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Implications for Minimal Supersymmetry from Grand Unification and the Neutralino Relic Abundance

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Abstract

We examine various predictions of the minimal supersymmetric standard model coupled to minimal supergravity. The model is characterised by a small set of parameters at the unification scale. The supersymmetric particle spectrum at low energy and the spontaneous breaking of the standard model itself are then generated radiatively. The previously considered predictions of the model now include the neutralino relic density which in turn provides bounds on the scale parameters. We find a remarkable consistency among several different constraints which imply all supersymmetric particle masses preferably within the reach of future supercolliders (LHC and SSC). The requirement that the neutralino be the dominant component of (dark) matter in the flat Universe provides a *lower* bound on the spectrum of supersymmetric particles beyond the reach of LEP, and most likely also the Tevatron and LEP 200.

1 Introduction

The minimal extension of the standard model [1] which corresponds to a softly-broken supersymmetric $SU(3) \times SU(2)_L \times U(1)_Y$ at the scale M_X where the gauge couplings unify (as recently confirmed by LEP [2]) provides a very attractive and economic description of physics beyond the standard model. It is possible to specify a small number of parameters at the unification scale and the low energy effective theory is then determined simply by the radiative corrections. In particular the spontaneous breaking of electroweak symmetry is radiately generated due to the presence of supersymmetry soft-breaking terms through the mass squared of one of the two Higgs doublets being driven negative at the scale $Q \simeq \mathcal{O}(m_Z)$ by the Yukawa top-quark coupling [3]. In terms of the starting parameters at the GUT scale a detailed spectrum of the supersymmetric (SUSY) states is completely determined. Even in the simplest SUSY scenario one meets considerable uncertainty related to the presence of both the superheavy states around the GUT scale and, more importantly, new supersymmetric states above m_Z [4, 5, 6]. Clearly the corresponding threshold corrections around M_X depend on which unified group or superstring scenario the minimal SUSY model is embedded into. This inherent uncertainty would weaken the predictive power of the theory and, as in a previous paper [6], we perform a minimal analysis where such corrections are ignored but corrections from supersymmetric states above m_Z are treated with particular care [6]. Similarly, the important constraint coming from the limits on the proton decay [7] depends on the choice of a specific GUT model and will not be discussed here.

In this letter, we extend the previous analysis [6] to include the predictions for the relic abundance of the lightest neutralino χ which is typically the lightest supersymmetric particle (LSP) of the model. The neutralino LSP has long been identified [8] as one of the leading candidates for dark matter in the Universe [9, 10]. It is neutral, weakly interacting, stable (if R-parity is valid) particle and its relic density is typically consistent with present cosmological expectations. We examine the predictions for χ from the minimal supersymmetric standard model (MSSM) and compute the annihilation cross sections which requires the detailed knowledge of the whole SUSY spectrum. Consequently we can relate values of the neutralino relic abundance to values of the parameters $m_{1/2}, m_0, \mu_0$ — the common gaugino mass, the common scalar mass and the higgsino mass at M_X. The lower

limit on the age of the Universe provides an upper bound on the relic abundance of matter, and in particular of dark matter which is believed to be a dominant mass component of the Universe. We can therefore use the dark matter abundance constraint to derive bounds on the ranges of $m_{1/2}, m_0, \mu_0$ and in turn get constraints on the masses of all the SUSY particles.

In fact we can combine the dark matter constraint with others which are either phenomenological (values of $\alpha_s(m_Z), m_t, m_b$) or theoretical (avoiding mass hierarchy problem) and examine the consistency of trying to satisfy several of these constraints simultaneously. We conclude that one can indeed achieve such consistency quite naturally. More interestingly, we find that this happens for the ranges of the fundamental parameters $m_{1/2}$, m_0, μ_0, and thus also masses of the supersymmetric particles, all preferably within the few hundred GeV mass range and thus *well within the reach* of the SSC and the LHC but typically *above the reach* of LEP, the Tevatron, and LEP 200. The lower limit on supersymmetric particle masses comes from the dark matter constraint as will be discussed in section 3.

