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Approximate Determination of the Gluon Density at Low-x From the F_2 Scaling Violations

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Abstract. A method to obtain an approximate relation between the unintegrated gluon density and the F_2 scaling violations at low-x is presented. The resulting formula can be used to determine the gluon density from the first HERA data, taken at low-x.

It was shown in ref. [1] that the gluon density at low-x can be obtained in a convenient way by analysing the longitudinal structure function. In this paper a similar method is applied using the Q^2 derivative of F_2 to obtain the gluon density to good accuracy. The basic idea rests on the fact that the scaling violation of F_2 arises, at low-x, from the gluon density alone and does not depend on the quark densities. This is illustrated in fig. 1 showing the scaling violation of the sea quark distribution as a function of x. At low-x, actually already at $x = 10^{-2}$, the quarks can be neglected in the Altarelli-Parisi equation, i.e. (for $n_f = 4$)

$$\frac{dF_2}{dlogQ^2} \approx \frac{5\alpha_s}{9\pi} \int_0^{1-x} G(\frac{x}{1-z}) P_{qg}(z) dz \tag{1}$$

where in lowest order

$$P_{qg}(z) = (1-z)^2 + z^2 \tag{2}$$

When applying (1) to experimental data the problem arises of determining the gluon distribution $G(x)$ ($= xg(x)$ where g is the gluon density) over the complete x-range. At low-x this problem can be avoided since the integral in (1) can then be performed approximately. For this purpose the gluon distribution is expanded in the following way

$$G(\frac{x}{1-z}) \approx G(z=1/2) + (z-1/2)G'(z=1/2) + (z-1/2)^2 \frac{G''(z=1/2)}{2} \tag{3}$$

This expression is then inserted in (1) and approximating the upper integration limit to 1 the second term will vanish in view of the symmetry of $P_{qg}(z)$ around $z = 1/2$. The third term is expected to give a small contribution compared to the first and is neglected. As a result one therefore obtains

$$\frac{dF_2}{dlogQ^2}(x) \approx \frac{5\alpha_s}{9\pi} G(2x) \int_0^1 P_{qg}(z) dz \tag{4}$$

This result is valid to all orders of the coupling constant. It shows that the effective value of the gluon momentum fraction during the integration of the Altarelli-Parisi equation is approximately $2x$ for low-x.
For a numerical study (4) is evaluated using the leading order expression (2) for $P_{qg}(z)$ to give:

$$\frac{dF_2}{dlogQ^2}(x) \approx \frac{5\alpha_s}{9\pi}\frac{2}{3}G(2x) \tag{5}$$

In fig. 2 the approximate form (5) is compared with the exact formula (1) for three different gluon distributions, MRS D_0 and D_- [2] and a distribution suggested by ref. [3] for the pomeron. Below $x = 10^{-3}$ the approximation is good to the level of 10%. The approximation is, however, slightly dependent on the shape of the gluon distribution. The reason for this is that the expansion is less accurate at the endpoints of the integration. If the distribution is large at these points the approximation is worse. In fig. 2a and 2c the distributions are rather large at low and high x respectively and the approximation is therefore worse compared to the result in fig. 2b.
The numerical study can be extended by assuming a gluon distribution of the general form $w^\delta(1-w)^a$ where w is the gluon momentum fraction. Using (2) one obtains

$$\int_0^{1-x} G(\tfrac{x}{1-z})P_{qg}(z)dz \approx (2x)^\delta(1-2x)^a\tfrac{2}{3} + \tfrac{1}{30}(2x)^\delta[4\delta(\delta+1)(1-2x)^a - 8\delta ax(1-2x)^{a-1}$$

$$- 8(\delta+2)ax(1-2x)^{a-1} + 16x^2a(a-1)(1-2x)^{a-2}] \tag{6}$$

Since x is small ax can be neglected and $1-2x$ approximated by 1 so that effectively $a = 0$ and the gluon distribution is proportional to w^δ, i.e.

$$\int_0^{1-x} G(\frac{x}{1-z})P_{qg}(z)dz \approx (2x)^\delta\frac{2}{3} + \frac{2}{15}(2x)^\delta\delta(\delta+1) \tag{7}$$

