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Damping and absorption of high-frequency waves in dusty plasmas

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Abstract

Extending the results of Forlani et al. (*Physica Scripta* **45**, 509, 1992), it is shown how the ensemble averages over statistical distributions of charged massive grains in a plasma can result in "average" dispersion relations for high-frequency electromagnetic and electrostatic waves. The dispersion relations admit solutions for complex frequencies and wave numbers, suggesting that the waves can be damped ($Im\,\omega \neq 0$) or absorbed ($Im\,k \neq 0$) in a dusty plasma.

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I. INTRODUCTION

The physics of dusty plasmas - plasmas in the presence of massive charged dust particles - has been the subject of many recent investigations (e.g., *Physica Scripta* Vol. **45**, pp. 465-544, 1992), and there is a growing interest for these systems due to the ubiquitous presence of charged dust grains in laboratory, space, and astrophysical plasmas.

The effects of charged dust particulates on the wave propagation have been considered by a number of authors[1-4]. De Angelis et al.[1] and Forlani et al.[2] found some changes in the dispersion relation of small amplitude waves in dusty plasmas, in which a spatial inhomogeneity is created by distribution of immobile dust grains. Thus, the charged dust grains, with the associated plasma particles' perturbations, contribute a statistical (random, if the grains are not interacting) background which can affect the properties of propagating waves. On the other hand, very low-frequency dust acoustic waves are shown to exist in dusty plasmas when the grains are either mobile[3] or stationary[4].

The wave propagation in "random media" has been thoroghly investigated in the past[5,6]. The wave "localization by disorder " (e.g. a fluctuating density) has been shown by Escande and Souillard[7]. The modification of the dispersion relation for spin-waves in a medium with random magnetic parameters has been discussed by Ignatchenko and Iskhatov[8], whereas damping of plasma waves in a metal with random inhomogeneities has been investigated by Ignatchenko et al.[9].

In this paper, we extend the model of Forlani et al.[2] and consider the propagation of small amplitude high-frequency waves in the presence of the electrostatic potential $\phi(\vec{r})$ (due to the plasma particles and the charged grains) which is a random function of position due to the random distribution of the charged dust grains in unmagnetized dusty plasmas. The manuscript is organized as follows: In Sec. II we discuss the statistical properties of a distribution of charged dust particles and obtain the ensemble averages for the quantities $< \phi(\vec{r}) >$, and $< \phi(\vec{r})\phi(\vec{r}') >$, where the bracket $< >$ denotes the statistical average. These are then used in Sec. III, where dispersion relations for small amplitude electromagnetic and Langmuir waves are derived in the high-frequency domain when any changes in either the grains' charge or positions can be ignored. Numerical results and discussions are contained in Sec. IV.

II. STATISTICS OF DUSTY PLASMAS

In this Section, we consider the stationary properties of dusty plasmas. For a system of N_g identical point-grains of equal charge eZ_g at positions $\left\{\vec{R}\right\} = \left(\vec{R}_1, \ldots, \vec{R}_{N_g}\right)$ in a volume V, the dust charge density is given by

$$\rho_g\left(\vec{r}, \left\{\vec{R}\right\}\right) = \frac{eZ_g}{V} \sum_i \delta\left(\vec{r} - \vec{R}_i\right). \tag{1}$$

We also assume that the dusty plasma system, whose constituents are charged dust grains, electrons and ions, is overall neutral. That means

$$\sum_\alpha q_\alpha \overline{n}_\alpha + \overline{\rho}_g = 0, \tag{2}$$

where $q_\alpha, \overline{n}_\alpha$ are the charge and mean number densities of the plasma particles α, and

$$\overline{\rho}_g = \frac{1}{V} \int \rho_g d^3r = eZ_g \frac{N_g}{V} \tag{3}$$

is the mean dust charge density. Here e is the magnitude of the electron charge.

Clearly, in dusty plasma systems, $\overline{n}_e \neq \overline{n}_i$, and the plasma is non-neutral due to the charging processes of the dust grains. For negatively charged grains ($Z_g < 0$), we may have $\overline{n}_e << \overline{n}_i$ and $\omega_{pe} \approx \omega_{pi}$. Thus, the electron plasma frequency can be substantially reduced in some cases.

