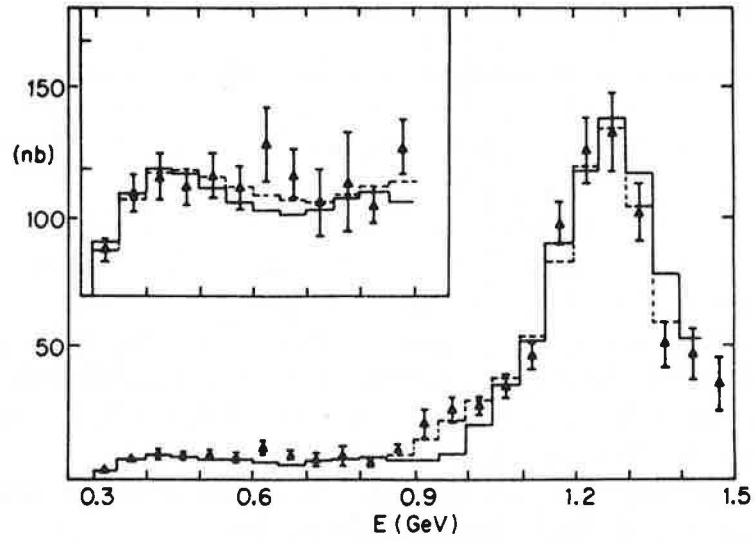


Fig 6. Amplitude analysis of $\gamma\gamma \rightarrow \pi\pi$ (ref [1]): Integrated cross-section from Fit A compared to $\gamma\gamma \rightarrow \pi^0\pi^0$ data from Crystal Ball³.

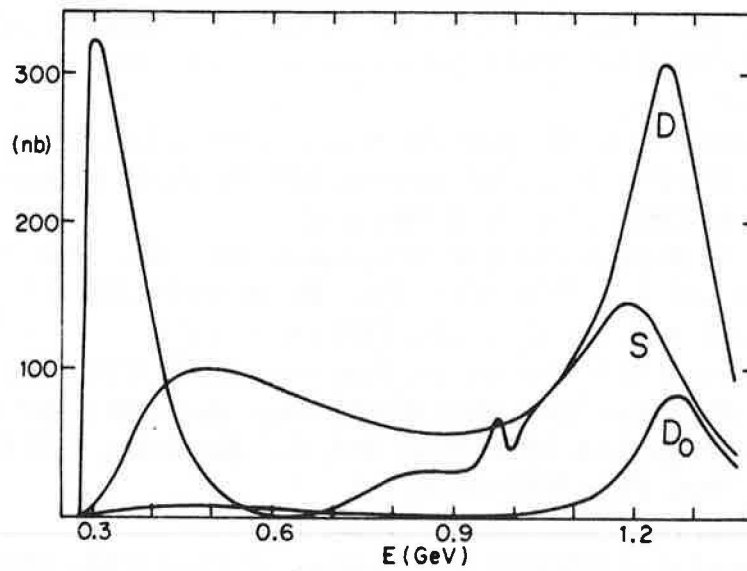


Fig 7. Amplitude analysis of $\gamma\gamma \rightarrow \pi\pi$ (ref [1]): $I = 0$ partial cross-sections corresponding to Fit A (cf. Figs 5 and 6).

References

1. D. Morgan and M.R. Pennington, *Z. Phys* **C48** (1990) 632.
2. J. Boyer et al. (Mark II collaboration), *Phys. Rev.* **D42** (1990) 1352.
3. H. Marsiske et al. (Crystal Ball collaboration), *Phys. Rev.* **D41** (1990) 3324.
4. J. Harjes, Univ. of Hamburg thesis, *DESY report* FCE-91- 01 (1991).
5. J.K. Bienlein (Crystal Ball collaboration) - these Proceedings.
6. Z.P. Li, F.E. Close and T. Barnes, *Phys. Rev.* **D43** (1991) 2161.
7. M. Chanowitz, *Proc VIII International Workshop on photon- photon collisions*, Shoresh, Israel, 1988: Ed. U. Karshon, World Scientific, 1988, p205.
8. M. Feindt and J. Harjes, Proceedings of the Rheinfels Workshop on the Hadron Mass Spectrum, St Goar, Germany, Sep 3-6, 1990, Ed. E. Klempt and K. Peters, *Nucl. Phys. B (Proc. Suppl.)* **21** (1991) 61.
9. J.K. Bienlein, *DESY report* 91-145A - to appear in Proceedings of Hadron' 91 Conference, College Park, Md., Aug 12-16 (1991).
10. T. Barnes, these Proceedings – first contribution.
11. E.S. Ackleh and T. Barnes, *Phys. Rev.* **D45** (1992) 232; T. Barnes, these Proceedings – second contribution. See also ref [6].
12. Proceedings of the Workshop on Physics and Detectors for DAΦNE (the Frascati Φ factory), Frascati, April 9-12, 1991, Ed. G. Pancheri, published by Servizio Documentazione dei Laboratori Nazionali di Frascati, P O Box 13 - I00044 Frascati, Italy.
13. P. Kessler and S. Ong, *College de France preprint*, LPC-92- 10, May 1992.
14. F.E. Low, *Phys. Rev.* **96** (1954) 1428; M. Gell-Mann and M.L. Goldberger, *Phys. Rev.* **96** (1954) 1433; H.D.I. Arbarbanel and M.L. Goldberger, *Phys. Rev.* **175** (1968) 1594.
15. J.J. Hernández et al, (Particle Data Group): *Phys. Lett.* **239B** (1990) 1.
16. W. Ochs, *Max Planck Institute preprint*, MPI-Ph/Ph 91-35, June 1991 - to appear in 'πN News Letter', Eds. G. Höhler et al.
17. K. L. Au, D. Morgan and M.R. Pennington, *Phys. Rev.* **D35** (1987) 1633.
18. J. Bijnens and F. Cornet, *Nucl. Phys.* **B296** (1988) 557; J.F. Donoghue, B.R. Holstein and Y.C. Lin, *Phys. Rev.* **D37** (1988) 2423.
19. D. Morgan and M.R. Pennington, *Phys. Lett.* **B272** (1991) 134.
20. S. Adler, *Phys. Rev.* **137** (1965) B1022, *Phys. Rev.* **139** (1965) B1638.
21. S.M. Roy, *Phys. Lett.* **36B** (1971) 352; J.L. Basdevant, C.D.Froggatt and J.L. Petersen, *Nucl. Phys.* **B72** (1974) 413.
22. S. Weinberg, *Phys. Rev. Lett.* **17** (1966) 616.
23. H.J. Behrend et al. (CELLO collaboration), *Z. Phys.* **C46** (1990) 583; see also B. Boštjančič (ARGUS collaboration) *Nucl. Phys. B (Proc. Suppl.)* **21** (1991) 92.
24. M. Feindt, Proc. III Int. Conference on Hadron Spectroscopy, Ajaccio (France), 1989, ed. F. Binon et al. (Editions Frontières, 1989) p. 133.
25. D. Antreasyan et al. (Crystal Ball collaboration), *Phys. Rev.* **D33** (1986) 1847.

