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Neutrino Masses Beyond the Standard Model*

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Abstract

We give a brief review of some aspects of the physics of massive neutrinos. In the first part, in addition to the main phenomenological consequences of lepton mixing, are shown some of the principal models of neutrino masses based on $SU(2)_L \times U(1)$, in the context of left-right symmetric models, and in the "standard" GUT's (SU(5), SO(10)). In the second part, we review the lepton sector in the framework of a particular Universal Seesaw Model which may also include the 17 keV neutrino. In the last part, some recent results obtained in the context of the "flipped" SU(5)×U(1) and the SU(4)×O(4) Superstring Inspired GUT models are discussed.

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* Commentary and Transparencies of Talk given at Rutherford Appleton Laboratory, 27 May and 3 June 1992.

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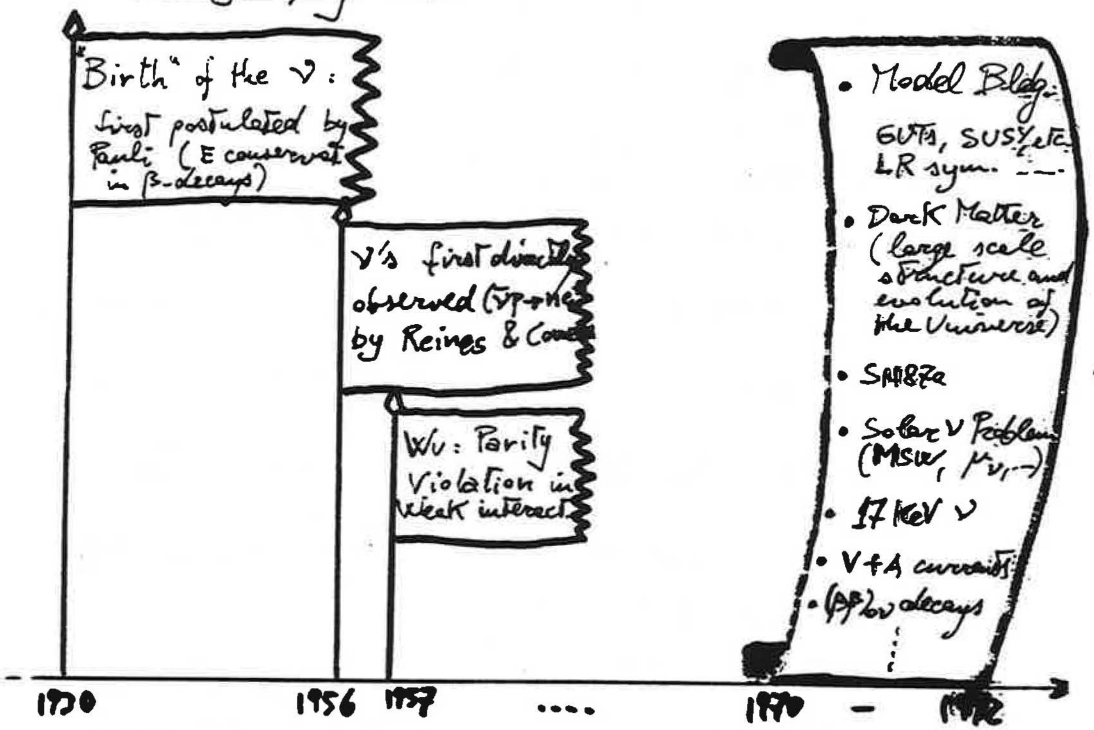
1. PRELIMINARIES

1.1) "Biography" of the Neutrino :



• Most of the present problems concerning the neutrinos are related to the main question:

- does the ν have a non-vanishing rest mass ?

... and of which type (Dirac or Majorana) ?

These questions are quite meaningful since (in spite of the exper. indications) there is not any "gauge" (i.e., local) symmetry principle ...

• Apart from this there are many indications (though not yet in the terrestrial laboratory experiments) for having $m_\nu \neq 0$:

a) Solar Neutrino Problem (MSW, μ_ν, ...) $\mu_\nu \sim 10^{-10}\mu_B$; MSW $\delta m^2 \sim 10^{-5}\,eV^2$

b) Dark Matter : (galaxy dark halo:

c) " " : M/L grows at larger scales → D.M. ($\Omega = 1$ in inflationary models, etc)

⇒ ν's dominate the dynamics of the Universe if $m_\nu \sim 10 - 50$ eV.

Other physical implications (but independent on m_ν) are :

d) ν's are responsible for the amount of [redacted] the light elements in the Universe : $He^4 \sim .25$ (with $n_\nu \approx 3$, consistent with LEP data).

e) SN87a .

• Also the distinction between Dirac and Majorana ν's is very important for most of the problems related to the ν's. Remembering that a mass term in $\mathcal{L}$ couples fields of different helicities, we may have either :

• MAJORANA (2 independ. states): $\nu = \nu^c$ $(\nu^c = C\bar{\nu}^T)$

$$\mathcal{L}^M = m_L(\nu_L^T C \nu_L + h.c.) + (L \leftrightarrow R) = m_L(\bar{\nu}^c_R \nu_L + h.c.) + (L \leftrightarrow R)$$

$$= m_L \bar{\chi}\chi + m_R \bar{\omega}\omega \,, \quad \text{where} \begin{cases} \chi = \nu_L + \nu^c_R \\ \omega = \nu_R + \nu^c_L \end{cases} \to \begin{pmatrix} \chi^c = \chi \\ \omega^c = \omega \end{pmatrix}$$

DIRAC (4 independ. states): ($\nu^c \neq \nu$)

$$\mathcal{L}^D = m_D(\bar\nu_L \nu_R + h.c.) = m_D \bar\nu\nu$$

where $\nu=\nu_L+\nu_R$ ($\nu_{L,R}=\frac{1}{2}(1\mp\gamma_5)\nu$).

In general: $D \oplus M$ ⇒

$$\mathcal{L}^{M+D} = m_D\bar\nu_L\nu_R + m_L \bar\nu_L \nu_R^c + m_R \bar\nu^c_L \nu_R + h.c. =$$

$$= (\bar\chi \;\; \bar\omega)\begin{pmatrix} m_L & m_D/2\\ m_D/2 & m_R\end{pmatrix}\begin{pmatrix}\chi\\ \omega\end{pmatrix},$$

where diagonalisation yields in general 2 Majorana mass eigenstates:

$$\begin{cases} \eta_1 = \chi\cdot\cos\vartheta - \omega\cdot\sin\vartheta \\ \eta_2 = \chi\cdot\sin\vartheta + \omega\cdot\cos\vartheta \end{cases}$$

with $\operatorname{tg} 2\vartheta = \frac{m_D}{m_R - m_L}$, and masses:

$$M_{1,2} = \frac{1}{2}\left\{(m_L+m_R) \pm \left[(m_R-m_L)^2 + m_D^2\right]^{1/2}\right\}$$

Majorana, Dirac, and lepton numbers:

- DIRAC ν's: ⇒ L_ℓ ($\ell=e,\mu,\tau$) are NOT conserved separately (because of lepton mixing) but $L=\Sigma_\ell L_\ell$ is conserved (and in particular, also B−L is conserved.

⇒

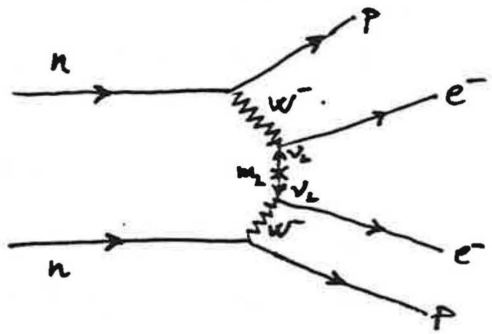
- MAJORANA ν's: ⇒ not only each L_ℓ is violated, but also the (i.e., general case) Total lepton #: $L=\Sigma_\ell L_\ell$, which is violated by 2 units ($|\Delta L|=2$), yielding, e.g. $(\beta\beta)_{0\nu}$-decays, such as:

$$(A,Z) \to (A,Z+2) + e^- + e^-$$

and processes such as:

$$K^+ \to \pi^- + e^+ + \mu^+$$

Majorana masses ⇒



$= (\beta\beta)_{0\nu}$

$(\beta\beta)_{0\nu}$ processes are characterised by the "effective" ν_e mass:

$$\langle m_\nu^{eff}\rangle_{\beta\beta_{0\nu}} = \sum_{i=1}^{n_g} [U_{1i}]^2 m_{\nu_i},$$

where U_{ij} is the [redacted] neutrino mixing matrix. Experimentally:

$$\langle m_\nu^{eff}\rangle \lesssim 0.9 \text{ eV}.$$

Lepton Mixing

- In both the Dirac and the Majorana case, there is a "lepton mixing" analogous to the quark KM mixing.

(e.g., for 3 generations):

$$\mathcal{L}_m = \left(\bar{\nu}_{iL}\,,\,\bar{\nu}^{c}_{iL}\right) \underbrace{\begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix}}_{\mathcal{M}_\nu\ (6\times 6)\ \text{"complex" symmetric matrix}} \begin{pmatrix} \nu^c_{iR} \\ \nu_{iR} \end{pmatrix} \qquad (i=e,\mu,\tau)$$

⇒ The diagonalisation of $\mathcal{M}_\nu$ through a "congruent" Transf.:

$$\tilde{V}^T \mathcal{M}_\nu \tilde{V} = diag(m_1, m_2, \ldots, m_6)\,,$$

(where $\tilde{V}^+ = \tilde{V}^{-1}$ is a 6×6 "unitary" matrix) ⇒ the standard (weak-eigenstates) ν's may be written as:

$$\nu_a = \sum_{\alpha=1}^{6} V_{a\alpha}\,\chi_\alpha \qquad \left(a=1,2,3;\ \nu_1=\nu_{eL},\ \nu_2=\nu_{\mu L},\ \nu_3=\nu_{\tau L}\right)$$

χ_α ($\alpha=1,\ldots,6$) are the "physical" mass eigenstates; and V is the 3×6 "upper-half" matrix of $\tilde{V}$, such that: $VV^+=1$ but: $V^+V\neq 1$ → whose consequence is the presence of NON-diagonal neutral currents.

If we analogously express the charged lepton weak eigenstates (e_L, μ_L, τ_L) in terms of the physical mass eigenstates e_a:

$$l_a = \sum_{b=1}^{3} \Omega_{ab}\, e_b \qquad (a,b=1,2,3)$$

($\Omega\Omega^+ = \Omega^+\Omega = 1$), then the "charged-current" lepton MIXING (à la KM) ⇒

⇒ $$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} W^-_\mu \sum_{a=1}^{3} \bar{l}_{aL}\gamma^\mu \nu_{aL} = -\frac{g}{\sqrt{2}} W^-_\mu \sum_{\substack{a=1,3\\ \alpha=1,6}} \bar{e}_{aL}\gamma^\mu K_{a\alpha}\chi_{\alpha L} + h.c.$$

where: $$K_{a\alpha} = \sum_{b=1}^{3} \Omega^+_{ab} V_{b\alpha} \qquad (3\times 6)$$

is such that: $KK^+=1$ but $K^+K\neq 1$, and is the lepton analogous to the KM matrix.

- But a new feature, present in the Majorana (or general) case, is the presence of NON-diagonal neutral current interactions for the mass eigenstate neutrinos χ_α:

$$\mathcal{L}_{NC} = -\frac{g}{4\cos\theta_W} Z_\mu \sum_{a=1}^{3} \bar{\nu}_a \gamma^\mu(1-\gamma_5)\nu_a = -\frac{g}{4\cos\theta_W} Z_\mu \sum_{\alpha,\beta=1}^{6} \bar{\chi}_\alpha \gamma^\mu(1-\gamma_5) P_{\alpha\beta}\chi_\beta$$

where: $P\ (6\times6) = V^+V \equiv K^+K \quad (\neq 1)!$ ⇔ FCNC !

$\left(P_{\alpha\beta} = \sum_{a=1}^{3} V^*_{a\alpha} V_{a\beta} \quad (\alpha,\beta=1,\ldots,6)\right)$ is such that: $\begin{cases} P^+ = P, \\ P^2 = P. \end{cases}$

Example: "NEUTRINO COUNTING" (e^+e^- exper. at $\sqrt{s} \gtrsim M_Z$ they look for single γ's of a few GeV)

(e.g. in: Giudice, S.R., Giuliani: PLB 212(88)181)

- Due to the ν-mixing, the measured "effective" # of ν's, N_ν, is not an integer but is given by:

$$N_\nu = \sum_{a,b=1}^{3} |P_{ab}|^2$$

(P_{ab} is the upper-left 3×3 block submatrix of $P_{\alpha\beta}$); ⇒

> e.g., suppose $\nu_{eL} = \chi_1$, $\nu_{\mu L} = \chi_2$; but $\nu_{\tau L}$ is mixed with a sterile state, say $\nu_{\tau R}$; that is: $V\,(3\times 6)$ is: $(c \equiv \cos\theta,\ s \equiv \sin\theta)$:

$$\tilde{V}(6\times 6) = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & c & s & & \\ & & -s & c & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix} \Rightarrow V\,(3\times 6) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & s & 0 & 0 \end{pmatrix},$$

and then:

$$P = V^+ V = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & c^2 & sc & & \\ & & sc & s^2 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}, \quad \text{which gives:}$$

$$N_\nu = \sum_{a,b=1}^{3} |P_{ab}|^2 = 1 + 1 + c^4 = 3 - 2s^2 + s^4 .$$

($\Rightarrow$ LEP sensitivity to N_ν is $\sim 15\%$).