The LSP for which we find sufficiently large values of the relic abundance to explain at least DM in the galactic halos ($\Omega h_0^2 \gtrsim 0.025$) invariably comes out to be almost gaugino-like (bino-like) consistent with the conclusions of some previous analyses [11, 12, 13]. It was first noticed in Ref. [11] that a higgsino-like LSP is somewhat disfavoured as it corresponds to a high scale of supersymmetry breaking, typically exceeding 1 TeV, and thus a gaugino-like LSP was selected as a unique candidate for DM. More recently, it has been shown [13, 14] that for the higgsino-like neutralinos additional effects (co-annihilation with the next-to-lightest neutralino and the lightest chargino, see sect. 3) have a dramatic effect of reducing the LSP relic abundance below any interesting level. Here we find that higgsino-like LSPs are also largely excluded by the current lower bound on the mass of the top quark.

Overall, the LSP relic abundance constraint, combined with the other constraints narrows down the allowed ranges of $m_{1/2}, m_0, \mu_0$ considerably. We find that the region $m_{1/2} \gg m_0$ is excluded by the lower bound on the top mass, while in the region $m_{1/2} \ll m_0$ the LSP relic abundance is too large ($\Omega h_0^2 > 1$). Furthermore, the requirement that the LSP provide enough missing mass in the flat ($\Omega = 1$) Universe can be fulfilled only in a relatively narrow band of comparable values of $m_{1/2}$ and m_0 and for $1 < \mu_0/m_0 \lesssim a\ few$.

In the next section we briefly review and update the procedure used in

deriving the low-energy spectrum from a limited number of basic parameters at the GUT scale. In section 3 we calculate the neutralino relic density and compare it with other constraints on the parameter space. We conclude with final remarks in section 4.

2 Solutions of the MSSM

We consider the MSSM in the context of a unified theory. At the compactification scale M_X where the three couplings of $SU(3), SU(2), U(1)$ have a common value α_X the SUSY parameter space is characterised by the common values of the gaugino masses $m_{1/2}$, the common value of the soft mass terms of the squarks, sleptons and Higgs bosons m_0, and by μ_0, the mass parameter of the Higgs/higgsino bilinear term in the superpotential. (The suffix 0 denotes values at M_X.) In addition there are two parameters characterising the soft terms proportional to the superpotential terms: B_0 in the bilinear term $B_0\mu_0$, and a common trilinear soft parameter A_0 which multiplies the Yukawa terms. Also one should include at least the Yukawa couplings $h_{t0}, h_{b0}, h_{\tau 0}$ at M_X to consider as parameters in principle.

However we can reduce this apparently unmanageable host of parameters down to a manageable set as follows. Firstly the coefficients A_0 and B_0 are set to zero, in the spirit of string-derived versions of the model [6]. Below the scale M_X, B grows to a finite positive value and generally reaches a maximum and may even decrease to negative values. The values of M_X and α_X are determined by the unification of the gauge couplings. Their precise values for each solution are computed by an iterative procedure discussed in Ref. [6] since the running of the gauge couplings depends on knowing the individual SUSY thresholds which in turn depend on all the parameters including M_X and α_X themselves. This procedure requires the measured values of $\alpha_1(m_Z), \alpha_2(m_Z)$ from LEP but the value of $\alpha_3(m_Z)$ must be adjusted to achieve the required unification for each solution. Another adjustment is to choose h_{t0} such that the running Higgs mass squared $m_2^2(Q)$ takes on the precise value (negative and $\mathcal{O}(m_Z^2)$) at $Q = m_Z$ needed to give the required spontaneous breaking of electroweak symmetry, i.e.,

$$(m_1^2 - m_2^2) + (m_1^2 + m_2^2)\cos 2\beta = -m_Z^2 \cos 2\beta \qquad (1)$$

where m_1, m_2 are the running masses of the Higgs doublets coupling to down-

3

and up-type quarks respectively. Here the ratio of the Higgs v.e.v.s $v_2/v_1 = tan\beta = \cot\theta$ with β related to μ and B by $\sin 2\beta = 2B\mu/(m_1^2 + m_2^2)$. The running of m_2^2 and therefore the satisfying of eq. (1) is controlled by the value of h_{t0}. Actually the other significant Yukawa couplings h_b, h_τ should be included in this running of m_2^2 but in order to achieve eq. (1) in a controllable way we drop them which is justified as long as $\tan\beta$ is not too large ($\tan\beta \ll m_t/m_b$).