The factor $[1+\delta(\delta+1)/5]$, i.e. the change to the leading order result due to the second term of the expansion, is shown as a dashed line in fig. 3. One may compare it to the ratio of (1) to the approximation (5) which is shown as a solid line. The agreement between the two lines shows that the further terms of the expansion can reliably be neglected. The smallness of the second term verifies (5).
It is interesting to note that distributions with $\delta \sim -0.5$ give less accurate result than for $\delta < -0.5$ although the latter distributions are more singular than the former. Due to the symmetry of $P_{qg}(z)$ the approximation (4) is forced to be exact at $\delta = -1$ (if the upper integration limit is set to 1) since

$$\int_0^1 \frac{1-z}{x}P_{qg}(z)dz = \frac{1}{2x}\int_0^1 P_{qg}(z)dz \tag{8}$$

changing the otherwise decreasing function with decreasing δ in fig. 3 to increase. This shows that the essential ingredient for this method to work is the symmetry property of P_{qg}.
As long as $-1.2 < \delta < 0.2$ the approximation is better than 10%. This range of δ comprises the predictions from the DGLAP, the Lipatov and the GLR equations. Adding a 5% uncertainty in the experimental measurement of the Q^2-dependence of F_2, 10% for α_s and 10% for neglecting typical low-x effects calculated beyond the DGLAP approximation such as gluon recombination into quarks and relaxation of the strong ordering of the transverse momenta [4], one obtains a total accuracy of around 18%, summing all sources in quadrature. This is accurate enough to distinguish between several different

gluon parametrisations now available on the market [5].
Of particular interest would be to distinguish between a gluon distribution behaving as a constant at low-x, as is conventionally assumed and can be explained through gluon recombination effects described by the GLR-MQ equation [6], and one proportional to $1/\sqrt{x}$ which is approximately predicted by the Lipatov equation [7]. MRS have included both alternatives in their fit of existing experimental data resulting in equally good descriptions. However, at low-x, outside the range of existing data, the two functions differ dramatically as is shown in fig. 4.
The first HERA data, occurring mainly at low-x, could be used for this purpose with the method described here. $dF_2/dlogQ^2$ has been estimated [8] to be measured with a high enough precision at the design luminosity down to $x = 5 \cdot 10^{-4}$.
Another interesting application is in the measurement of the pomeron structure function at HERA as described in ref. [9]. It was shown there that assuming the pomeron to be purely gluonic, gluon recombination is expected to occur at a significant level provided the gluon density is not too small at low-x. This could quickly be investigated using eq. (4). Having knowledge of the pomeron gluon density at the level of $20 - 40\%$ at low-x might be sufficient to establish the occurrence of gluon recombination.
In summary, a convenient method to determine the gluon distribution from the Q^2 dependence of F_2 has been introduced. The essential result is (4) which is valid to all orders of α_s since only the symmetry of P_{qg} has been used in its derivation. This is favourable compared to ref. [1] where the approximation of F_L was given only in lowest order.
The argument of G, $2x$, as obtained in (4) is also a useful result in another respect. Having knowledge of the effective gluon momentum fraction simplifies the usage of the gluon to quark transition formula in the transverse momentum scheme (see ref. [4] and references therein for a review and a numerical analysis of the subject). The result obtained here is a hint on what has to be used for the argument in this case. It depends of course on the actual gluon distribution, but the best value of the argument should be close to $2x$.

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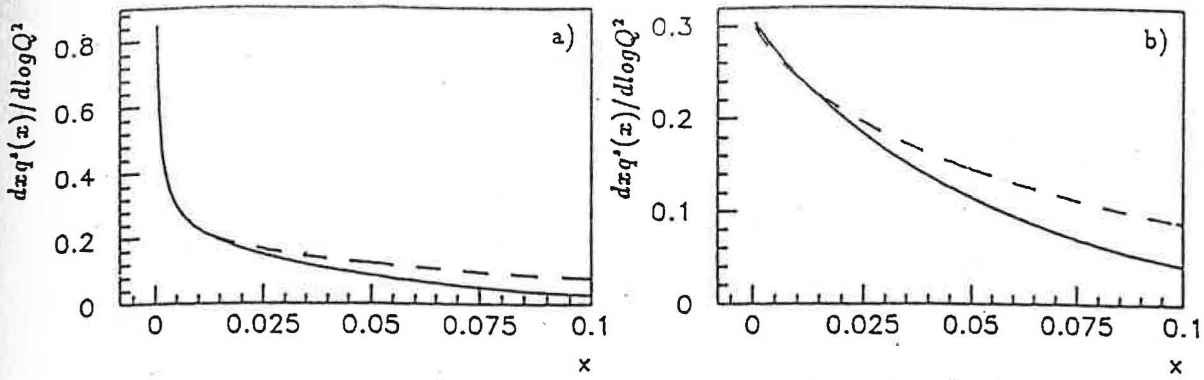