Other Consequences of ν-mixing (and $m_\nu \neq 0$): "NEUTRINO OSCILLATIONS":

e.g., with:

$$\nu_{e(L)} = \sum_i V_{ei} \underset{\text{i-th mass eigenstate}}{\chi_i},$$

$$\chi_i(x) = \chi_i(0)\, e^{ikx} = \chi_i(0)\, e^{i(kx + Et)};$$

for $E_i = (k_i^2 + m_i^2)^{1/2} \gg m_i$ we get: $E_i t - k_i x = \frac{m_i^2}{2E} x$, $(x \simeq t)$

$\Rightarrow$ $\chi_i(t) \simeq \chi_i(0)\, e^{i m_i^2 t/2E}$

$\Rightarrow$ if $\underline{\nu(t=0) = \nu_{eL}}$ $\Rightarrow$ $|\nu(t)\rangle = \sum_i V_{ei}\, e^{i m_i^2 t/2E} |\chi_i\rangle$,

and $\Rightarrow$ $\langle \nu_\beta | \nu(t)\rangle = \ldots = \sum_i V_{ei} V^*_{i\beta}\, e^{i m_i^2 t/2E}$

$$\Rightarrow \quad P(\nu_e \to \nu_\beta\,; t \simeq x) = \sum_{i,j} V_{ei} V^*_{ej} V_{j\beta} V^*_{i\beta}\, e^{i\Delta m^2_{ij} t/2E}$$

or for the simple 2-gen. case; with $V = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$

(i.e., $\nu_e = \cos\theta\, \chi_1 + \sin\theta\, \chi_2$, $\nu_\mu = -\sin\theta\, \chi_1 + \cos\theta\, \chi_2$), $\Rightarrow$ and with $|\nu(0)\rangle \equiv \nu_{eL}$, we get:

$$|\nu(t)\rangle = \sum_i V_{ei}\, e^{i m_i^2 t/2E} |\chi_i\rangle \Rightarrow$$

$$\Rightarrow \quad \langle \nu_{\beta\neq e} | \nu(t)\rangle = sc\left(e^{i m_2^2 t/2E} - e^{i m_1^2 t/2E}\right)$$

$$\Rightarrow \quad \underbrace{P(\nu_e \to \nu_{\beta = \mu}\,; x) = \sin^2 2\theta \cdot \sin^2\!\left(\frac{\pi x}{L}\right)}_{\text{(VACUUM OSCILLATIONS)}}$$

where $\quad L = \frac{4\pi E}{\Delta m^2} \simeq 2.5 \left[\frac{E(\text{MeV})}{\Delta m^2 (\text{eV}^2)}\right]$ meters.

$(\Delta m^2 \equiv m_2^2 - m_1^2)$.

- Similarly, oscillations may occur (resonantly) in matter (as a result of a level crossing) as in the MSW (solar ν's)...

Exper. Limits on Neutrino Masses:

- Kinematical limits (end-point of β-spectrum)

$$\begin{cases} m_{\nu_e} < 18 \text{ eV} & (\text{Tritium } \beta\text{-decay}) \\ m_{\nu_\mu} < 250 \text{ KeV} & (\pi \to \mu \nu_\mu) \\ m_{\nu_\tau} < 35 \text{ MeV} & (\tau \to 5\pi + \nu_\tau) \end{cases}$$

- "Indirect" limits:

$\sum_i m_{\nu_i} (U_{1i})^2 \lesssim 0.9$ eV, $(\beta\beta)_{0\nu}$

$\sum_i m_{\nu_i} \lesssim 100$ eV (cosmology, energy...)

Models for ν-masses.

A. m_ν in $SU(2)_L \otimes U(1)$

A1) SM $\Rightarrow$ $m_\nu = 0$ (by "construction") [B-L conserved]

A2) SM $\oplus$ $N_{\ell R}$ $\quad (\ell = e, \mu, \tau)$:

("sterile" singlet states of the SM)

$$-\mathcal{L}'_Y = \sum_{\ell,\ell'} f_{\ell\ell'} \bar{\Psi}_{\ell L} \phi^c N_{\ell' R} + h.c.$$

$(\phi^c = i\tau_2 \phi^*)$ $\Rightarrow$ $M_{(DIRAC)\ell\ell'} = f_{\ell\ell'} \frac{v}{\sqrt{2}}$ $\quad (\equiv M_D)$

$\Rightarrow$ diagonalization of M_D $\Rightarrow$ DIRAC (physical) neutrinos.

$\Rightarrow$ L_e, L_μ, L_τ are NOT conserved, but $L \equiv \sum_i L_i$ is conserved (as well as B-L). (e.g.: $\mu \to e\gamma$)

$\Rightarrow$ lepton mixing in c.c. inter.'s but diagonal n.c. inter.'s.

NOTE: ... but the only way of getting light ν's is to assume incredibly small Yukawa couplings $f_{\ell\ell'}$.

A2') (A2) $\oplus$ BARE (Majorana) MASSES for the RH states $N_{\ell R}$:

$$M_R = \sum_{\ell,\ell'} M_{R_{\ell\ell'}} N^T_{\ell R} C N_{\ell' R} + h.c.$$

(violating "explicitly" B-L).

$$\Rightarrow \mathcal{M}_\nu = (\bar{\nu}_L \;\; \bar{N}^c_L) \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ N_R \end{pmatrix} + h.c.$$

$\Rightarrow$ whose diagonaliz. leads (in general) to: $2 n_g$ Majorana neutrinos.

$\Rightarrow$ both c.c. lepton mixing as well as n.c. "flavor-changing" inter.'s

- In particular, if $M_{R_{\ell\ell'}} \gg M_{D_{\ell\ell'}}$ then (with M_R non singular!):

$$M^{light}_{eff} \simeq - \underbrace{M_D^T M_R^{-1} M_D}_{\text{SEESAW term}} \qquad (n_g \times n_g)$$

e.g., with $n_g = 1$: $\quad m_1 \simeq M_D^2 / M_R$, $\quad m_2 \simeq M_R$ $\quad \begin{pmatrix} \nu_1 \simeq \nu_{eL} \\ \nu_2 \simeq \nu_{eR} \end{pmatrix}$

i.e., with $M_D \simeq m_e$ $\Rightarrow$ $m_1 \simeq m_e^2 / M_R \ll m_e$, $\quad m_2 \gg m_e$

$$\Rightarrow \underbrace{m_1 \cdot m_2 \simeq M_D^2 \sim m_e^2}$$ $\Leftrightarrow$

(SEESAW)

LIGHT ν's $\Rightarrow$ $M_R \gtrsim 10^{7-8}$ GeV $\begin{pmatrix} \to \text{"hierarchy" problem} \\ \text{in the SM; better} \\ \text{in SUSY GUT context} \end{pmatrix}$

$\Rightarrow$ NON-singular SEESAW (3 ν_R's) $\Big\}$ $\Rightarrow$ $\Big\{$ 3 extra-light Majorana ν's

A3) SM with "minimal" fermionic content but with extended Higgs sector.

$\Downarrow$

Only $\nu_{\ell L}$, $\nu^c_{\ell R}$ are present $\Rightarrow$ only B-L violating Majorana masses are possible, at the price of extending the Higgs sector.

Since the possible fermion bilinears must be made by using $\psi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \sim (2,-1)$, $\ell_R \sim e^-_R \sim (1,-2)$ and their antiparticles:

$\psi^c_R = C\gamma_0 (i\tau_2)\psi^*_L \sim (2,1)$ $\left(= \begin{pmatrix} e^+_R \\ -\nu^c_R \end{pmatrix}\right)_L$, $\ell^c_L = C\gamma_0 \ell^*_R \sim (1,2)$ $= e^+_L$.

$\Rightarrow$ 2 fermion bilinears with $B-L \neq 0$:

$$\begin{cases} \bar{\psi}_L \psi^c_R \sim (2,1) \otimes (2,1) = (3,2) \oplus (1,2) \\ \bar{\ell}^c_L \ell_R \sim (1,-2) \otimes (1,-2) = (1,-4) \end{cases}$$

$\Rightarrow$ (A3.α): $\Delta_L \sim (3,-2) = \begin{pmatrix} \Delta^0 \\ \Delta^- \\ \Delta^{--} \end{pmatrix}$; $\vec{\tau}\cdot\Delta_L \doteq \begin{pmatrix} \Delta^- & \Delta^0 \\ \Delta^{--} & \Delta^- \end{pmatrix}$

(A3.β): $h^- \sim (1,-2)$, [Zee-Models; μ_ν ---]

(A3.γ): $h^{++} \sim (1,4)$ (doubly-charged singlet)

A3.α): $\Delta_L \sim (3,-2)$

$$-\mathcal{L}'_Y = \sum_{\ell,\ell'} f_{\ell,\ell'} \bar{\psi}_{\ell L} \frac{1}{\sqrt{2}} \vec{\tau}\cdot\vec{\Delta}_L \psi^c_{\ell' R} + h.c.$$

$\left(\vec{\tau}\cdot\vec{\Delta}_L = \begin{pmatrix} \Delta^- & \Delta^0 \\ \Delta^{=} & \Delta^- \end{pmatrix} \right)$; when $\langle \phi^0_{SM} \rangle = \frac{v_2}{\sqrt{2}}$, $\langle \Delta^0_L \rangle = \frac{v_3}{\sqrt{2}}$

$$\Rightarrow \underbrace{M_L^{(Major.)} = \frac{v_3}{\sqrt{2}} f_{\ell\ell'}}$$

($\underline{v_3 \ll v_2}$ in order to preserve $\rho \equiv M_W^2 / M_Z^2 \cos^2\theta_W = 1$)

$\hookrightarrow$ [as a temptative explanation of the lightness of ν's

... but: $\rho = \frac{1+2\omega^2}{1+4\omega^2} \Rightarrow \omega \equiv v_3/v_2 < 0.07 \Rightarrow$ still too large

- B-L ("global" symm. of SM) is here "spontaneously" broken by $\langle \Delta^0_L \rangle \neq 0$ and therefore leads to a TRIPLET Majoron J (massless Goldstone boson) which is ruled out by exper. CEP.

A3.β): h^- (Zee) Model

$$-\mathcal{L}'_Y = \sum_{\ell,\ell'} f_{\ell\ell'}\, \bar{\Psi}_{\ell L} \Psi^c_{\ell' R}\, h^- + h.c. =$$

$$= \sum_{\ell,\ell'} f_{\ell\ell'}\, \Psi^T_{\ell L}\, \mathbf{C}(i\tau_2)\, \Psi_{\ell' L}\, h^+ + h.c. =$$

$$= \sum_{\ell,\ell'} f_{\ell\ell'}\, (\nu^T_{\ell L}, e^{-T}_{\ell L})\, C \begin{pmatrix} e^-_{\ell' L} \\ -\nu_{\ell' L} \end{pmatrix} h^+ + h.c. =$$

$$= \sum_{\ell,\ell'} f_{\ell\ell'} \left[\nu^T_{\ell L} C\, e^-_{\ell' L} - e^{-T}_{\ell L} C\, \nu_{\ell' L} \right] + h.c.$$

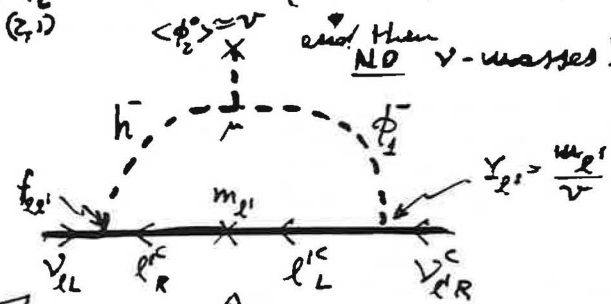
$\Rightarrow \underline{f_{\ell\ell'} = -f_{\ell'\ell}}$ $\Rightarrow$ at a single generation level there cannot be any contribution to m_ν (ν-masses are "symm."!), but O.K. for μ_ν! (Zee → SMP ...).

- Apart from that, ν-masses (Majorana) can only be generated if there are at least 2 Higgs doublets, otherwise B-L cannot be broken:

$$\mu\, \underset{(2,1)}{\phi_1^T} (i\tau_2) \underset{(2,1)}{\phi_2}\, h^- + h.c. \quad (= 0 \quad \text{if } \phi_1 = \phi_2 \text{ !})$$

and then NO ν-masses!

but if $\underline{\phi_1 \neq \phi_2}$ $\Rightarrow$



CONVERGENT (because of the Renormalizability of the Theory)

$$\underline{M_{L_{\ell\ell'}} \simeq (\text{const.})\, f_{\ell\ell'} (m^2_\ell - m^2_{\ell'})}$$

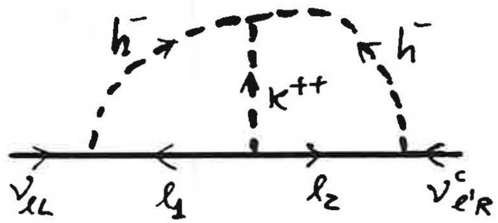
A3.γ): K^{++} (doubly-charged singlet)

$$-\mathcal{L}'_Y = \sum_{\ell,\ell'} \tilde{f}_{\ell\ell'}\, \ell^T_R\, C\, \ell'_R\, K^{++} + h.c.$$

$\Rightarrow \underline{m_\nu = 0}$ with any # of Higgs doublets, because B-L remains "unbroken".

However, if also h^- is present (Babu's model):

$\tilde{\mu}\, h^- h^- K^{++} + h.c.$ → breaks $\underline{\text{B-L}}$ and $\underline{m_\nu \neq 0}$ at the 2-loop



↳ $m_\nu \neq 0$ only at 2-loop ⇔ explanat. of lightness of ν's.

- **COMMENT:** Since $\tilde{f}_{\ell\ell'}$ is $\underline{\text{antisymmetric}}$, it turns out that, in the case of $\underline{\text{3 generations}}$, $\det M_{L_{\ell\ell'}} = 0$, implying the vanishing of the mass of one neutrino at the 2-loop level; since we expect smaller corrections from higher-loop levels, this means that (for $n_g = 3$) the model $\underline{\text{A3.γ}}$ should give:

$$m_{\nu_1} \ll m_{\nu_2} \ldots !$$

About the Majoron

• 2 Types of B−L breaking $\begin{cases} \text{explicit (in } \mathcal{L}) \\ \text{SPONTANEOUS } (\langle \text{VEV} \rangle) \end{cases}$

↓

MAJORON (J)
(massless Goldstone boson of the global B−L symmetry).