The non-neutrality of the plasma introduces a mean potential

$$\overline{\phi} = \frac{1}{V} \int \phi(\vec{r})\, d^3 r = -\frac{4\pi}{k_D^2} \sum_\alpha q_\alpha \overline{n}_\alpha = \frac{4\pi}{k_D^2} \overline{\rho}_g, \tag{4}$$

where $k_D = \lambda_D^{-1}$ is the plasma Debeye length[10,11], which allows a linearization of the particle distribution functions and the Poisson equation provided that

$$\frac{|q_\alpha \delta\phi(\vec{r})|}{T_\alpha} << 1, \tag{5}$$

where T_α is the temperature of the plasma species α, and

$$\delta\phi(\vec{r}) = \phi\left(\vec{r}, \left\{\vec{R}\right\}\right) - \overline{\phi}. \tag{6}$$

We note that (5) is, in general, a much weaker requirement than the condition that the potential $\phi(r)$ itself be "small": the latter requires that the potential fluctuations (6) around the mean value be "small".

Subsequently, the particle distribution functions that are taken as solutions of the stationary Vlasov equation and which are of the form

$$F_\alpha^0(\vec{r}, \vec{v}) = \overline{n}_\alpha f_{\alpha 0}(\vec{v}) \frac{\exp\left(-q_\alpha \delta\phi(\vec{r}) / T_\alpha\right)}{\frac{1}{V} \int \exp\left(-q_\alpha \delta\phi(\vec{r}) / T_\alpha\right) d^3 r}, \tag{7}$$

can be expanded, upto second order in $\delta\phi$, as

$$F_\alpha^0(\vec{r}, \vec{v}) = \overline{n}_\alpha f_{\alpha 0}(\vec{v}) \left(1 - \frac{q_\alpha}{T_\alpha} \delta\phi(\vec{r}) + \frac{1}{2}\left(\frac{q_\alpha}{T_\alpha}\right)^2 \left[\delta\phi^2(\vec{r}) - \overline{\delta\phi^2(\vec{r})}\right]\right), \tag{8}$$

where the condition (from the definition of $\overline{\phi}$)

$$\frac{1}{V} \int \delta\phi(\vec{r})\, d^3 r = 0 \tag{9}$$

has been used, and the last term in (8), namely,

$$\overline{\delta\phi^2(\vec{r})} \equiv \frac{1}{V} \int \delta\phi^2(\vec{r})\, d^3 r \tag{10}$$

comes from the expansion of the denominator.

We note that the function $f_{\alpha 0}(\vec{v})$ is an (arbitrary) isotropic function of velocity, because the solutions of the stationary Vlasov equation are (arbitrary) functions of the particles' energy

$$\frac{1}{2}m_\alpha v^2 + q_\alpha \phi\left(\vec{r}\right) \ .$$

The solution of the linearized Poisson equation

$$\nabla^2 \phi\left(\vec{r}\right) - k_D^2 \phi\left(\vec{r}\right) + 4\pi \rho_g\left(\vec{r}\right) = -\left(4\pi \sum_\alpha q_\alpha \overline{n}_\alpha + k_D^2 \overline{\phi}\right), \tag{11}$$

corresponding to the particular choice (4) is given by (in Fourier components):

$$\delta\phi_k = \frac{4\pi \delta\rho_k}{k^2 + k_D} \tag{12}$$

where $\delta\rho_k = \rho_{gk} - \overline{\rho}_g \delta\left(\vec{k}\right)$ and

$$\rho_{gk} = eZ_g \sum_i e^{-i\vec{k}\cdot\vec{R}_i} \ . \tag{13}$$

It is clear that the potential distribution in the system and the associated plasma particle perturbations (through (8)) depend on the grains' positions $\left\{\vec{R}\right\}$. The latter are, in general, unknown. In the following, we consider cases when the grains have a statistical distribution and establish the ensemble averages of the potential and its correlator.

For high charges ($Z_g >> 1$) and small inter-grain distance $d \sim n_g^{-1/3} (d < \lambda_D, n_g = N_g/V =$ number density of grains), the electrostatic interactions between the charged grains will establish a correlation between their positions. In the opposite limit of small charges or large distances ($d > \lambda_D$), the grain charge is totally screened by the plasma particles and any correlation will vanish. Accordingly, the positions will be totally random and $\phi(\vec{r})$ will be a random function of position in space.

For any function $f\left(\vec{r},\left\{\vec{R}\right\}\right)$, the ensemble average is defined as[12]

$$< f(\vec{r}) >= \int f\left(\vec{r},\left\{\vec{R}\right\}\right) W\left(\left\{\vec{R}\right\}\right) d^3R_1 \ldots d^3R_N, \tag{14}$$

where $W\left(\left\{\vec{R}\right\}\right) d^3\left\{R\right\}$ is the probability of a configuration with particle 1 in d^3R_1, particle N in d^3R_N, and is given by:

$$W\left(\left\{\vec{R}\right\}\right) d^3\left\{R\right\} = \frac{1}{Z_N} e^{-\beta U(\{\vec{R}\})} d^3\left\{R\right\}, \tag{15}$$

where

$$Z_N = \int e^{-\beta U(\{\vec{R}\})} d^3\left\{R\right\} \tag{16}$$

is the configurational integral and $\beta U\left(\left\{\vec{R}\right\}\right)$ is the ratio of the potential to thermal energy of the system of interacting particles.