Thus each solution is specified by the values of the three parameters $m_{1/2}$, m_0, μ_0. Each solution then provides at low energies specific values for the quantities $\alpha_s(m_Z)$, $\tan\beta, \mu$, gaugino masses M_1, M_2, M_3, squark masses, slepton masses, Higgs masses, Higgsino masses and top-quark mass $m_t = (\sqrt{2}h_t m_W/g)\sin\beta$. Relaxing the constraints $A_0 = 0, B_0 = 0$ affects the resulting value of $\tan\beta$ mostly — see the analysis of Ref. [6], and so quantities which depend sensitively on $\tan\beta$ at low energies are, in principle, less precise, in our procedure. From Ref. [6] we see that, in general, $\tan\beta > 2$ even when A_0, B_0 are allowed to vary within values of $\mathcal{O}(m_0)$.

Another quantity associated with each solution is the ratio m_b/m_τ, assuming that this ratio is unity at M_X. Thus we include also $h_{b0} = h_{\tau 0}$ as another parameter in the running of the Yukawa couplings, and obtain a specific value for m_b for each solution. Apart from the above phenomenological constraints on the solutions we have the standard constraints that the Higgs potential be bounded, i.e.,

$$|\sin 2\beta| < 1, \quad m_1^2 m_2^2 < \mu^2 B^2 \tag{2}$$

and that all the physical mass squared be positive.

The strongest constraint for insisting that the SUSY spectrum is relatively light comes from the 'naturalness' argument [6, 15] which regards the need to tune the value of h_{t0} to a very high precision in order for m_2^2 to take the exact value given by eq. (1) at the scale $Q = m_Z$. A measure of this 'fine tuning' problem is the fine tuning constant c defined by [6]

$$c = \frac{\delta m_W^2}{m_W^2} / \frac{\delta h_t^2}{h_t^2} \tag{3}$$

so that absence of fine tuning means $c \approx 1$. Approximately we have [6]

$$c \approx \frac{m_0^2 + \mu_0^2 + k m_{1/2}^2}{m_Z^2} \tag{4}$$

4

A reasonable limit to the degree of precision needed would be $c \lesssim \mathcal{O}(10)$ and consequently the typical SUSY mass cannot be many times greater than m_Z.

We illustrate the various constraints by showing the values of m_t and m_b in Fig. 1a and $\alpha_s(m_Z)$ and c in Fig. 1b as a function of $m_{1/2}$ and m_0 for a fixed ratio $\mu_0/m_0 = 2$. The variation with μ_0/m_0 will be discussed later. The regions marked CDF and LEP are excluded by the CDF searches for the top ($m_t \gtrsim 91$ GeV) and the LEP searches for charginos ($m_{\chi_1^\pm} \gtrsim 46$ GeV), respectively. We see from Fig. 1a that the current 'experimental' value for m_b (in the $\overline{MS}$ scheme), $m_b(2m_b) = 4.25 \pm 0.1$ GeV [16], implies a rather heavy top quark ($m_t \gtrsim 150$ GeV) for the values of the input parameters $m_{1/2}, m_0$ and μ_0 roughly within the 1 TeV limit. On the other hand, beyond that range the resulting value of m_b is consistent with $m_t \lesssim 150$ GeV. Larger values of the input mass parameters are, however, disfavoured by the fine-tuning constraint and the current bounds on $\alpha_s = 0.122 \pm 0.010$ (based on analysis of jets at LEP) [17] as we can see from Fig. 1b. We also note that the uncertainty on $\tan\beta$ arising from allowing A_0 and B_0 to be non-zero (discussed above) would imply that m_t could be smaller by a further 10%.

One can see immediately that demanding $c \lesssim \mathcal{O}(10)$ forces one to consider only values of $m_{1/2}, m_0$ up to a few hundred GeV. This was the conclusion of the previous analysis [6]. Thus unification of the couplings demands a value of $\alpha_s(m_Z)$ close to the values extracted from jet analyses at LEP.