• $\begin{cases} \text{singlet Majoron (the original)} \\ \text{Triplet Majoron (Roncadelli; Gelmini)} \Rightarrow \text{ruled out at LEP (by data on Z-width)} \end{cases}$

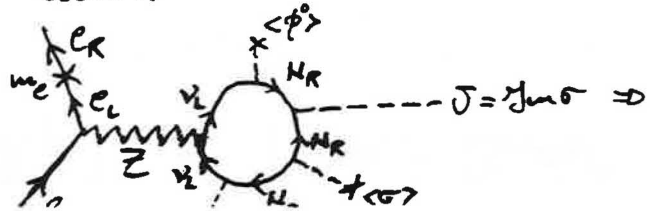
(singlet Higgs)

– Example: $-\mathcal{L}'_Y = \sum_{\ell,\ell'} Y_{R\ell\ell'} N^T_{R\ell} C N_{\ell' R}\, \sigma + h.c.$

$\Rightarrow$ if $\langle\sigma\rangle \neq 0$, then: $M_{R\ell\ell'} = Y_{R\ell\ell'} \langle\sigma\rangle$,

and: $J \simeq \mathrm{Im}\,\sigma$

• in principle, one must check that J couples very weakly to ordinary matter: e.g., the strongest bounds come from limits on the star energy loss via J-production $(\gamma + e \to e + J)$, which gives $g_{eJ} < 10^{-12}$ (Red Giants). In the case above:

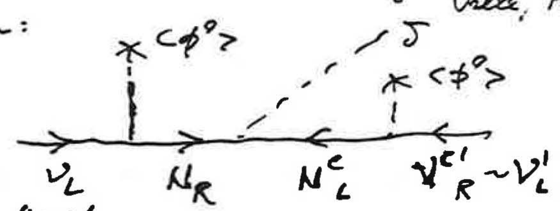


$\Rightarrow$ $g_{eJ} \doteq \frac{1}{16\pi^2} G_F m_e m_\nu$ $(m_\nu \sim M_D^2/M_R)$

$\simeq \frac{1}{16\pi^2} G_F m_e \frac{M_D^2}{M_R}$

• However, probably the main "application" of the Majoron is in allowing "fast" neutrino decays (e.g. the 17 keV neutrino; (e.g. Allen, Johnson, S.R. Schaefer, Valle, Mod. Phys. Lett. 6 (91) 1961) via the diagram:

$(\nu \to \nu' J)$



$\mathrm{Ampl.}(\nu \to \nu' J) \sim \left(\frac{M_D}{M_R}\right)^4 \sim$ very slow in general!

[The situation improves in the SINGULAR seesaw ...].

Other examples:

• $\Delta_L \to$ Triplet Majoron (Roncadelli & Gelmini) $\to$ RULED OUT at LEP

$Z \to J + \mathrm{Re}\,\Delta^0_L$ contributes as a pair of $\nu\bar\nu$!

• combination "eaten" by Z: $\frac{v_2\, \mathrm{Im}\,\phi^0 + 2 v_3\, \mathrm{Im}\,\Delta^0}{\sqrt{v_2^2 + 4 v_3^2}}$

• orthogonal combination is the (triplet) MAJORON: $J = \frac{2 v_3\, \mathrm{Im}\,\phi^0 - v_2\, \mathrm{Im}\,\phi^0}{\sqrt{v_2^2 + 4 v_3^2}} \simeq$

$\simeq -\mathrm{Im}\,\Delta^0 + \frac{2 v_3}{v_2} \mathrm{Im}\,\phi^0$

gives a direct coupling to matter (eJ)

since $g(\phi e) \sim \sqrt{2} \frac{m_e}{v_2}$ we get:

$g(eJ) \sim \sqrt{2} \frac{m_e}{v_2} \cdot \frac{2 v_3}{v_2} < 10^{-12}$

$\Rightarrow \frac{v_3}{v_2} < 10^{-6} \Rightarrow v_3 \lesssim 20\,\text{keV} \Rightarrow m(\mathrm{Re}\,\Delta^0) \sim v_3 \Rightarrow Z \to J + \mathrm{Re}\,\Delta^0$

B. Left-Right Symmetric Models.

$\not{P} \leftrightarrow$ "Spontaneous"

("exact" LR symmetry prior symmetry Breaking)

$\Rightarrow$ SM fermions $\oplus$ ν_R $\Rightarrow$

$\Rightarrow$ automatically to m_ν

$$G_{LR} = \overset{g_L = g}{SU(2)_L} \otimes \overset{g_R = g}{SU(2)_R} \otimes \overset{g'}{U(1)_{B-L}}$$

$$\Psi_{L\ell} = \begin{pmatrix} \nu_{\ell L} \\ e^-_{\ell L} \end{pmatrix} \sim (2,1,-1), \quad \Psi_{R\ell} = \begin{pmatrix} \nu_{\ell R} \\ e^-_{\ell R} \end{pmatrix} \sim (1,2,-1)$$

$(e = g \sin\theta_W = g'\sqrt{\cos 2\theta_W}\,;\ g_L = g_R = g)$

$$Q = T_{3L} + T_{3R} + \tfrac{1}{2}(B-L) \quad \Rightarrow \quad Y_{SM} = 2\,T_{3R} + (B-L)$$

$\Rightarrow$ at energies where $SU(2)_L$ is unbroken (but $SU(2)_R$ is broken), i.e. $M_{W_L} < E < M_{W_R}$: $\Delta T_{3R} = -\frac{1}{2}\Delta(B-L)$;

e.g., with $\Delta B = 0$ $\Rightarrow$ $\Delta L = 2\,|\Delta T_{3R}|$ $\Rightarrow$

$|\Delta L| = 2$, $(\Delta B = 0)$ $\Rightarrow$ $\begin{cases} \text{MAJORANA } \nu\text{'s} \\ (\beta\beta)_{0\nu} \text{ decays} \end{cases}$

(or: $\Delta L = 0 \rightarrow |\Delta B| = 2 \Rightarrow n\bar{n}$ oscillations).

S.S.B.: $G_{LR} \xrightarrow{M_R} SM \xrightarrow{M_L} SU(3)_c \otimes U(1)_{em}$ in 2 possible ways:

$\begin{cases} \text{a)}\ \Delta_L \sim (3,1,2),\ \Delta_R \sim (1,3,2);\ \phi(2,2,0) = \begin{pmatrix} \phi^0 & \bar\phi^+ \\ \phi^- & \bar\phi^0 \end{pmatrix} \\ \text{b)}\ \phi_L \sim (2,1,1),\ \phi_R \sim (1,2,1)\ \therefore \text{ odd-parity singlet } \sigma \sim (1,1,0) \text{ (breaking à la CMP)} \end{cases}$

$\Rightarrow$ $\begin{cases} \text{ⓐ more common : better for } \nu\text{-seesaw} \\ \text{ⓑ better for USM-Type of models } (\rightarrow \text{in 2nd part}) \end{cases}$

We shall examine here the "standard" choice ⓐ :

$$G_{LR} \xrightarrow{\langle\Delta_R\rangle = v_R} SM \xrightarrow{\langle\phi\rangle = \kappa,\kappa' \ll v_R} U(1)_{em} \qquad (v_L \ll \kappa,\kappa' \ll v_R)$$

(because of $\rho \simeq 1$)

$\langle\phi\rangle = \begin{pmatrix} \kappa & \\ & \kappa' \end{pmatrix}$ $\quad (\kappa,\kappa' \sim M_{W_L})$

- $-\mathcal{L}'_Y = \sum_{\ell,\ell'} Y_{\ell\ell'}\,\bar\Psi_{\ell L}\,\phi\,\Psi_{\ell' R} + h.c. \Rightarrow$ $\begin{cases} \text{DIRAC masses to all fermions} \end{cases}$

- c.c.: $\begin{cases} W_1 = W_L \cos\zeta + W_R \sin\zeta \quad (\simeq W_L) \\ W_2 = -W_L \sin\zeta + W_R \cos\zeta \quad (\simeq W_R) \end{cases}$

$$\mathrm{tg}\,2\zeta = \frac{2\kappa\kappa'}{v_R^2 - v_L^2} \simeq \frac{2\kappa\kappa'}{v_R^2} \ll 1.$$

- n.c.: $(A, Z_L, Z_R) \rightarrow \begin{cases} \gamma: \ m_\gamma = 0 \\ Z_1 \simeq Z_L \\ Z_2 \simeq Z_R \end{cases}$

Present bounds (accuracy on neutral current data) $\Rightarrow$

$\Rightarrow$ $M_{Z_2} \gtrsim 370$ GeV; $\begin{cases} M_{W_2} \gtrsim 432 \text{ GeV, (for arbitrary } \zeta) \\ \zeta \lesssim 0.035, \ (\text{for } M_{W_R} \to \infty) \end{cases}$

- SN87a : $\begin{cases} M_{W_R} \gtrsim 22 \text{ TeV} \\ \zeta \lesssim 10^{-5} \end{cases}$ (but, perhaps, still controversial)

B1. The "Seesaw" Mechanism in LR symm.

lightness of ν's $\Leftrightarrow$ SEESAW

$$-\mathcal{L}_Y = h\,\bar{\Psi}_L \phi \Psi_R + \tilde{h}\,\bar{\Psi}_L \tilde{\phi} \Psi_R + f\left[\Psi_L^T C (i\tau_2)\vec{\tau}\cdot\vec{\Delta}_L \Psi_L + (L\to R)\right] + \text{h.c.}$$

$\Rightarrow \quad \langle \Delta_R \rangle = v_R \quad \Rightarrow \quad \underline{M_R = f\, v_R}$ (heavy),

$\Rightarrow \quad \langle \phi \rangle = \begin{pmatrix} \kappa & \\ & \kappa' \end{pmatrix} \quad \Rightarrow \quad \underline{m_D = h\kappa + \tilde{h}\kappa'}$,

$\langle \Delta_L \rangle = v_L \quad \Rightarrow \quad \underline{m_L = f\, v_L \ll m_D \ll M_R}$

$\Rightarrow$ $2n_g \times 2n_g$ (complex symmetric) mass matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$\Rightarrow \quad m_\nu^{light} \simeq \underset{\substack{\text{mass-triplet} \\ \text{term} \propto v_L}}{m_L} - \underbrace{m_D M_R^{-1} m_D^T}_{\substack{\text{"SEESAW"} \\ \text{term}}}$$

$\Rightarrow$ tipically: $\quad m_{\nu_\ell}^{light} \simeq \dfrac{m_\ell^2}{M_{N_\ell}}$

e.g., if $M_{N_\ell} = M_R$ (independent on generation), then:

$$\underline{m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_e^2 : m_\mu^2 : m_\tau^2}$$

$\Rightarrow \quad \underline{\underline{v_R \to \infty}} \quad \Rightarrow \quad \underline{\underline{m_\nu \to 0}} \quad \Leftrightarrow$ smallness of $\underline{\underline{V+A}}$ c.c. $\Leftrightarrow$

$\Leftrightarrow$ lightness of neutrinos.

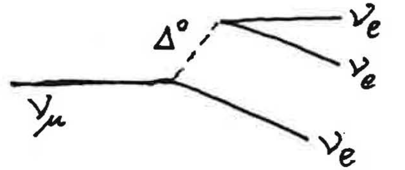
e.g. if $\underline{v_R \sim \text{TeV}} \Rightarrow m_\nu \sim 10\,\text{eV}$ (ν_e), $\sim 100\,\text{KeV}$ (ν_μ), $\sim 100\,\text{MeV}$ (ν_τ)

therefore (since cosmology: $\Sigma m_\nu < 100\,\text{eV}$) $\Rightarrow \nu_\mu\, \nu_\tau$ must decay! $\boxed{m_\nu \left(\frac{\tau_\nu}{}\right)^{1/2} < 10^{..}}$

$\Rightarrow$ but: $\quad \nu_\mu \to \nu_e \gamma$ is too slow: ($\sim 10^{21}$ sec instead of $< 10^{11}$ sec!)

Other possibilities:

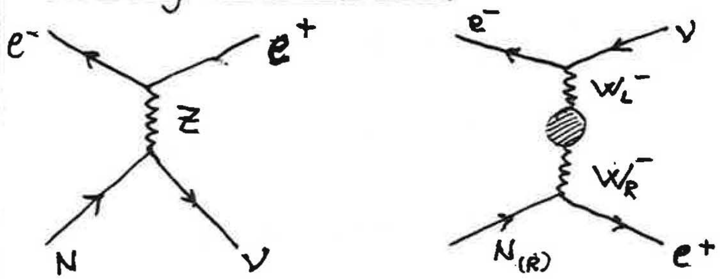
e.g. ⓐ $\underline{\nu_\mu \to 3\nu_e}$ ..

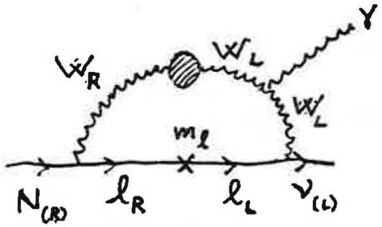


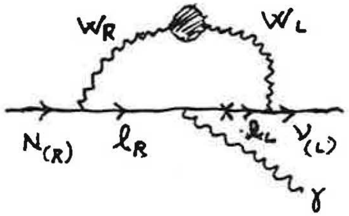
which however is connected to the process ⓑ. $\mu \to 3e$ (exp.: <10

but, "in general", ⓐ $\Rightarrow M(\Delta^0) \lesssim 100$ GeV, which $\Rightarrow$ ⓑ too fast, (since Δ^{++}-Δ^0 splitting $\simeq O(M_W)$, since they belong to the same $SU(2)_L$-multiplet). There are, however, some ways out of this problem.

- Decays of N_R's:







Problem of "naturalness" of Seesaw:

$\Rightarrow$ Even if $v_L \equiv \langle \Delta_L^0 \rangle = 0$ at the tree-level,

a $v_L \neq 0$ is induced radiatively at 1-loop:

$$v_L^{rad} \simeq \gamma \frac{K^2}{v_R}$$

(γ is a combinat. of Higgs potential parameters).