Figure 1: *The scaling violations of the sea quark distribution using KMRS B_- (a) and B_0 (b) parton parametrisations. The full line corresponds to the complete Altarelli-Parisi equation whereas the dashed line was obtained neglecting the quark distributions.*

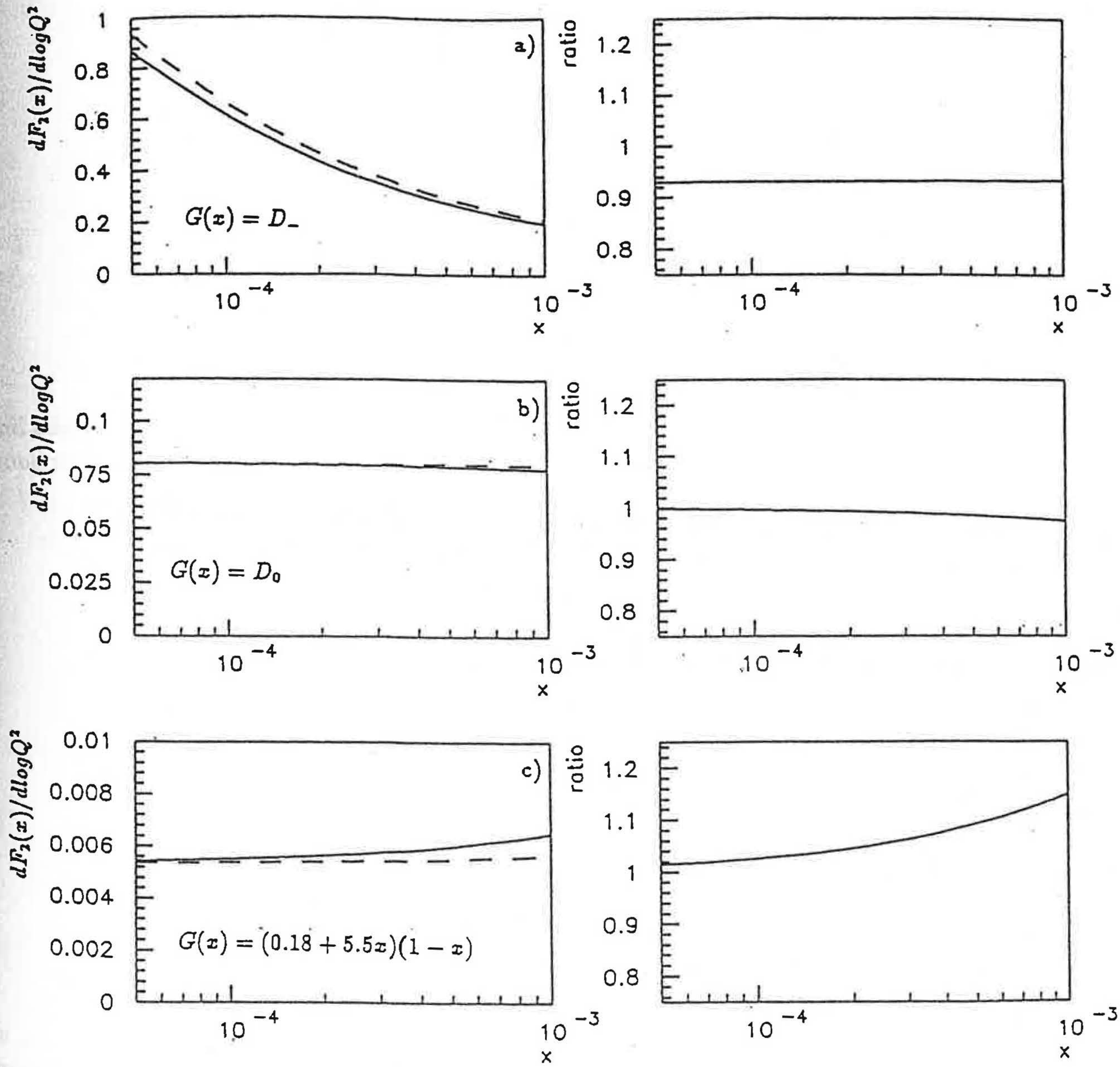

Figure 2: *Illustration of the accuracy of the approximation (5). The F_2 scaling violations is shown on the left side with a dashed line for the approximation (5) and a solid line for the exact formula (1). The right-hand plots show the ratio of the corresponding two lines. The gluon distributions used were MRS D_- (a), MRS D_0 (b) [2] and the calculated pomeron gluon distribution [3] (c).*