To summarise so far, the solutions obtained for the MSSM with the inclusion of electroweak symmetry allow a fairly restricted region of the parameters $m_{1/2}, m_0, \mu_0$ which is consistent with all the above constraints, *i.e.*, $m_{1/2}, m_0 \lesssim 200$ GeV, $\mu_0 \lesssim 400$ GeV. We will comment on the restrictions on the ratio μ_0/m_0 later.

3   The Neutralino Relic Abundance

The knowledge of the whole mass spectrum of both the ordinary and super-symmetric particles allows one to reliably compute the relic abundance of the lightest supersymmetric particle (LSP) as a candidate for the dark matter in the Universe.

At the outset we note that, in the parameter space not already excluded by LEP and CDF, we find that it is the lightest of the four neutralinos that is *always* the LSP. Another potential candidate for the LSP, the sneutrino,

5

has been now constrained by LEP to be heavier than about 42 GeV and, if it were the LSP, its contribution to the relic abundance would be of the order of 10^{-4}, and thus uninterestingly small. In the analysis presented here, the sneutrino is typically significantly heavier than the lightest neutralino. Typically, it is not even the lightest sfermion.

The actual procedure of calculating the relic abundance has been adequately described in the literature and will not repeated here. We use the technique developed in Ref. [18] which allows for a reliable (except near poles and thresholds) computation of the thermally averaged annihilation cross section in the non-relativistic limit and integrating the Boltzmann equation.

In the early Universe the LSP pair-annihilated into ordinary matter with total mass not exceeding $2m_\chi$. In calculating the LSP relic density one needs to include all possible final states. Lighter χs annihilate only (except for rare radiative processes) into pairs of ordinary fermions via the exchange of the Z and the Higgs bosons, and the respective sfermions. As m_χ grows new final states open up: pairs of Higgs bosons, gauge and Higgs bosons, ZZ and WW, and $t\bar{t}$. We include all of them in our analysis.

Generally one considers $\Omega h_0^2 > 1$ as incompatible with the assessed lower bound of about 10 Gyrs on the age of the Universe or, in more popular terms, as corresponding to too much mass in the Universe [10]. Many astrophysicists strongly favour the value $\Omega = 1$ (or very close to one), corresponding to the flat Universe, either because of cosmic inflation or for aesthetical reasons. Moreover, there is growing evidence that, on a global scale, the mass density indeed approaches the critical density, as well as that most of the matter in the Universe is non-shining and non-baryonic [10]. If one assumes that the LSP is the dominant component of dark matter in the flat ($\Omega = 1$) Universe then one typically expects

$$0.25 \lesssim \Omega h_0^2 \lesssim 0.5, \tag{5}$$

where the biggest uncertainty lies in our lack of knowledge of the Hubble parameter h_0 to better than a factor of two. As we will see shortly, varying somewhat the bounds in eq. (5) will not significantly alter our conclusions.

We present in Fig. 1c the relic abundance of the LSP and compare it with the other results shown before in Figs. 1a and 1b. Several features can be immediately noticed.

Firstly, most of the region corresponding to larger values of m_0 (roughly $m_0 \gtrsim 500$ GeV) is cosmologically excluded as it corresponds to $\Omega h_0^2 > 1$.

6

The relic abundance generally decreases with decreasing m_0 reaching very low values of Ωh_0^2 (0.025, or less) for m_0 roughly below 200 GeV, especially for $m_{1/2} > m_0$. It is worth noting that the region favoured by cosmology, eq. (5), takes a shape of a relatively narrow band running roughly parallel to the border of the area excluded by $\Omega h_0^2 > 1$. The contour $\Omega h_0^2 = 0.1$ shows how quickly Ωh_0^2 decreases with decreasing m_0 but also limits from below the region where the LSP relic abundance is reasonably large.