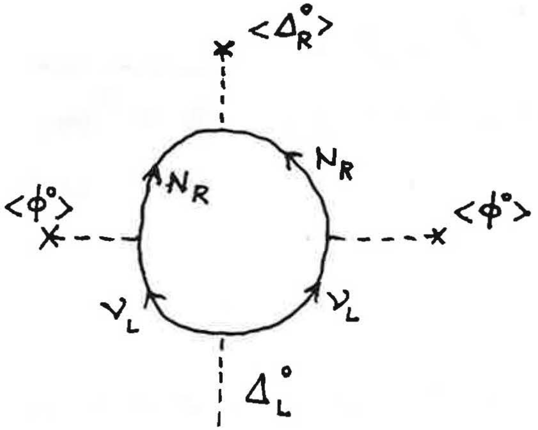
leading to a: $m_L \simeq f v_L^{rad} \simeq f \gamma \frac{K^2}{v_R}$, i.e.,

$$\Rightarrow \underline{m_{\nu_L} = \gamma \frac{f K^2}{v_R} - \frac{m_D^2}{f v_R}}$$

; therefore, with $v_R \sim 10$ TeV, $f \sim 10^{-2}$, one must require: $\gamma < 10^{-7}$ (naturalness problem)

- This problem can be solved by introducing an odd-parity real scalar field σ ($\langle \sigma \rangle = M_P \gg M_{W_R}$) $\Rightarrow$ "discrete" Parity breaking scale

then: $\Rightarrow$ $v_L \simeq \gamma K^2 \frac{v_R}{M_P^2}$ is naturally very small (explanation more natural in SO(10))



C. Neutrino Masses in GUTs

("old"-fashion GUTs): $G = SU(5), SO(10), E_6$

C1) SU(5) $\xrightarrow{\Phi = 24}$ SM $\xrightarrow{H = \bar{5}}$ $SU(3)_c \times U(1)_{em}$

24 gauge bosons: $A_\mu = \underset{G_\mu}{(8,1)_0} \oplus \underset{X,Y}{(3,2)} + \underset{\bar X, \bar Y}{(\bar 3, 2)} + \underset{W_\mu^i}{(1,3)} + \underset{\tilde B}{(1,1)}$

$$\psi_a = \bar 5 = (\bar 3, 1)_{2/3} + (1,2)_{-1} = \begin{pmatrix} d^c \\ e^- \\ \nu \end{pmatrix}_L \qquad (a = 1, \dots, 5)$$

$$\psi^{ab} = -\psi^{ba} = 10 = (3,2)_{1/3} + (\bar 3, 1)_{-4/3} + (1,1)_2 = \begin{pmatrix} & u^c & -u^c & u & d \\ & & u^c & u & d \\ & & & u & d \\ & & & & e^c \\ & & & -e^c & \end{pmatrix}_L$$

- at M_G: $g_1 = g_2 = g_3 = g$ $\Rightarrow$ $\underline{M_G \simeq 10^{15}\ \text{GeV}}$, ($g_{M_G} \simeq 0.02$)

$$\langle \Phi \rangle = \text{diag}(1, 1, 1; -\tfrac{3}{2}, -\tfrac{3}{2}) \cdot V, \quad \Rightarrow M_X^2 = M_Y^2 = \frac{25}{8} g^2 V^2$$

($\sin^2\theta_W|_{M_G} = 3/8$)

$$\langle H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v/\sqrt{2} \end{pmatrix}, \quad \Rightarrow M_W = \frac{g v}{2}, \quad M_Z = \frac{g v}{2 \cos\theta_W}.$$

$\Rightarrow$ $\tau_p(p \to e^+ \pi^0) \simeq \dfrac{192 \pi^3 M_G^4}{\alpha_G^2 m_p^5} \sim 10^{31}$ yrs $\Rightarrow$ "minimal" SU(5) is ruled out by experiments

• Neutrino Masses in SU(5).

$$\mathcal{L}_Y = h_1 \Psi^{Tab} C \Psi_a H_b + h_2 \varepsilon_{abcde} \Psi^{Tab} C \Psi^{cd} H^e + h.c.$$

$$\Rightarrow \langle H_a \rangle = v/\sqrt{2} \cdot \delta_{a5} \Rightarrow M_u = h_2 \frac{v}{\sqrt{2}}, \ \underbrace{M_d = M_\ell}_{\Downarrow} = h_1 \frac{v}{\sqrt{2}} \text{ at } M_G$$

giving: $\begin{cases} m_b = r\, m_\tau \\ m_s = r\, m_\mu \\ m_d = r\, m_e \end{cases}$ at low-E $(r \simeq O(3))$

BUT:

$\underline{m_\nu = 0}$ (because there are not ν_R and B−L is a good global symmetry).

• The simplest way of getting $m_\nu \neq 0$ in SU(5) is (apart from considering the "FLIPPED" case) via the introduction of a 15-dim. Higgs $S_{ab} = S_{ba}$ (sym.):

$$\to \mathcal{L}'_Y = f\, \Psi^T_a C \Psi_b S^{ab} + h.c.$$

$$\Rightarrow \text{since } S_{ab} \supset \Delta_L(3,2) \Rightarrow \langle S_{55} \rangle = \frac{u}{\sqrt{2}} \Rightarrow \begin{cases} m_{\nu_L} = f \frac{u}{\sqrt{2}} \\ (\text{Majorana}) \\ \oplus \\ \text{Triplet Majoron} \\ (\text{ruled out at LEP}) \end{cases}$$

C2) SO(10)

• it incorporates **L-R** symmetry;

⇒ "natural" $m_\nu \neq 0$ (both D. & M. mass terms)

⇒ "B−L" is a gauged symmetry ⇒ NO Majoron

• There are several possible "chains" of SSB down to the SM ⇔ therefore it does not give in general UNIQUE predictions.

Fermions:

$$\Psi \sim \underline{16} = 10 \oplus \bar{5} \oplus 1 \quad (\text{under } SU(5))$$

$$1 \to \nu^c_L \sim \nu_R \quad (\sim (1,2) \text{ of } SU(2)_L \otimes SU(2)_R)$$

$$= (4,2,1) \oplus (\bar{4},1,2) \quad (\text{under } G_{PS} = SU(4) \otimes SU(2)_L \otimes SU(2)_R)$$

• Since $16 \otimes 16 = 10_s \oplus 120_A \oplus 126_s$ the most general Higgs sector contains all these three multiplets:

$\underline{10 \to S}$, $\underline{126 \to S'}$, $\underline{120 \to A}$

(A, S, S' are $n_g \times n_g$ Yukawa matrices times VEVs)

[Johnson, S.R., Schechter: PL 179B(86)355, PRD 35 (87) 282, PRD 37 (88) 2597

$$\Rightarrow \begin{cases} M_u = S + \varepsilon S' \\ M_d = \alpha S + S' + \delta A = \alpha M_u + (1 - \alpha\varepsilon) S' + \delta A \\ r M_e = \alpha S - 3S' + A = \alpha M_u - (3 + \alpha\varepsilon) S' + A \\ s M_\nu^D = S - 3\varepsilon S' = M_u - 4\varepsilon S' \end{cases}$$

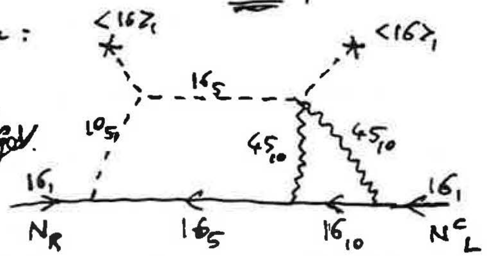
⇒ • "10" (S) reproduces the mass relations of the Standard SU(5): $m_{d_i} = m_{l_i}$ at M_G, plus: $m_\nu^D = m_u$ (at M_G).

• "126" (S') modifies these mass relations and furnishes the source of L and R Majorana masses for the ν's:

$$\begin{cases} M_R \sim \langle SU(2)_L\text{-singlet in } \underline{126} \rangle = \gamma S' \\ m_L \sim \langle SU(2)_L\text{-Triplet in } \underline{126} \rangle = \beta S' \end{cases};$$

so that the 126 is also essential for producing the mass suppression of the ν's à la "seesaw", avoiding unplausible Dirac ν's with $m_{\nu_i} \sim m_{u_i}$!

Comment: even if the 126 were absent, it is still possible (only in the NON-SUSY case ! → [[BAHEZ]) to generate large radiative M_R, via the Witten Mechanism:



$$M_{N(R)} \sim \frac{f_{10}\, v_{BL}^2\, g^4}{(16\pi^2)^2\, M_G} \sim 10^{-7} M_G \sim 10^8 \text{ GeV}$$

⇒ In any case, in general $\underline{\mathcal{M}_\nu}$ takes the form:

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \simeq \begin{pmatrix} \beta S' & m_D \\ m_D^T & \gamma S' \end{pmatrix},$$

$$\Rightarrow \quad m_\nu^{light} \simeq \underbrace{\beta S'}_{\text{"Triplet"-term}} - \frac{1}{\gamma} \underbrace{m_D S'^{-1} m_D^T}_{\text{"seesaw" term}}$$

$$\Rightarrow \text{in general: } \begin{cases} 3 \text{ superlight } \nu\text{'s } (\simeq \nu_{iL}), \\ 3 \text{ superheavy } \nu\text{'s } (\simeq \nu_{iR}). \end{cases}$$

But in 1986 we introduced the idea of the "SINGULAR" SEESAW [Johnson, S.R., Schechter: PL 179B (86) 355, " " " PRD35 (87) 282, Grossman, J., S.R., S., PRD37 (88) 2597, Giudice, Giuliani, S.R., PL 212B (88) 181]. $\det M_R = \gamma \det S' = 0$, in the context of SO(10)-"Fritzsch-Stech" model. There, we first suggested that this kind of seesaw could explain the 17 KeV Simpson neutrino, an idea "used" by Glashow four years later! [See also, [Allen, J., S.R., S., Valle, Mod. Ph. Let. 6 (91) 1967]

↓

The "singularity" of M_R seemed to be suggested by the charged fermion sector: in fact, the eqs. given in the last Transparence, using (assumed) the FRITZSCH form of the Yukawa's give:

$$\det S' = 0 \iff S'_{33} = \mathrm{Tr}\, S' = \frac{1}{4} \mathrm{Tr}(M_d - r M_e) = 0$$

i.e.: $\underline{\det M_R = 0}$ for: $r = r^* = \dfrac{\mathrm{Tr}\, M_d}{\mathrm{Tr}\, M_e} = \dfrac{m_b - m_s + m_d}{m_\tau - m_\mu + m_e} \simeq \dfrac{m_b}{m_\tau} + \epsilon \ldots$

that is $r^* \simeq r$ (calcul. from RGE) $\simeq 3.0$!

⇒ So the [illegible]

· Then, it is easy to show that $\underline{\det M_R = 0}$ results in ⇒

$$\begin{cases} \text{only } 2 \text{ super-light } \nu\text{'s} \\ \quad '' \quad 2 \text{ super-heavy } \nu\text{'s} \\ \qquad \oplus \\ 2 \text{ degenerate intermediate-mass } \nu\text{'s,} \\ \text{which form a single Dirac state} \\ (\text{mostly directed along the } \nu_\tau\text{-direction!}) \end{cases}$$

---

(3) E_6 (only a short comment)

$$\text{fermions}: \ \underline{27} = 16 + 10 + 1 \quad (\text{of } SO(10))$$
$$= (10+\bar{5}+1) + (5+\bar{5}) + 1 \quad (\text{of } SU(5))$$

$$\nu_L,\ \nu_L^c \qquad \begin{pmatrix} g \\ E_1^0 \\ E_1^- \end{pmatrix} \begin{pmatrix} g^c \\ E_2^0 \\ E_2^+ \end{pmatrix} \quad n$$

⇒ $\mathcal{M}_\nu$ is $(\underline{5n_g \times 5n_g})$ sym. matrix

⇒ in general, many possible patterns!

Alternatively, under $\underline{E_6 \to [SU(3)]^3}$ (as suggested from STRING)

$$27 = (3,\bar{3},1) \oplus (\bar{3},1,3) \oplus (1,3,\bar{3})$$
$$\begin{pmatrix} u \\ d \\ g \end{pmatrix} \qquad \begin{pmatrix} u^c \\ d^c \\ g^c \end{pmatrix} \qquad \begin{pmatrix} E_u^0 & E_u^+ & e^+ \\ E_d^- & E_d^0 & \nu^c \\ e^- & \nu & n \end{pmatrix}$$

• Comment on large μ_ν and small m_ν

- The motivation for having a large neutrino magnetic moment, $\mu_\nu \sim 10^{-10}\mu_B$, is the sol.n of the solar neutrino problem via the ν helicity flipping: $\nu_{eL} \to \nu_R$ in the magnetic field of the Sun.

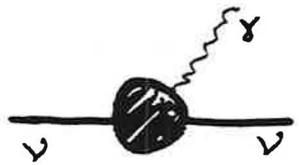
- Recall that "diagonal" μ_ν is possible only for Dirac neutrinos; on the other hand, off-diagonal "transition" $\mu_{\nu_{ij}}$ $(i \neq j)$ are also possible for the "more general" case of Majorana ν's.

- But, in general, it is difficult to get large μ_ν while maintaining small m_ν:

$$= \ m_\nu = G\,M$$

characteristic mass in the loop

combination of coupl., etc.



$$= \ \mu_\nu = \frac{eG}{M},$$

so that: $\mu_\nu \sim \dfrac{e}{M^2} m_\nu = \mu_B \cdot \dfrac{2 m_e m_\nu}{M^2}$

⇒ for $M \simeq 100$ GeV, $\underline{\mu_\nu \simeq 10^{-10}\cdot\mu_B}$ ⇒ need of: $\underline{m_{\nu_e} \gtrsim 10\text{ KeV}}$

(which is ~~unacceptable experimentally~~)

A sol.n is the "VOLOSHIN" model, which introduces the so-called "CUSTODIAL" symmetry:

- it introduces a $\underline{SU(2)_\nu}$ symmetry, under which ν and N^c belong to a same doublet, $\begin{pmatrix} \nu \\ N^c \end{pmatrix}_L$;

then: a Dirac mass $\bar{N}_R \nu_L$ transforms as a <u>TRIPLET</u> under $SU(2)_\nu$.