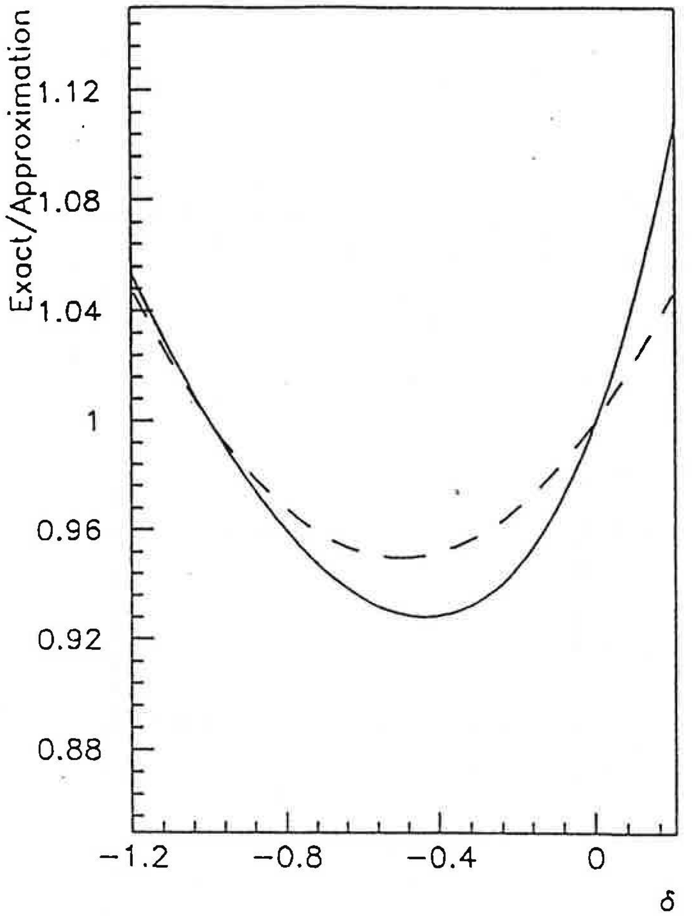

Figure 3: *The dashed line shows the ratio of (7) to its first term on the RHS. The solid line is the ratio of (1) to (4) at* $x = 5 \cdot 10^{-4}$*. The gluon distribution was taken as proportional to* x^{δ}*.*

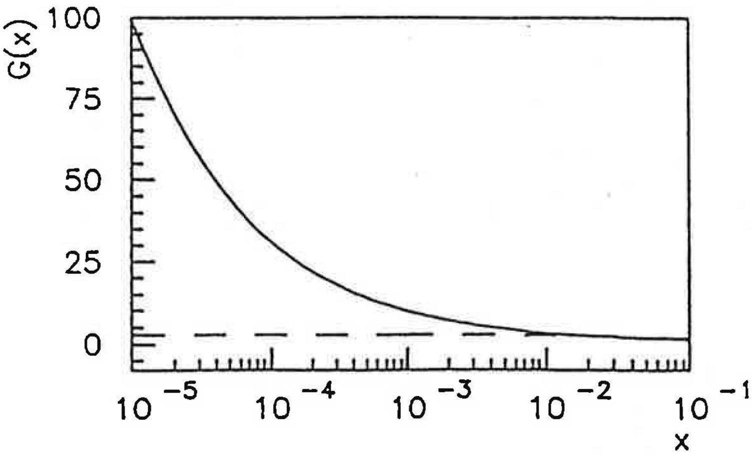

Figure 4: *The gluon distributions obtained by ref. [2]. The dashed curve is the* D_0 *and the solid curve the* D_- *distribution. The today existing data cannot distinguish between these distributions.*