For non-interacting grains $U = 0, Z_N = V^N, W\left(\left\{\vec{R}\right\}\right) = V^{-N}$ and

$$< f(\vec{r}) >= \frac{1}{V^N} \int f\left(\vec{r},\left\{\vec{R}\right\}\right) d^3R_1 \ldots d^3R_N\ , \tag{17}$$

which corresponds to a random average.

The probability to find particle 1 in d^3R_1 independently of where the remaining $N-1$ particles are, is clearly:

$$W^{(1)}\left(\vec{R}_1\right) = \frac{1}{Z_N} \int e^{-\beta U} d^3R_2 \ldots d^3R_N = \frac{1}{V}\ , \tag{18}$$

while the conditional probability to find particle 1 in d^3R_1 when particle 2 is in d^3R_2, independently of where the remaining $N-2$ particles are, is defined as the pair correlation function of the system:

$$g(\vec{R}_1, \vec{R}_2) = \frac{V^2}{Z_N} \int e^{-\beta U} d^3R_3 \ldots d^3R_N\ . \tag{19}$$

For translationally invariant systems this function only depends on $\vec{R}_{12} = |\vec{R}_2 - \vec{R}_1|$, and it is useful to introduce the structure factor $S(k)$ of the system defined as the Fourier transform:

$$S(k) = 1 + \frac{N}{V} \int \left(g(\vec{R}) - 1 \right) e^{-i\vec{k}\cdot\vec{R}} d^3 R \, . \tag{20}$$

It is well known that $g(\vec{R})$, or $S(k)$, contains all the statistical information for a given system i.e., their knowledge allows the calculation of all its statistical properties like averages, variance, and correlators[13].

For the case of random distributions, we have

$$g(\vec{R}) = 1, \qquad S(k) = 1 \tag{21}$$

i.e., there is no correlation in the absence of interactions. We shall establish the ensemble averages for dusty plasmas in general (interacting grains) and then use (21) in the limit of random grains.

First consider the ensemble averaged potential

$$\begin{aligned} < \phi(\vec{r}) > =& \frac{1}{Z_N} \int e^{-\beta U} \phi \left(\vec{r}, \left\{ \vec{R} \right\} \right) d^3 \{R\} \\ =& \frac{1}{Z_N} \int d^3 \{R\} \, e^{-\beta U} \int \phi_k \left(\left\{ \vec{R} \right\} \right) e^{i\vec{k}\cdot\vec{R}} \frac{d^3 k}{(2\pi)^3} \, . \end{aligned} \tag{22}$$

Using (12) and 13) we can write (22) as

$$< \phi(\vec{r}) > = \frac{4\pi e Z g}{(2\pi)^3} \int \frac{d^3 k}{k^2 + k_D^2} N_g \int e^{-i\vec{k}\cdot(\vec{R}_1 - \vec{r})} d^3 R_1 \left(\frac{1}{Z_N} \int e^{-\beta U} d^3 R_2 \ldots d^3 R_N \right) \, , \tag{23}$$

which on using (18) becomes

$$< \phi(\vec{r}) > = \frac{4\pi e Z g}{(2\pi)^3} n_g \int \frac{d^3 k}{k^2 + k_D^2} \int e^{-i\vec{k}\cdot(\vec{R}_1 - \vec{r})} d^3 R_1 = \frac{4\pi e Z_g n_g}{k_D^2} \, , \tag{24}$$

where we have used $(2\pi)^3 \delta(\vec{k})$ for the last integral.

Recalling (4), we see that

$$< \phi(r) >= \overline{\phi} \ , \tag{25}$$

which proves the ergodicity of the function $\phi(\vec{r})$ (its ensemble average coincides with the space integral) and, therefore,

$$< \delta\phi(\vec{r}) >= 0 \ , \tag{26}$$

showing that the average of the potential fluctuations vanishes.