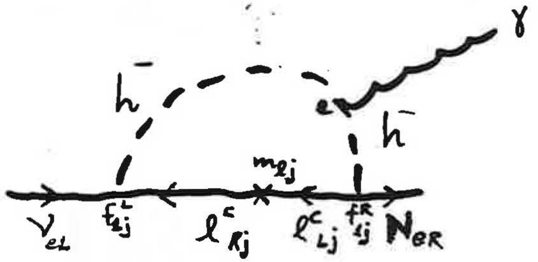
(since $m_{ab} = m_{ba}$!)

but: a Magnetic moment μ_ν is a <u>SINGLET</u>

(since $\mu_{ab} = -\mu_{ba}$!)

$\Rightarrow$ therefore it allows a large μ_ν (which is $SU(2)_\nu$-invariant) while suppresses the $SU(2)_\nu$-violating m_ν.
(example: identification of $SU(2)_\nu$ with an horizontal e-μ symmetry; note that this is a good global symmetry in the limit $m_e = m_\mu = 0$

Particular Models: "Zee - Type" of models:



($SM \oplus N_R \oplus h^+$, but with a single LH Higgs doublet in order to preserve $B-L$)

$$\mathcal{L}'_Y = \sum_{a,b} \left(f^L_{ab} \, \Psi^T_{aL} C (i \tau_2) \Psi_{bL} h^+ + f^R_{ab} N^T_{aR} C \ell_{bR} h^+ \right) + h.c.$$

$(f^L_{ab} = - f^L_{ba})$ $\quad \mu_\nu = \dfrac{f^L_{13} f^R_{13}}{16\pi^2} \dfrac{m_e m_\tau}{M_h^2} \left[\ln\left(\dfrac{M_h^2}{m_\tau^2}\right) - 1 \right] \mu_B$

[illegible]

Neutrino Masses Beyond the Standard Model

S. Ranfone (R.A.L)

2nd part (3 June 1992; R.A.L):

i) **USM** and the 17 KeV neutrino.

ii) "Flipped" $SU(5) \otimes U(1)$ and $SU(4) \otimes O(4)$ GUT models.

iii) Implications of the charged fermion masses on the neutrino sector in $SU(4) \otimes O(4)$ and recent results.

D. Neutrino Masses in the "UNIVERSAL SEESAW MECHANISM" (USM)

D1) USM : A Review

[Davidson & Wali: PRL (87); D.S.R., W: PRD 41 (90) 208; S.R.: PRD 42 (90) 3819] [redacted]

- **Motivation**: Two of the main "problems" of the SM, which do not find a satisfactory explanation even in the context of GUTs, are related to:
  1) the "family" replication of fermions;
  2) the "generational" fermion mass hierarchy.

⇒ **USM**: Extension of the "seesaw" idea to all fermions (in order to explain why $m_f \ll M_W$, without the need of very small Yukawa's).

- The USM is based on L-R symm. models but:

"USUAL" L-R symm. models:

fermions → minimal (in L-R) SM + ν_R

Higgs → extended (for obtaining a mass suppression à la seesaw):

$\phi(2,2) \oplus \Delta(3,1) \oplus \Delta(1,3)$

USM:

fermions → extended:

$$q_{L(R)},\ \ell_{L(R)} \ \oplus \begin{cases} U_{L,R}(1,1)_{4/3},\ D_{L,R}(1,1)_{-2/3} \\ E_{L,R}(1,1)_{-2},\ N_{L,R}(1,1)_{0} \end{cases}$$

(each standard ferm. has its own L-R singlet partner)

Higgs → minimal (ext. of SM):

$\phi_L(2,1) \oplus \phi_R(1,2) \oplus \sigma(1,1)$ ...

- Because of the absence of $\phi(2,2)_0$ there are NOT the ordinary Dirac masses ($m_f \bar{f}_L f_R$); but the fermions get their masses through the mixing with the new singlet partners:

$$\begin{aligned} -\mathcal{L}_Y = {} & Y_{dL}\, \bar{q}_L \phi_L D_R + Y_{uL}\, \bar{q}_L \tilde{\phi}_L U_R + (L \to R) + h.c. \\ & + Y_{eL}\, \bar{\ell}_L \phi_L E_R + Y_{\nu L}\, \bar{\ell}_L \tilde{\phi}_L N_R + (L \to R) + h.c. \\ & + \tilde{Y}_{\nu L}\, \ell_L^T C (i\tau_2) \phi_L N_L + \tilde{Y}_{\nu R}\, \ell_R^T C (i\tau_2) \phi_R N_R + h.c. \\ & + \left[Y_N \bar{N}_L N_R + Y_1 N_L^T C N_L + Y_2 N_R^T C N_R \right] \sigma + h.c. \\ & + \left[Y_U \bar{U}_L U_R + Y_D \bar{D}_L D_R + Y_E \bar{E}_L E_R \right] \sigma + h.c. \end{aligned}$$

where, after the "minimization" of the Higgs potential ⇒

$$\langle \phi_L \rangle \equiv v_L \ll \langle \phi_R \rangle = v_R \ll \langle \sigma \rangle \equiv X$$

Then: (e.g. for the "electron" field),

$$M_e = \begin{array}{c} \\ \bar{e}_L \\ \bar{E}_L \end{array} \!\!\begin{array}{c} \begin{array}{cc} e_R & E_R \end{array} \\ \begin{pmatrix} 0 & Y_e v_L \\ Y_e v_R & Y_E X \end{pmatrix} \end{array} = \begin{pmatrix} 0 & L \\ R & X \end{pmatrix}$$

$$\Rightarrow \quad m_e \simeq \left(\frac{Y_e^2}{Y_E} \right) \frac{v_L v_R}{X} \ll v_L \sim M_W \quad ! \quad \underline{\text{O.K.}}$$

⇒ • $\underbrace{m_e \sim m_u \sim m_d \ll M_W}$

↳ good explanation for the lightness of the 1st generation (charged) fermions, without need of very small Yukawas!

On the other hand, for (each generation of) neutrinos:

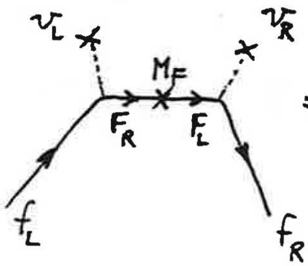
$$
M_\nu = \begin{array}{c} \\ \bar\nu_L \\ \bar\nu_L^c \\ \bar N_L \\ \bar N_L^c \end{array}
\begin{array}{c}
\begin{array}{cccc} \nu_R^c & \nu_R & N_R^c & N_R \end{array} \\
\left(\begin{array}{cc:cc}
\multicolumn{2}{c:}{\multirow{2}{*}{0}} & Y_{\nu L} v_L & \tilde Y_{\nu L} v_L \\
 & & Y_{\nu R} v_R^* & \tilde Y_{\nu R}^* v_R^* \\ \hdashline
Y_{\nu L} v_L & Y_{\nu R} v_R^* & \chi_1 & \chi \\
\tilde Y_{\nu L} v_L & \tilde Y_{\nu R} v_R^* & \chi & \chi_2
\end{array}\right)
\end{array}
\Rightarrow
\begin{cases}
m_{\nu_L} \simeq Y_{\nu_L} \tilde Y_{\nu_L} \dfrac{v_L^2}{\chi} \\[2ex]
m_{\nu_R} \simeq Y_{\nu_R} \tilde Y_{\nu_R} \dfrac{v_R^2}{\chi}
\end{cases}
$$

which recovers the "standard seesaw" formula:

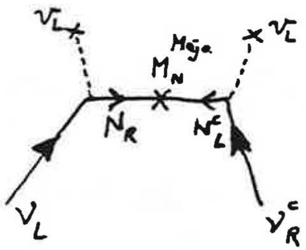
$$m_{\nu_L}\, m_{\nu_R} \simeq m_e^2$$

$\Leftrightarrow$ [The USM relates therefore the lightness of the charged fermions with respect to M_W with the smallness of m_ν with respect to m_e.

Graphically:



$$\Rightarrow\ m_f \simeq Y_f^2\, \frac{v_L v_R}{M_F}$$
$(f = e, u, d)$



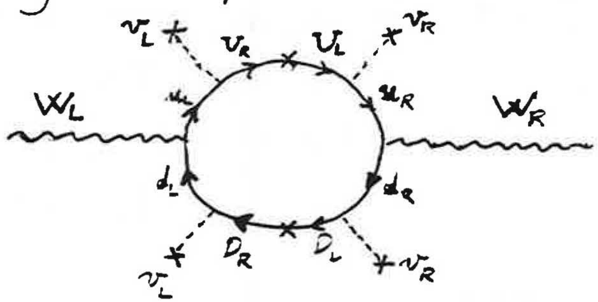
$$\Rightarrow\ m_{\nu_L} \simeq Y_\nu^2\, \frac{v_L^2}{M_N^{Maj.}} \qquad (m_{\nu_R} = m_{\nu_L}(L \leftrightarrow R)).$$

Phenomenology of 1^{st}-generation USM:

$$\left.\begin{array}{l} m_{\nu_L} < 18 \text{ eV} \\ 10^{8(10)}\text{GeV} < \chi < 10^{12}\text{GeV (Peccei-Quinn Scale)} \end{array}\right\} \Rightarrow \begin{cases} M_{W_R} \gtrsim 10^6 \text{ GeV} \\ m_{\nu_R} > 15 \text{ GeV} \end{cases}$$

$(m_{\nu_L} = 18\text{ eV} \Rightarrow \chi = 10^{12}\text{ GeV})$

, USM is also consistent with the data on the RH (V+A) currents, since the W_L-W_R mixing (ξ) arises only at 1-loop [S.R., PRD 42(90)3819]:



"Multi-generation" USM:

- If in the SM the problem was to understand why $m_{e,u,d} < M_W$, in the USM the problem is:

$$\text{WHY} \quad m_t \sim M_W \ !?$$

$\Rightarrow$ its sol.n requires: "HORIZONTAL" (family) AXIAL SYMMETRY

$$U(1)_A \equiv U(1)_{PQ}$$
Peccei-Quinn

which prevents the singlet masses; in fact:
if X in $\begin{pmatrix} 0 & L \\ R & X \end{pmatrix}$ is present one gets, as usual, $m_1 \sim \frac{LR}{X} \ll L$, $m_2 \sim X$;
but if $X = 0$ because of the new symm., then: $\begin{pmatrix} 0 & L \\ R & 0 \end{pmatrix}$, which gives: $m_1 \sim L \sim M_W$! $m_2 \sim R$.

$\Rightarrow$ (generational) mass hierarchy in the multi-generation case requires $\mathbf{X}$ to be SINGULAR: $\det X = 0$; in its turn, this implies the U(1) symm. to be AXIAL (if it had been "vectorial", diagonal terms $\bar F_{Li} F_{Ri}$ would be allowed, giving a non-singular $\underline{X}$).

• $U(1)_{PQ} \Rightarrow$ the "doubling" of the doublet-Higgs sector:

$$\begin{cases} \phi_{L1} = \begin{pmatrix} \phi_{L1}^{+} \\ \phi_{L1}^{\circ} \end{pmatrix} \sim (2,1)_{1}^{1 \leftarrow U(1)_{PQ}} & \phi_{L2} = \begin{pmatrix} \phi_{L2}^{+} \\ \phi_{L2}^{\circ} \end{pmatrix} \sim (2,1)_{1}^{-1} \\ \phi_{R1} \sim (1,2)_{1}^{1} & , \quad \phi_{R2} \sim (1,2)_{1}^{-1} \end{cases}$$

with VEV's: v_{L1}, v_{L2} ; v_{R1}, v_{R2} $\qquad \left((v_{L1}^2 + v_{L2}^2)^{1/2} \equiv v_{SM} \right)$

Definitions: $U(1)_{PQ}$-charges of fields:

$$\begin{cases} f^{i}_{L(R)} \longrightarrow e^{\pm i \vartheta x_i} f^{i}_{L(R)} \\ F^{i}_{L(R)} \longrightarrow e^{\pm i \vartheta y_i} F^{i}_{L(R)} \end{cases}$$

$x_i \neq x_j$, $y_i \neq y_j$ for $i \neq j$ (in order to be a good horizontal symm.)

By imposing "L" and "R" to be of FRITZSCH Type: $\begin{pmatrix} & A & \\ A & & B \\ & B & C \end{pmatrix}$ fixes the PQ-charges in terms of a single "free" parameter x $(\equiv x_1)$ which only affects the form of the "singular" matrix $\underline{X}$.

We have studied [D., S.R., W., 89; 89 bis; S.R. '90] the quark sector, obtaining the KM matrix in terms of the q-masses:

• $\sin\vartheta_c \simeq \sqrt{\dfrac{m_d}{m_s}} - \sqrt{\dfrac{m_u}{m_c}}$;

• the understanding of the suppression of $|V_{ub}|$;

$|V_{td}| \simeq |V_{cb}| \cdot \sin\vartheta_c$

$|V_{ts}| \simeq |V_{cb}|$

D2) "The Mass-Hierarchy puzzle and The 17 KeV in the USM"

[E. Papageorgiu & S.R.: NPB369 (92) 99].