It is interesting to see what mass and compositions of the LSP correspond to its relic abundance favoured by cosmology. We remind the reader that, in minimal supersymmetry, the lightest neutralino and its three heavier partners χ_i^0 $(i = 1, ..., 4)$ are the physical (mass) superpositions of higgsinos $\widetilde{H}_1^0$ and $\widetilde{H}_2^0$, the fermionic partners of the neutral Higgs bosons, and of two gauginos $\widetilde{B}^0$ and $\widetilde{W}_3^0$, the fermionic partners of the neutral gauge bosons

$$\chi \equiv \chi_1^0 = N_{11}\widetilde{W}^3 + N_{12}\widetilde{B} + N_{13}\widetilde{H}_1^0 + N_{14}\widetilde{H}_2^0. \tag{6}$$

In distinguishing the gaugino-like and higgsino-like regions it is convenient to use the gaugino purity $p = N_{11}^2 + N_{12}^2$. In particular, the LSP is almost a pure bino where $p_{bino} \equiv N_{12}^2$ is close to one. In Fig. 1d we show the bino purity of the LSP. (The gaugino purity is almost identical.) Remarkably, we find that the band favoured by cosmology corresponds to the LSP being almost a pure bino ($\sim 95\%$) up to very large values of $m_{1/2}$. We also find that that higgsino-like LSPs are incidentally almost entirely excluded by the lower bound on the top quark of 91 GeV. (The contour of equal gaugino and higgsino contributions almost coincides with the contour $m_t = 91$ GeV.) It was also noticed in Ref. [13] that for a heavy top constraints from radiative gauge symmetry breaking exclude higgsino-like LSPs. (With the expectation for m_t to be actually much heavier than 91 GeV a larger cosmologically uninteresting region is likely to be ruled out.) The LSP mass contours are almost vertical in the gaugino region with m_χ growing with $m_{1/2}$, and almost horizontal in the higgsino region with m_χ increasing with m_0. Again, the lines meet in the narrow sub-diagonal region where the LSP is both a gaugino and a higgsino.

Since higgsino-like LSPs in our analysis not only give very little DM but also are practically excluded by the CDF top searches, we need not worry about the additional effect of the higgssino-like LSP 'co-annihilation' [19] with the next-to-lightest neutralino and the lightest chargino which has been

recently shown to significantly reduce the LSP relic density [13, 14]. We have explicitly verified that *all* solutions for which co-annihilation of the LSP with χ_2 and $\chi_1^\pm$ is important lie in the region excluded by $m_t \geq 91$ GeV. Thus neglecting the effects of co-annihilation is justified.

The LSP relic abundance in the allowed region is mostly dominated by its annihilation into fermionic final states, although in a few cases the Higgs final states contributed comparably. We thus do not expect that the radiative corrections to the Higgs masses due to the heavy top [20] would noticeably modify our results [12]. We also found that the lightest sfermion is either $\tilde{t}_R$ or $\tilde{l}_R$, in agreement with Ref. [13], except in the (mostly excluded by LEP) region of small m_0 and $m_{1/2}$ where it is the sneutrino.

We now pass to combine the band favoured by cosmology with the mass contours of the top and the bottom quarks. This is shown in Fig. 2. We see that the region where the LSP gives the dominant contribution to the matter density of the flat Universe (marked $\Omega = 1$) crosses the estimated value of the bottom quark mass ($m_b = 4.25 \pm 0.1$ GeV) for m_t broadly between 160 GeV and 180 GeV. Remarkably, this happens for 150 GeV $\lesssim m_{1/2}$, $m_0 \lesssim$ 400 GeV, the range also strongly favoured by constraints from α_s and fine tuning.

When the ratio μ_0/m_0 is decreased, the relic abundance contours generally move towards larger values of m_0 as do the contours for m_t and m_b. For $\mu_0/m_0 = 1$ the favoured range of the bottom quark mass of about 4.25 GeV lies entirely within the cosmologically excluded region $\Omega h_0^2 > 1$. It also becomes harder to reconcile this region with the fine tuning constraint and with a value of $\alpha_s(m_Z)$ close to 0.122. On the other hand as μ_0/m_0 increases, $m_b = 4.25$ takes us to a region of larger $m_{1/2}$ and lower m_0 while the contours relic abundance remain relatively unchanged. The area consistent with the constraints of m_b, m_t correspond to lower values of the relic abundance, $\Omega h_0^2 \lesssim 0.25$. If we increase μ_0/m_0 still further the region of overlap for the constraints of m_b, m_t, and $\Omega = 1$ shrinks and leads us the region of larger fine tuning and smaller α_s. We thus conclude that the combination of all the above constraints selects the range $1 \lesssim \mu_0/m_0 \lesssim a\ few$.