• Detailed Study of the Three-generat. lepton sector, for the model corresponding to the "choice" $\underline{x=5}$, which gives a LR-singlet mass matrix of the form

$$X \equiv M_\sigma = \begin{pmatrix} & & K \\ & & \\ K & & \end{pmatrix} , \quad K \sim O(\chi)$$

a) Charged Leptons:

$$\underset{(6\times6)}{M_\ell} = \begin{pmatrix} 0 & M_L \\ M_R & M_\sigma \end{pmatrix}, \text{ where: } M_L = \begin{pmatrix} & L & \\ L & & aL \\ & aL & bL \end{pmatrix} , \quad M_R = M_L (L \leftrightarrow R) ;$$

$$x=5: \quad M_\sigma = \begin{pmatrix} & & K \\ & & \\ K & & \end{pmatrix} \qquad \begin{cases} L = Y^{\ell}_{12} \dfrac{v_L}{\sqrt{2}} , & R = Y^{\ell}_{12} \dfrac{v_R}{\sqrt{2}} \\ K = Y^{E}_{13} \chi , & a = Y^{\ell}_{23}/Y^{\ell}_{12} \\ & b = Y^{\ell}_{33}/Y^{\ell}_{12} \end{cases}$$

$\Rightarrow M_\ell$ not being hermitian $\Rightarrow$ charged lepton masses squared are the eigenvalues of $\underset{6\times6 \text{ sym. matrix}}{\underline{M_\ell M_\ell^{+}}}$

$\Rightarrow$ Secular Eq.:

$$\lambda^6 - 2\lambda^5 K^2 + \lambda^4 K^4 - \lambda^3 K^4 R^2 (1+a^2) + \lambda^2 K^4 R^2 L^2 (1+a^2)^2 - 2\lambda K^2 L^4 R^4 (1+a^2)(2a^2+b^2+2a^4) + L^6 R^6 b^4 = 0$$

$\rightarrow$ it can be solved perturbatively, giving:

$$m_e \simeq \frac{LR}{K}\left(\frac{\varepsilon}{2a}\right) , \quad m_\mu \simeq \frac{LR}{K}(2a) , \quad m_\tau \simeq \sqrt{1+a^2}\, L$$

with, e.g., $a=1$ $(Y^{\ell}_{12} = Y^{\ell}_{23})$ we also get: $\underline{Y_{12} (= Y_{23}) \sim 10^{-2}}$, $\underline{Y^{\ell}_{33} \sim 2 \cdot 10^{-3}}$

·) Neutrino Sector:

M_ν (12×12 symmetric) in the basis:

$(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}; \nu^c_{eL}, \nu^c_{\mu L}, \nu^c_{\tau L}; N_{1L}, N_{2L}, N_{3L}; N^c_{1L}, N^c_{2L}, N^c_{3L})$

Takes the form:

$$M_\nu = \begin{pmatrix} 0 & M_0 \\ M_0^T & M_N \end{pmatrix},$$

where:

$$M_0 = \left(\begin{array}{ccc|ccc} & dL & & & cL & \\ dL & & L & cL & & bL \\ & L & aL & & bL & eL \\ \hline & cR & & & dR & \\ cR & & bR & dR & & R \\ & bR & eR & & R & aR \end{array}\right) \qquad M_N = \left(\begin{array}{ccc|ccc} & & hK & & & K \\ hK & & & K & & \\ \hline & & K & & & hK \\ & & & & & \\ K & & & hK & & \end{array}\right)$$

of course M_N is <u>singular</u>!

a,…,h are Yukawa ratios)

⇒ Semi-analitical evaluation of the eigenvalues of M_ν:

- in the limit $L \to 0$ ⇒ rank(M_ν) reduces (from 12) to 9 ⇒ $\{L \to 0 \Rightarrow m_{1,2,3} \to 0$; i.e.: $m_{1,2,3} \propto L\}$;
- when also $R \to 0$ rank$(M_\nu) \to 4$, ⇒ 4 very heavy states: $m_{9,10,11,12} \sim O(K)$ ➡

⇒ $\boxed{m_{1,2,3} \ll m_{4,5,6,7,8} \ll m_{9,10,11,12}}$

⇒ These considerations and the use of some of the "INVARIANTS" of M_ν: $\Delta_1 \equiv \mathrm{Tr}\, M_\nu, \ldots, \Delta_{12} \equiv \det M_\nu$, whose leading terms are:

$$\begin{cases} \Delta_{12} \simeq f_{12} R^6 L^6 & \Delta_{11} = f_{11} K R^6 L^4 \quad \Delta_{10} = f_{10} K^2 R^6 L^2, \\ \Delta_9 \simeq f_9 K^3 R^6, & \Delta_8 = f_8 K^4 R^4, \end{cases}$$

allows the construction of the following ratios: ➡

⇒
$$\left\{\begin{array}{lll} \Delta_{12}/\Delta_{11} \simeq \dfrac{m_1 m_2 m_3}{m_1 m_2 + m_1 m_3 + m_2 m_3} & \sim \dfrac{L^2}{K} \\ \Delta_{11}/\Delta_{10} \simeq \dfrac{m_1 m_2 + m_1 m_3 + m_2 m_3}{m_1 + m_2 + m_3} & \sim \dfrac{L^2}{K} \\ \Delta_{10}/\Delta_9 = m_1 + m_2 + m_3 & \sim \dfrac{L^2}{K} \end{array}\right\} \Rightarrow \begin{array}{l} \underline{\text{all}} \text{ three} \\ m_1 \sim m_2 \sim m_3 \sim \dfrac{L^2}{K} \\ (\text{"seesaw" masses}) \end{array}$$

$$\left\{\begin{array}{l} \Delta_9/\Delta_8 \simeq \left(\dfrac{1}{m_4} + \dfrac{1}{m_5} + \dfrac{1}{m_6} + \dfrac{1}{m_7} + \dfrac{1}{m_8}\right)^{-1} \sim \dfrac{R^2}{K} \\ \Delta_{12} = \det M_\nu \sim R^6 L^6 \end{array}\right\}$$

⇓

$$\left\{\begin{array}{ll} \underline{\text{one}} \text{ eigenvalue:} & m_4 \sim \dfrac{R^2}{K} \\ \underline{\text{four}} \text{ eigenvalues:} & m_5 \sim m_6 \sim m_7 \sim m_8 \sim R \\ \ldots\text{and the remaining } \underline{\text{four}}: & m_9 \sim m_{10} \sim m_{11} \sim m_{12} \sim K \end{array}\right\}$$

⇒ • <u>Comment</u>: the surprising result is the <u>absence</u> of a <u>mass hierarchy</u> for the standard light ν's, to be compared to the hierarchy obtained in the charged sector ($m_{e,\mu} \ll m_\tau$)! This is due to the presence of Majorana mass terms in the neutral sector.

• "Phenomenology":

- A priori, the Yukawa couplings in the ν-sector are arbitrary, but if (for simplicity) we assume them to be of the same order of Y_ℓ (charged leptons) $\sim 5\cdot 10^{-3}$, then

$$m_{\nu_e} \sim m_{\nu_\mu} \sim m_{\nu_\tau} \sim \frac{L^2}{K} \sim Y_\nu \frac{v_L^2}{\chi}$$ gives, for $10^{10}\,\mathrm{GeV} < \chi < 10^{12}\,\mathrm{GeV}$

$$\boxed{3\,\mathrm{eV} \lesssim m_{\nu_{e,\mu,\tau}} \lesssim 30\,\mathrm{eV}}$$!

L.T. [illegible] for MSW in the Sun ⟶ sol.n for the DM

·) The 17 KeV ν in the USM:

- The assumption of the proportionality of the Dirac- and the Majorana-type Yukawa couplings (in generation space), i.e.

$$\underline{Y_{ij}/\tilde{Y}_{ij} = \text{constant}} \quad (\text{i.e., independent on } j,i)$$

$\Rightarrow$ leads to an INTERMEDIATE-MASS DIRAC ν-STATE.

in fact, the conditions: $\frac{Y_{ij}}{\tilde{Y}_{ij}} = \text{const.} \Rightarrow \frac{e}{a} = \frac{c}{d} = b$ changes the leading behaviour of the following invariants of M_ν:

$$\Delta_9 \simeq f_9\, K^3 L^2 R^4 \quad , \quad \Delta_8 = f_8\, K^2 R^6 \quad \rightarrow$$

$\rightarrow$ $\Delta_9 \to 0$ as $L \to 0$ $\rightarrow$ rank$(M_\nu) \to 8$ (instead of 9) as $L \to 0$

$\Rightarrow$ four eigenvalues ($m_{1,2,3,4}$) scale with L)!

· As before, if also $R \to 0$, then Rank$(M_\nu) \to 4$, suggesting that

$$m_{5,6,7,8} \propto R \quad , \quad m_{9,10,11,12} \propto K .$$

Then, using:

$$\begin{cases} \Delta_{12}/\Delta_8 \simeq m_1 m_2 m_3 m_4 \sim L^6/K^2 & (\alpha) \\ \Delta_9/\Delta_8 \simeq m_1 + m_2 + m_3 + m_4 \sim KL^2/R^2 , & (\beta) \\ \Delta_8/\Delta_4 \simeq m_5 m_6 m_7 m_8 \sim R^6/K^2 & (\gamma) \\ (\Delta_4 \sim K^4) & \end{cases}$$

it is possible to completely determine the structure of the ν-spectrum:

$(\alpha) \Rightarrow m_1, m_2 \sim L^2/K$; $m_3, m_4 \sim L$;

but $(\beta) \Rightarrow$ $m_3 + m_4$ cancels up to $\sim KL^2/R^2 \ll L$; i.e. $\Rightarrow$

$\Rightarrow$ $\boxed{\nu_3 \text{ and } \nu_4 \text{ behave (approx.ly) as a single effective Dirac state.}}$

Then, from (γ) and from $\mathrm{Tr}\, M_\nu = \Delta_1 = 0$, we obtain 2 pairs of

· 17 KeV Simpson NeuTrino:

- In order to identify the 17 KeV Simpson ν with our Dirac state "$\nu_{3,4}$", we must have: $\underline{Y_\nu \sim 10^{-7}}$ ($m_{\nu_{(\tau)}} = 17$ KeV $\simeq Y_\nu \cdot v_L$).
- Furthermore, since we find: $\nu_{1,2} \simeq \nu_{e,\mu(L)}$, in order to have the MSW in the Sun, we require: $m_{\nu_{1,2}} \simeq 10^{-3}$ eV, which gives: $\underline{\chi \sim 10^9 - 10^{10}\ \text{GeV}}$ (consistent as a PQ-scale!).

On the other hand, the "choice" $m_{\nu_{1,2}} \simeq 10$ eV, i.e., the "DM sol.n", would yield a too small χ

$\Rightarrow$ So, using $R/K \sim 0.04$ ($v_R \sim 6 \cdot 10^8$ GeV), we get the following ν-spectrum:

Mass [KeV]	NeuTrino Spectrum
$L^2/K \sim 10^{-7}$	$\nu_1 \simeq \cos\theta \cdot \nu_{eL} - \sin\theta \cdot \nu_{\tau L}$ $\nu_2 \simeq \nu_{\mu L}$
$L \simeq 17$	$\nu_{3,4} \simeq \frac{1}{\sqrt{2}}(\cos\theta \cdot \nu_{\tau L} + \sin\theta \cdot \nu_{eL}) \pm \frac{1}{2}(N_{2L} - N$
$R^2/K \simeq 3 \cdot 10^6$	$\nu_5 \simeq \cos\theta \cdot \nu_{eR} - \sin\theta \cdot \nu_{\tau R}$ $\nu_6 \simeq \nu_{\mu R}$
$R \simeq 6 \cdot 10^7$	$\nu_{7,8} \simeq \frac{1}{\sqrt{2}}(\cos\theta \cdot \nu_{\tau R} + \sin\theta \cdot \nu_{eR}) \pm \frac{1}{2}(N_{2L} \pm N_2$
$K \simeq 10^9$	$\nu_{9,10} \simeq \frac{1}{2}[(N_{1L} - N_{1R}) \pm (N_{3L} - N_{3R})]$ $\nu_{11,12} \simeq \frac{1}{2}[(N_{1L} + N_{1R}) \pm (N_{3L} + N_{3R})]$

$\rightarrow$ the 17 KeV ν is: $\boxed{\nu_S \equiv \frac{1}{\sqrt{2}}(\nu_3 + \nu_4) = \nu_{\tau L} \cos\theta + \nu_{eL} \sin\theta}$; $\sin\theta$- degenerate with: $\tilde{N}_2 \equiv (\nu_3 - \nu_4)/\sqrt{2} \simeq (N_{2L} - N_{2R})/\sqrt{2}$.

$\Rightarrow$

$$\begin{cases} \nu_{eL} \simeq \nu_1 \cdot \cos\theta + \nu_S \cdot \sin\theta \\ \nu_{\mu L} \simeq \nu_2 \\ \nu_{\tau L} \simeq -\nu_1 \cdot \sin\theta + \nu_s \cdot \cos\theta \end{cases}$$

Comment: The 17 KeV-Type spectrum arises only in the $x = 5$ case $\Rightarrow$ if the 17 KeV ν_s will be confirmed, it will fix the PQ charges of all fermions in the context of the present model.

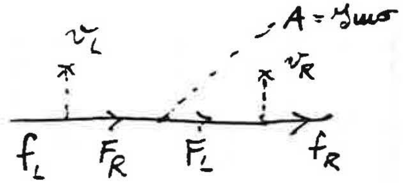
Decay Modes: COSMOLOGY $\Rightarrow$ $\tau_{\nu_S} \lesssim 1.5 \cdot 10^{12}$ sec.

- $\nu_S \to 3\nu$ possible only in some models;
- $\nu_S \to \nu\gamma$ usually too small; in any case ruled out by the lack of CBR distortion.
- $\nu_S \to \nu\, J$(Majoron) is usually the best candidate, but... ... in the present model B-L is a "local" symmetry (broken SPONTANEOUSLY) $\Rightarrow$ therefore there is NO Majoron (Goldstone) boson!

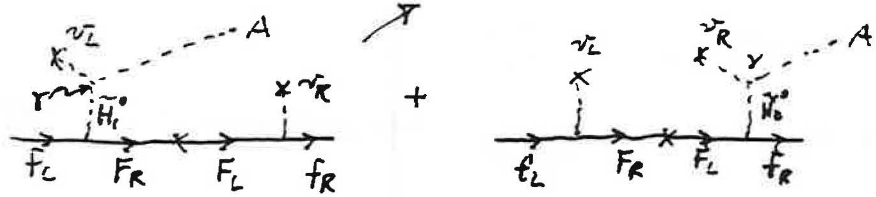
.. Nevertheless, we have a $U(1)_{PQ}$ global symm. spontaneously broken by $\langle\sigma\rangle = \chi$ at $\sim 10^{10}$-10^{12} GeV, which therefore gives rise to an AXION: $A \simeq \mathrm{Im}\,\sigma$, which only couples to the heavy states $N_{1,3}$ (since $M_K = (\kappa^0 \kappa)$).

- Assuming $\tilde{H}^0_{1,2}$ (p.s. Higgses) have a mass $\sim v_R$, and using for $\underline{\gamma}_1$ in $\gamma A \phi^0_L \phi^0_L$, the value $\gamma \simeq v_R^2/\chi$ obtained from the minimization of the ...

... to "standard" fermions in the usual way:

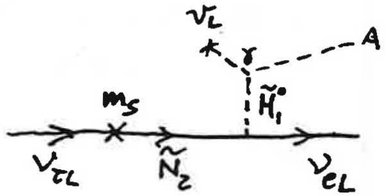


$$: \quad Y^2 \frac{v_L v_R \gamma}{M_F M^2_{\tilde{H}}} \bar{f}\gamma_5 f \cdot A \simeq \frac{m_f}{\chi} \bar{f}\gamma_5 f A$$

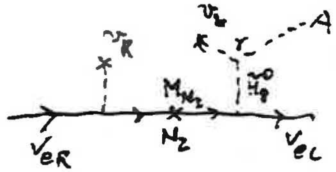


17 KeV ν decay: $\tau(\nu_{\tau L} \to \nu_{eL} A) = \frac{32\pi}{m_{\nu_{\tau L}}} \left(\frac{\sqrt{2} M^2_{\tilde{H}}}{Y v_L \gamma}\right)^2 \simeq \frac{32\pi}{m_{\nu_{\tau L}}} \left(\frac{\chi}{m_{\nu_{\tau L}}}\right)^2 \simeq 1.3 \cdot 10^{12}$ sec.

$\Updownarrow$



- Analogously:



etc.

$$\begin{cases} \tau(\nu_{\tau R} \to \nu_{eL} A) \simeq \tau(\nu_{\tau L} \to \nu_{eL} A) \cdot \left(\frac{m_{\nu_{\tau L}}}{m_{\nu_{\tau R}}}\right) \simeq 3.8 \cdot 10^5 \text{ sec}, \\ \tau(\nu_{eR} \to \nu_{eL} A) \simeq \tau(\nu_{\tau L} \to \nu_{eL} A) \cdot \left(\frac{m_{\nu_{\tau L}}}{m_{\nu_{eR}}}\right) \simeq 9.2 \cdot 10^6 \text{ sec} \\ \tau(\nu_{\mu R} \to \nu_{eL} A) \simeq \frac{\pi}{m_{\nu_{\mu R}}} \left(\frac{\chi}{v_R}\right)^2 \left(\frac{\chi}{m_{\nu_{\tau R}}}\right)^2 = 7.7 \cdot 10^7 \text{ sec} \end{cases}$$

"Neutrino Masses in the "Flipped $SU(5)\otimes U(1)$ and in $SU(4)\otimes O(4)$ GUT Models"

[E. Papageorgiu & S.R.: [PLB (92) 289, 89.] ⊗ RAL 92-035 ;.. ; S.R.: RAL 92-030 (PLB (92))]

Generalities:

String theories with $K=1 \Rightarrow$ "absence" of large self-conjug. or adjoint Higgs repres.

$$\Rightarrow \begin{cases} \underline{NO}: \text{"STANDARD" GUT's}: \; SU(5),\ SO(10),\ E(6),\ \text{etc.}; \\ \underline{YES}: \text{"flipped"}-SU(5)\otimes U(1) \;\text{ and }\; SU(4)\otimes O(4). \end{cases}$$

E1) FLIPPED $SU(5)\otimes U(1)$:

"flipped" fermionic repres.: $u^{(c)} \leftrightarrow d^{(c)},\ \nu^{(c)} \leftrightarrow e^{(c)} \Rightarrow$ (with respect to "standard" SU(5)).

$$F_i = (10,1) = \left(\begin{array}{ccccc} & d^c & -d^c & d & u \\ & & d^c & d & u \\ & & & d & u \\ \hline & & & & \nu^c \\ & & & -\nu^c & \end{array}\right)_{iL},\quad \bar f_i = (\bar 5,-3) = \begin{pmatrix} u^c \\ u^c \\ u^c \\ \hline e \\ \nu \end{pmatrix}_{iL},\quad l_i^c = (1,5) = e^c_{iL}$$

SSB: $\quad SU(5)\otimes U(1) \xrightarrow[M_G]{\langle H,\bar H\rangle} \underline{SM} \xrightarrow[M_W]{\langle h,\bar h\rangle} SU(3)_c \otimes U(1)_{em}$

$H = (10,1) = (u_H, d_H;\ d^c_H, \nu^c_H)$; $\quad \bar H = (\overline{10},-1) = (\bar u_H, \bar d_H;\ \bar d^c_H, \bar\nu^c_H)$;

$h = (5,-2) = (D_3, h^-, h^0)$; $\quad \bar h = (\bar 5, 2) = (\bar D_3, \bar h^+, \bar h^0)$;

and: $n_g+1\ (=4)$ singlets: $\phi_m = (1,0)$, of which one has a VEV M_{W}: $\langle\phi_0\rangle = X \simeq v = \langle h,\bar h\rangle$ (in order to produce the correct

- The ϕ's are also essential for suppressing the ν-masses;
- The model is then completely determined by the <u>SUPERPOTENTIAL</u> (satisfying the discrete symm.: $H\to -H$):

$$W_{(5)} = \lambda_1 FFh + \lambda_2 F\bar f\bar h + \lambda_3 \bar f l^c h + \lambda_4 HHh + \lambda_5 \bar H\bar H\bar h + \lambda_6 F\bar H\phi + \lambda_7 h\bar h\phi + \lambda_8 \phi\phi\phi$$

After <u>SSB</u>:

$$\left.\begin{array}{ll} \lambda_1\text{-Term}: & M_u = m_\nu^D = \lambda_1 \langle h^0\rangle^{v_u} \\ \lambda_2\text{-Term}: & M_d = \lambda_2 \langle \bar h^0\rangle^{=v_d} \\ \lambda_3\text{-Term}: & M_e = \lambda_3 \langle \bar h^0\rangle^{=v_d} \end{array}\right\} \Rightarrow$$

(*) λ_4- and λ_5-Terms give the well-known "doublet-triplet" mass splitting needed to avoid a fast proton decay.

$\Rightarrow$ [In "flipped" $SU(5)\otimes U(1)$ one <u>looses</u> the "standard" $SU(5)$ mass relation: $M_e = M_d$ at M_G.

E2) $SU(4)\otimes O(4)$

$\hookrightarrow$ <u>isomorphic</u> to: $\underline{SU(4)\otimes SU(2)_L\otimes SU(2)_R}$ $\hookrightarrow$ L-R symm.

<u>SSB</u>: $SU(4)\otimes SU(2)_L\otimes SU(2)_R \xrightarrow[M_G]{\langle H,\bar H\rangle} SU(3)_c\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$

$$\xrightarrow[M_G]{\langle H,\bar H\rangle} \underline{SM} \xrightarrow[M_W]{\langle h\rangle} SU(3)_c\otimes U(1)_{em}$$

$$\begin{cases} F_i = (4,2,1) = q_L\,(3,2,1)_{1/3} + l_L\,(1,2,1)_{-1} = \begin{pmatrix} u \\ d\end{pmatrix}_{Li} + \begin{pmatrix}\nu \\ e\end{pmatrix}_{Li} \\ \bar F_i = (\bar 4,1,2) = q^c_L\,(\bar 3,1,2)_{-1/3} + l^c_L\,(1,1,2)_1 = \begin{pmatrix} d^c \\ u^c\end{pmatrix}_{Li} + \begin{pmatrix} e^c \\ \nu^c\end{pmatrix}_{Li} \end{cases}$$

$(\ 16_{SO(10)} = F + \bar F \ !\ )$

$$H = (4,1,2) = \begin{pmatrix}\bar u_H^c\\ \bar d_H^c\end{pmatrix} + \begin{pmatrix}\bar \nu_H^c\\ \bar e_H^c\end{pmatrix}; \qquad \bar H = (\bar 4,1,2) = \begin{pmatrix}u_H^c\\ d_H^c\end{pmatrix} + \begin{pmatrix}\nu_H^c\\ e_H^c\end{pmatrix}$$

$$h = (1,2,2) = \begin{pmatrix}h^0 & \bar h^+\\ h^- & \bar h^0\end{pmatrix}; \qquad D = (6,1,1) = D_3 + \bar D_3; \qquad \phi_m = (1,1,1) \quad (m = 1,\dots,n_g+1)$$

and:

$$W_{(4)} = \lambda_1 F\bar F h + \lambda_2 \bar F H \phi + \lambda_3 HHD + \lambda_4 \bar H \bar H D + \lambda_5 hh\phi + $$
$$+ \lambda_6 \phi\phi\phi + \lambda_7 FFD + \lambda_8 \bar F \bar F D + \lambda_9 DD\phi$$

(invariant under $\bar H \to -\bar H$) ⇒

$$\Rightarrow \begin{cases} M_u = M_\nu^D = \lambda_1 \langle h^0 \rangle \\ M_d = M_e = \lambda_1 \langle \bar h^0 \rangle \end{cases}$$

The λ_3- and the λ_4-terms ensure that the $D_3, \bar D_3$ get mass terms $\sim M_G$, through mixing with d_H^c and $\bar d_H^c$ (this is NOT the "usual" doublet-triplet "mass splitting")

The $SU(4)\otimes O(4)$ model (naively) reproduces the same mass relations as in "standard" $SU(5)$

$m_d = r m_e'$, $m_s = r m_\mu$, $m_b = r m_\tau$ ($r \simeq 2.8$)

[E. Papag. & S.R.: P.B(92) 282; 8]

E3) The Neutrinos (in both models):

(e.g., in $SU(4)\otimes O(4)$):

$$\begin{cases} \lambda_2 \bar F \langle H \rangle \phi \Rightarrow \nu^c\phi \text{ mixing at } \lambda_2\langle H\rangle \sim M_G \\ \lambda_6 \phi^3 \Rightarrow \phi \text{ masses} \sim \lambda_6 \langle\phi\rangle \sim M_W\,(M_X) \\ \lambda_1 F\bar F h \Rightarrow \text{Dirac } \nu \text{ masses: } \lambda_1\langle h^0\rangle = m_\upsilon; \end{cases}$$

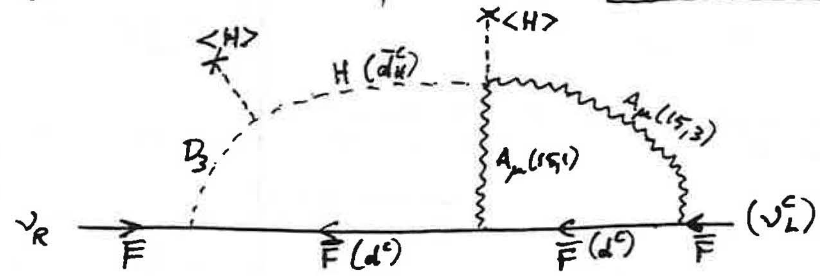
and define M_R the RH Majorana mass matrix ($\nu^c\nu^c$).

⇒ Then, in the basis (ν, ν^c, ϕ), M_ν (9×9 for 3 gener.'s) takes the form:

$$M_\nu = \begin{pmatrix} & m_\upsilon & \\ m_\upsilon & M_R & M_G \\ & M_- & M \end{pmatrix}$$

There are the following cases,

a) NON-SUSY case [Leontaris, et al. R]: then M_R ($\sim 10^8$ GeV) (radiatively) arises at 2-loop via the Witten Mechanism



$$M_R \sim \varepsilon \left(\frac{\alpha_G}{4\pi}\right)^2 \frac{3}{16} M_G$$

(ε = mixing$(D_3, d_H^c) \sim .1$); $\alpha_G \sim .02$.

⇒ In this case, when M_G is "singular" (because of discrete symmetries from the strings), one gets: some ν-masses very very suppressed:

$$m_\nu \sim \left(\frac{m_u}{M_G}\right)^2 M_W \sim 10^{-23} - 10^{-26}\ \text{eV} \,!$$

(i.e., ν's essentially MASSLESS)

and: some à la "STANDARD" seesaw:

$$m_\nu' \sim \frac{m_u^2}{M_R} \sim 0.1 - 10 \text{ KeV}.$$

- Unfortunately, this "radiative" mechanism is not available in the more interesting SUSY case:

→ Therefore, in general, in the SUPERSYMMETRIC versions of these models one expects the ordinary neutrinos to be essentially massless!

There are however two ways of getting ν-masses "phenomenologically" interesting, also in the SUSY case:

i) SUSY-case $\oplus$ non-renormalizable terms: e.g.:

$$\frac{1}{M_S}\,\bar{F}HH\bar{F} \qquad (M_S \sim 10^{18}\,\text{GeV}) \text{ (is the "string" unification scale)}$$

$$\Downarrow$$

$$M_R \sim \frac{\langle H\rangle^2}{M_S} \sim \frac{M_G^2}{M_S} \sim 10^{13}-10^{14}\,\text{GeV} \Rightarrow$$

$\Rightarrow$ some ν's are ~ massless: $\left(m_\nu \sim \left(\frac{m_u}{M_G}\right)^2 M_W\right)$

and some ν's have masses: $m'_\nu \sim \frac{m_u^2}{M_R} \sim 10^{-3}\,\text{eV}$

"good" spectrum for MSW mechanism.