In the selected range all the Higgs bosons, squarks and sleptons, as well as the gluino, are significantly lighter than 1 TeV and thus are bound to be found at the LHC and SSC.

However, the expectation that the LSP dominates the dark matter relic density is a natural one. (In minimal supersymmetry no other particle can

even significantly contribute to the missing mass.) It then implies a significant *lower* bound on the spectrum of supersymmetric particle masses. We see from Figs. 1d and 2 that the LSP masses favoured by all the constraints lie in the range

$$60 \; GeV \lesssim m_\chi \lesssim 200 \; GeV, \tag{7}$$

the upper limit being also expected in the minimal supersymmetric model [11, 15] on the basis of naturalness. Similarly, we find

$$150 \; GeV \lesssim m_{\chi_1^\pm} \lesssim 300 \; GeV \tag{8}$$

$$200 \; GeV \lesssim m_{\tilde{l}} \lesssim 500 \; GeV \tag{9}$$

$$250 \; GeV \lesssim m_{\tilde{q}} \lesssim 850 \; GeV \tag{10}$$

$$350 \; GeV \lesssim m_{\tilde{g}} \lesssim 900 \; GeV. \tag{11}$$

gluino

The heavy Higgs bosons are roughly in the mass range between 250 GeV and 700 GeV. Of course, lower values of all these masses correspond to less fine tuning and larger values of α_s. The lightest Higgs boson tree-level mass invariably comes out close to m_Z; its one-loop-corrected value [20] is then roughly in the range 120 to 150 GeV. By comparing Figs. 1b and 2 we also find $0.116 \lesssim \alpha_s(m_Z) \lesssim 0.120$. (Larger values of α_s are also disfavoured by considering threshold corrections at the GUT scale [5].)

Thus, if the LSP is indeed the dominant component of DM in the flat Universe, supersymmetric particles are probably beyond the reach not only of LEP but also the Tevatron and LEP 200 [21, 12, 13, 22]. We note, on the other hand, that smaller ranges of supersymmetric particles are not firmly excluded but would correspond to the LSP contributing only a fraction of the critical density. We also note that we do not claim to have done a fully exhaustive search of the whole parameter space. In fact, Drees and Nojiri [13] have found in certain extreme cases (rather large values of A_0) squarks even somewhat lighter that 200 GeV and a lower limit $m_0 > 40$ GeV. We find that the condition $\Omega = 1$ requires in our case significantly larger values of m_0 ($m_0 \gtrsim 150$ GeV), in agreement with Refs. [12, 22]. However, we do not consider it to be in contradiction with the mentioned results of Ref. [13] but a reflection of somewhat different assumptions at the GUT scale and methods of deriving the supersymmetric mass spectra. We do not expect that the procedure adopted here would produce substantially modified results by performing a finer search of the parameter space.

9

4 Conclusions

Our basic conclusions for the neutralino relic abundance and the associated implications for the supersymmetric mass spectra are generally consistent with several other recent analyses. We do find that cosmologically attractive LSP is almost purely bino-like ($\sim 95\%$) and lies in the range $60 \text{ GeV} \lesssim m_\chi \lesssim 150 \text{ GeV}$. Moreover, as first noted in Ref. [21] and confirmed in Refs. [12, 13, 22], if the LSP dominates the dark matter in the (flat) Universe then the expected ranges of chargino, slepton and Higgs boson masses lie beyond the reach of LEP 200. The associated ranges of gluino and squark masses then exceed the reach of the Tevatron but should be discovered at the SSC and/or the LHC.

Generally, we find it very reassuring that, in the simplest and most economic supersymmetric scenario, a careful analysis of the implications of several different (and independent) constraints, including the DM constraint, which result from the grand unification conditions, leads to a supersymmetric spectrum accessible to the next generation of accelerators.

Acknowledgments

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Figure Captions

Figure 1: In the plane $(m_{1/2}, m_0)$ for the fixed ratio $\mu_0/m_0 = 2$ we show: in window a) the mass contours of the top and the bottom quarks (solid and short-dashed lines, respectively); in window b) the contours of $\alpha_s(m_Z)$ (solid) and the measure c of fine-tuning (dots), as discussed in the text; in window c) the relic abundance Ωh_0^2 of the LSP; and in window d) the mass contours of the LSP (solid) and the lightest chargino (dashed) at 50, 100, 150, 200, 500, and 1000 GeV, starting from left, and the contribution (dots) of the bino to the LSP composition (bino purity, as discussed in the text). In all the windows thick solid lines delineate regions experimentally excluded by the CDF (marked CDF) where $m_t < 91$ GeV and by the LEP experiments (LEP) where the lightest chargino is lighter than 46 GeV. In window c) we also mark by $\Omega h_0^2 > 1$ the region cosmologically excluded (too young Universe). The thin band between the thick dashed lines in window c) corresponds to the flat Universe ($\Omega = 1$), as discussed in the text. In window d) the region excluded by CDF almost coincides with the bino purity of 50% or less.

Figure 2: We show a blow-up of the down-left portion of the plane $(m_{1/2}, m_0)$ from the previous figure for the same fixed ratio $\mu_0/m_0 = 2$. We combine the mass contours of the top and the bottom quarks with the ones of the LSP relic mass density. We use the same textures as in Fig. 1 but we also show (two medium-thick short-dashed lines) the contours $m_b = 4.15$ GeV and 4.35 GeV which reflect the currently favoured range of the mass of the bottom quark (see text). We see that they cross the cosmologically favored region (thick long-dashed lines) marked $\Omega = 1$ at roughly 150 GeV $\lesssim m_{1/2}, m_0 \lesssim$ 400 GeV and for m_t broadly between 150 GeV and 180 GeV.
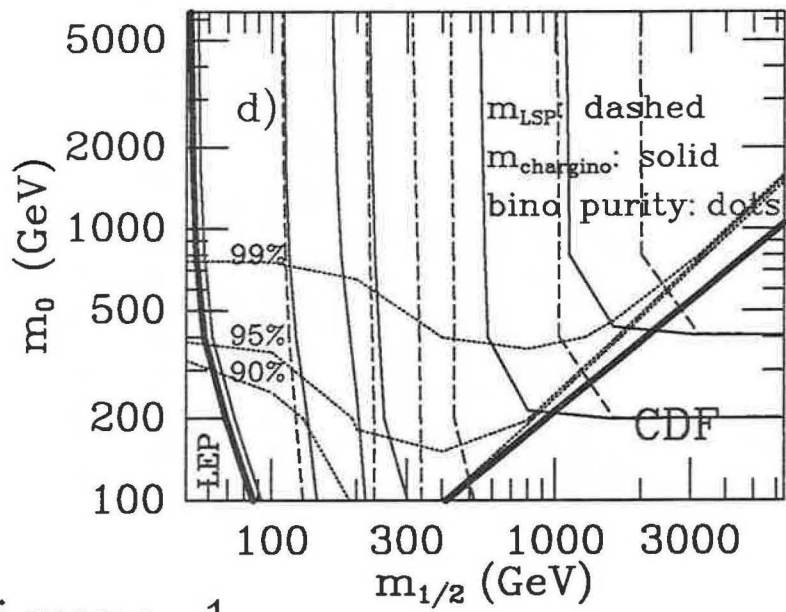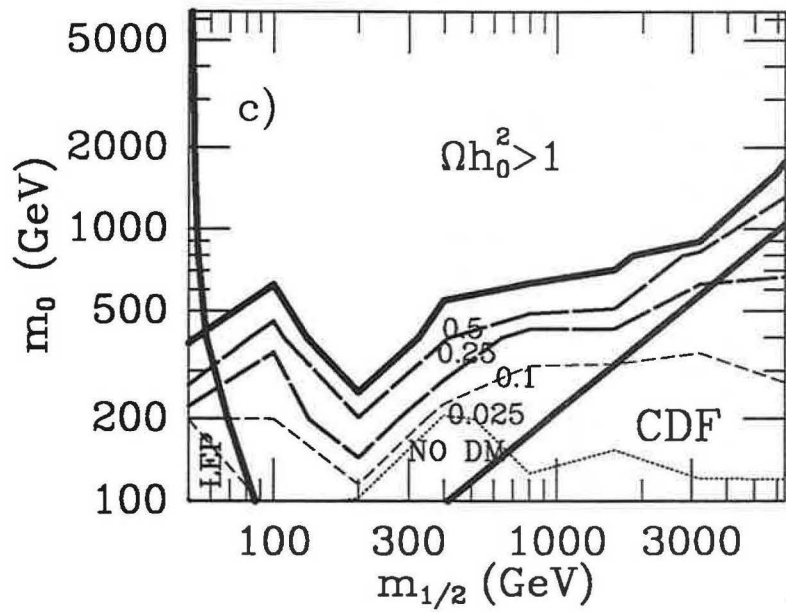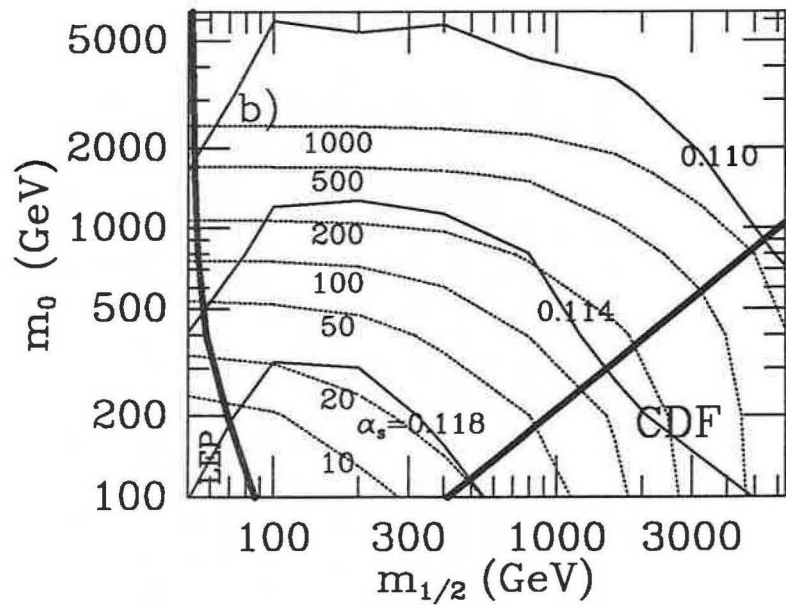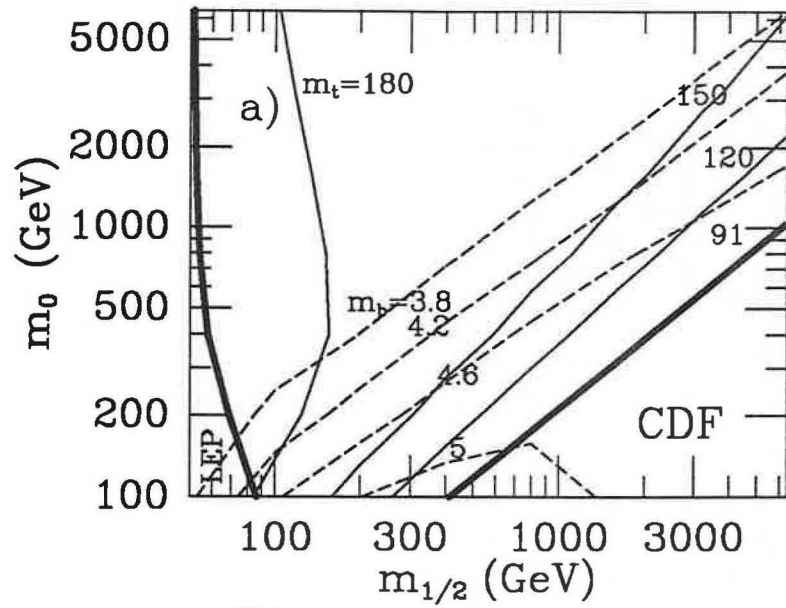
13

Figure 1

Figure 2