.. but the most interesting result [E.P. & S.R.: RAL 92-035] is that the study of the charged fermion mass relations allows us to "predict" the VEV's of the RH sneutrinos, $\langle \nu^c\rangle \sim M_G$ [S.R.: RAL-92-030 (PL(92))], which opens a new possibility of obtaining interesting ν-masses in the SUSY case, without requiring "arbitrary" non-renormalizable terms. →

COMMENT ON THE charged fermion masses in the SUSY-SU(4)⊗O(4) model [S.R.: RAL-92-030; PL(92)B]:

$$\begin{cases} M_u = M_\nu^D = \lambda_1 \langle h^\circ\rangle \\ M_d^{(0)} = M_e = \lambda_1 \langle \bar{h}^\circ\rangle \end{cases} \rightarrow M_d^{(0)} = r M_e \text{ at low energy } (r \simeq 2.7 \text{ (renormal.)})$$

but because of the mixing among d, $\bar{d}_H^c$, and D_3, the "full" d-type mass matrix is:

$$\mathcal{M}_d = \begin{array}{c} \\ \bar{d}_{Ri} \\ d_H^c \\ \bar{D}_3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} d_{Li} & \bar{d}_H^c & D_3 \end{array} \\ \begin{pmatrix} M_d^{(0)} & X & S \\ & & M_G \\ & M_G & X_q \end{pmatrix} \end{array} \qquad (5\times5 \text{ for 3 gen.})$$

$(X \sim X_q \sim O(M_W)$, $S \propto \langle \nu^c\rangle)$

$\Rightarrow$ "effective" d-type (mass)^2-matrix (3×3) for "ordinary" d (for $n_g = 3$)

$$M_d^2(\text{eff.}) \simeq r\hat{M}_e\left\{1 - \frac{1}{|S|^2+M_G^2}\,(SS^T)\right\} r\hat{M}_e$$

$(\hat{M}_e = \text{diag}(m_e, m_\mu, m_\tau)$; $|S| = (S_1^2+S_2^2+S_3^2)^{1/2}$; $r \simeq 2.7)$

$\Rightarrow$ e.g., in order to preserve the good relation $m_b \simeq r m_\tau$ but "improve" the relation: $m_s \simeq r m_\mu$, we must assume that $\langle \nu_\mu^c\rangle \neq 0$; this gives:

$$\begin{cases} m_d = r m_e & \text{(we do not solve it, but } m_d \text{ is probably dominated by non-perturbative QCD !!)} \\ m_s = \sqrt{Z}\, r m_\mu, & Z = \left[1 + (|S|/M_G)^2\right]^{-1} \\ m_b = r m_\tau & \text{(good !)} \end{cases}$$

↳ Prediction for $\langle \nu_\mu^c\rangle$: $\boxed{\frac{\langle \nu_\mu^c\rangle}{M_G} \simeq \frac{\lambda_3 v}{m_s}\left(\frac{m_b}{m_t}\right)\sqrt{1-\left(\frac{m_s}{r m_\mu}\right)^2}}$

<u>Note</u> that $\langle \nu_\mu^c \rangle$ is proportional to the "<u>departure</u>" from the "naive" mass relation $m_s = m_\mu$ at M_G, and would vanish if this relation had been "exact"!

<u>Comment</u>: The fact that $\langle \nu_\mu^c \rangle = \langle \overline{F}_2(\bar{4},1,2) \rangle \sim M_G$, means that the gauge group is <u>not</u> broken down to the SM only by the Higgs fields H and $\overline{H}$, but also by the "matter superfield"!

• Other Comment: Notice that if $\langle \nu^c \rangle$ were zero, M_u and M_d had been essentially proportional to each other $M_u \propto M_d \sim \lambda_1$, and we could not explain a large Cabibbo!

<u>Consequences in the ν-sector</u> (for <u>each</u> generation):

$$
M_\nu = \begin{array}{c|c}
 & \begin{matrix} \nu & \nu^c & \phi & \nu_H^c & \bar{\nu}_H^c \end{matrix} \\
\begin{matrix} \nu \\ \nu^c \\ \phi \\ \nu_H^c \\ \bar{\nu}_H^c \end{matrix} &
\begin{pmatrix}
 & M_\upsilon & & & \\
M_\upsilon & M_R & M_G & & \tilde{X} \\
 & M_G & X & & \lambda_2\langle \nu^c \rangle \\
 & & & M_G & \\
 & \tilde{X} & \lambda_2\langle \nu^c \rangle & & M_G
\end{pmatrix}
\end{array}
\qquad
\begin{pmatrix} \text{where} & \lambda_2 \simeq 0.1 \\ \tilde{X} \sim X \sim M_R \sim M_W & \\ M_G \sim 10^{16}\,\text{GeV} & \end{pmatrix}
$$

$$
\Rightarrow \quad m_\nu(\text{eff; light}) \simeq (M_\upsilon \;\; 0 \;\; 0 \;\; 0)
\begin{pmatrix}
M_R & M_G & 0 & \tilde{X} \\
M_G & X & 0 & \lambda_2\langle \nu^c \rangle \\
0 & 0 & M_G & 0 \\
\tilde{X} & \lambda_2\langle \nu^c \rangle & 0 & M_G
\end{pmatrix}^{-1}
\begin{pmatrix} M_\upsilon \\ 0 \\ 0 \\ 0 \end{pmatrix} \simeq
$$

$$
\simeq \frac{1}{M_G^4} (M_\upsilon \;\; 0 \;\; 0 \;\; 0)
\begin{pmatrix}
M_G \langle \nu^c \rangle^2 \lambda_2^2 M_\upsilon \\
M_G^3 M_\upsilon \\
0 \\
-M_G^2 \langle \nu^c \rangle \lambda_2 M_\upsilon
\end{pmatrix} \simeq
$$

$$
\simeq \underbrace{\left(\frac{\lambda_2 \langle \nu^c \rangle}{M_G} \right)^2 \frac{M_\upsilon^2}{M_G}}
$$

▷ Therefore, <u>for those families</u> for which <u>$\langle \nu^c \rangle \sim M_G$</u>, we get: $\boxed{m_{\nu_i} \sim m_{\upsilon i}^2 / M_G}$, which is a "standard-seesaw" [illegible] the other families of course.

• <u>Comment</u>: Our analysis also justifies the previous "naive" studies of the neutrino mass spectrum, for the case $\langle \nu^c \rangle = 0$.

In fact, in the limit $\langle \nu^c \rangle \to 0$, the evaluation of the "effective mass" gives:

$$
M_\nu^{\text{eff.}} \simeq (m_\upsilon \;\; 0 \;\; 0 \;\; 0)
\begin{pmatrix}
M_R & M_G & & X_2 \\
M_G & X & & \\
 & & M_G & \\
X_2 & & & M_G
\end{pmatrix}^{-1}
\begin{pmatrix} m_\upsilon \\ 0 \\ 0 \\ 0 \end{pmatrix} \simeq
$$

$$
\simeq \frac{1}{M_G^4} (m_\upsilon \;\; 0 \;\; 0 \;\; 0)
\begin{pmatrix}
M_G^2 X & M_G^3 & \cdots \\
M_G^3 & M_G^2 M_R & \cdots \\
0 & \cdots & \\
M_G X^2 & \cdots &
\end{pmatrix}
\begin{pmatrix} m_\upsilon \\ 0 \\ 0 \\ 0 \end{pmatrix} =
$$

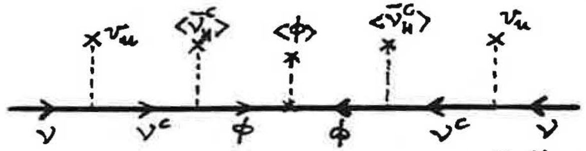
$$
= \frac{1}{M_G^4} (m_\upsilon \;\; 0 \;\; 0 \;\; 0)
\begin{pmatrix}
M_G^2 X m_\upsilon \\
M_G^3 m_\upsilon \\
0 \\
M_G X^2 m_\upsilon
\end{pmatrix} \simeq \left(\frac{m_\upsilon}{M_G} \right)^2 \cdot X \quad \Rightarrow
$$

$$
\Rightarrow \quad \boxed{m_\nu^{\text{light}} = \left(\frac{m_\upsilon}{M_G} \right)^2 \cdot M_W}
$$

, confirming the result obtained "neglecting" the effect of the mixing of ν_H^c and $\bar{\nu}_H^c$.

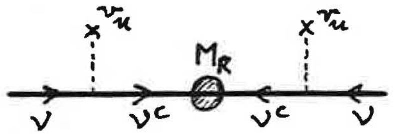
• **"Diagrammar"**: All the results obtained above in the various cases by the perturbative study of the ν-mass matrix may more easily obtained by means of the evaluation of the following graphs:

a) "general" case (with $\langle \nu^c \rangle \simeq 0$):



$$\Rightarrow \quad \underline{m_\nu} \sim \frac{(Y_u v_u)^2 \langle \phi \rangle^{\times \sim M_W}}{M_G^2} \sim \underbrace{\left(\frac{m_u}{M_G}\right)^2 \cdot X}$$

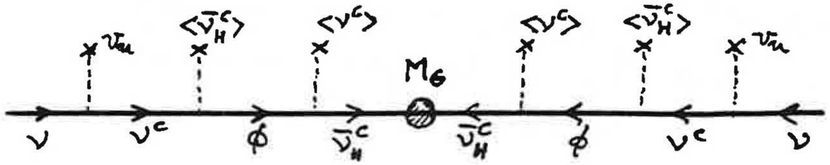
b) M_G "singular" $\Rightarrow$ $\nu^c\phi$-term absent (with $\langle \nu^c \rangle$ still zero):



$$\Rightarrow \quad \underbrace{m_\nu \sim \frac{m_u^2}{M_R}}$$

(where M_R is generated either radiatively in the non-SUSY case, or by the non-renormalizable terms).

c) New case: SUSY with $\langle \nu^c \rangle \neq 0$:



$$\Rightarrow \quad \underbrace{m_\nu \sim \frac{m_u^2 \langle \nu^c \rangle^2}{M_G^3}}$$

(which, for $\langle \nu^c \rangle \sim M_G$, reduces to: $\boxed{m_\nu \sim \frac{m_u^2}{M_G}}$).

→ So, we have shown that the study of the charged fermion masses, suggesting that $\langle \nu_\mu^c \rangle \sim M_G$, (but $\langle \nu_e^c \rangle \sim \langle \nu_\tau^c \rangle \sim 0$ (or $\ll M_G$)) implies:

$$m_{\nu_e} \simeq \left(\frac{m_u}{M_G}\right)^2 M_W \sim 10^{-25}\,\mathrm{eV}$$

$$m_{\nu_\tau} \simeq \left(\frac{m_t}{M_G}\right)^2 M_W \sim 10^{-17}\,\mathrm{eV}$$

} almost massless!

but:

$$m_{\nu_\mu} \simeq \left(\frac{\lambda_2 \langle \nu_\mu^c \rangle}{M_G}\right)^2 \frac{m_c^2}{M_G} \simeq \frac{m_c^2}{M_G} \sim \underline{10^{-6} - 10^{-7}\,\mathrm{eV}}$$

still quite small!

• Note, that if we assume that also $\langle \nu_\tau^c \rangle \sim M_G$, then: m_{ν_τ} would have been: $m_{\nu_\tau} \simeq \frac{m_t^2}{M_G} \sim 10^{-3}\,\mathrm{eV}$ suitable for the MSW mechanism.

– But the main "new" result is that, even in the SUSY case, where the radiative mechanisms are not available for producing a large M_R scale, it is possible to get non-negligible neutrino masses without the need of introducing non-renormalizable terms! This may be obtained by introducing VEV's at the GUT scale for the RH sneutrinos.

About the relevance of "Model Building" in Physics:

When you follow two separate chains of
thought, Watson, you will find some point
of intersection which should
approximate the truth.

(A. Conan Doyle)

References

1a.

[1] J.W.F. Valle, "Gauge Theories and the Physics of Neutrino Mass", in "Progress in Particle and Nuclear Physics, vol.26; ed. A. Faessler (Pergamon Press, 1991);

[2] R.N. Mohapatra and P.B. Pal, "Massive Neutrinos in Physics and Astrophysics", (World Scientific, 1991);

[3] G.F. Giudice, F. Giuliani, and S. Ranfone, Phys. Lett. **212B** (1988) 181.

1b.

[4] Most of the models for neutrino masses reviewed here have been more estensively discussed in Refs.[1,2];

[5] G.G. Ross, "Grand Unified Theories", (Benjamin/Cummings Publ. Co., 1984);

[6] R.N. Mohapatra, "Unification and Supersymmetry" (Springer Verlag, 1986);

[7] M. Gell-Mann, P. Ramond, and R. Slansky, in "Supergravity", ed. by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, 1979);

[8] R. Johnson, S. Ranfone, J. Schechter, Phys. Lett. **179B** (1986) 355, Phys. Rev. **D35** (1987) 282;

[9] M. Gronau, R. Johnson, S. Ranfone, and J. Schechter, Phys. Rev. **D37** (1988) 2597;

[10] See Ref.[3].

[11] S. Ranfone, "Some aspects of Seesaw Models for quarks and leptons", Ph.D. Thesis, Syracuse University, (1990); *(unpublished);*

[12] T.J. Allen, R. Johnson, S. Ranfone, J. Schechter, and J.W.F. Valle, Modern Phys. Lett. **A6** (1991) 1967.

2a.

[13] A. Davidson and K.C. Wali, Phys. Rev. Lett. **58** (1987) 2623, **59** (1987) 393, **60** (1988) 1813;

[14] A. Davidson, S. Ranfone, and K.C. Wali, Phys. Rev. **D41** (1990) 208; and in "Proceed. of the XI annual Montreal-Rochester-Syracuse-Toronto High-Energy Meeting", ed. by C. Rosenzweig and K.C. Wali, Syracuse, N.Y., (May 1989);

[15] See Ref.[11];

[16] S. Ranfone, Phys. Rev. **D42** (1990) 3819;

[17] E. Papageorgiu and S. Ranfone, Nucl. Phys. **B369** (1992) 99.

2b.

[18] I. Antoniadis, J. Ellis, J.S. Hagelin, and D.V. Nanopoulos, Phys. Lett. **194B** (1987) 231;

[19] I. Antoniadis and G.K. Leontaris, Phys. Lett. **216B** (1989) 333;

[20] G.K. Leontaris and J.D. Vergados, Phys. Lett. **258B** (1991) 111;

[21] E. Papageorgiu and S. Ranfone, Phys. Lett. **282B** (1992) 89; and RAL preprint, RAL-92-035 (June 1992);

[22] S. Ranfone, RAL preprint, RAL-92-030 (May 1992), *(to appear in Phys. Lett.B).*