As shown in the next section, this is the reason why it is necessary to go to the second order (in $\delta\phi$) in the perturbation theory, which is the first non-vanishing order. This requires the calculation of the "correlator"

$$< \phi(\vec{r}_1)\phi(\vec{r}_2) >= \frac{1}{Z_N} \int e^{-\beta U} \phi\left(\vec{r}_1, \left\{\vec{R}\right\}\right) \phi\left(\vec{r}_2, \left\{\vec{R}\right\}\right) d^3R_1 \dots d^3R_N \ . \tag{27}$$

Using again the Fourier components for ϕ and (12) we have

$$\begin{aligned} < \phi(\vec{r}_1)\phi(\vec{r}_2) >= & \frac{(4\pi)^2}{(2\pi)^6} (eZ_g)^2 \iint \frac{d^3k d^3k'}{\left(k^2 + k_D^2\right)\left(k'^2 + k_D^2\right)} \\ & < \sum_i e^{-i\vec{k}\cdot(\vec{R}_i - \vec{r}_1)} \sum_j e^{-i\vec{k}'\cdot(\vec{R}_j - \vec{r}_2)} > \ . \end{aligned} \tag{28}$$

After some tedious but straightforward algebra, using the definitions (19) and (20), it can be shown that the average of the double sum gives

$$\begin{aligned} & < \sum_i e^{-i\vec{k}\cdot(\vec{R}_i - \vec{r}_1)} \sum_j e^{-i\vec{k}'\cdot(\vec{R}_j - \vec{r}_2)} > \\ = & e^{-i\vec{k}\cdot(\vec{r}_2 - \vec{r}_1)} (2\pi)^3 \delta\left(\vec{k} + \vec{k}'\right) \left[S(k) + (2\pi)^3 n_g \delta\left(\vec{k}\right)\right] \ . \end{aligned}$$

Thus (28) becomes

$$< \phi(\vec{r}_1)\phi(\vec{r}_2) >= \frac{(4\pi e Z_g)^2}{(2\pi)^3} n_g \left\{ \int S(k) \frac{e^{-i\vec{k}\cdot(\vec{r}_2 - \vec{r}_1)}}{\left(k^2 + k_D^2\right)^2} d^3k + \frac{(2\pi)^3 n_g}{k_D^4} \right\} \tag{29}$$

where $S(k)$ is the structure factor of the system of charged grains.

The correlator of the potential fluctuations is given by

$$< \delta\phi\left(\vec{r}_1\right)\delta\phi\left(\vec{r}_2\right) >=< \phi\left(\vec{r}_1\right)\phi\left(\vec{r}_2\right) > -\overline{\phi}^2 , \tag{30}$$

which on using (4) and (29) can be written as

$$< \delta\phi\left(\vec{r}_1\right)\delta\phi\left(\vec{r}_2\right) >= \frac{\left(4\pi e Z_g\right)^2}{8\pi k_D} n_g C\left(\left|\vec{r}_2 - \vec{r}_1\right|\right) , \tag{31}$$

where the dimensionless function

$$C\left(r\right) = \frac{k_D}{\pi^2}\int \frac{S\left(k\right)}{\left(k^2 + k_D^2\right)^2} e^{-i\vec{k}\cdot\vec{r}} d^3k \tag{32}$$

is the "correlation function of the fluctuations". In the following, we shall also need the correlator of the Fourier components of the potential fluctuations; from (31) and (32) we find it to be

$$< \delta\phi_{\vec{k}}\delta\phi_{\vec{k}'} >= \left(4\pi e Z_g\right)^2 n_g \frac{S\left(k\right)}{\left(k^2 + k_D^2\right)^2}\delta\left(\vec{k} + \vec{k}'\right) . \tag{33}$$

Equations (26), (31) and (33) are general. For the particular case of random grains (i.e., no interactions),we have $S(k) = 1$ and the following results are obtained:

a) vanishing average of the potential fluctuations:

$$< \delta\phi_k >=< \delta\phi\left(\vec{r}\right) >= 0 , \tag{34}$$

b) correlation function of the fluctuations:

$$C\left(r_{12}\right) = \frac{k_D}{\pi^2}\int \frac{e^{-i\vec{k}\cdot\vec{r}_{12}}}{\left(k^2 + k_D^2\right)^2} d^3k = e^{-r_{12}/\lambda_D} , \tag{35}$$

which is the Booker-Gordon function with a correlation length given by the plasma Debye length[1,6].

c) correlator of the Fourier components of the potential fluctuations:

$$< \delta\phi_{\vec{k}}\delta\phi_{\vec{k}'} >= \frac{\left(4\pi e Z_g\right)^2}{8\pi k_D} n_g \tilde{C}\left(k\right)\delta\left(\vec{k} + \vec{k}'\right) , \tag{36}$$

where $\tilde{C}_k$, known as the "spectral function of the correlations", is the Fourier transform of (35) given by:

$$\tilde{C}(k) = \frac{8\pi k_D^{-3}}{\left(1 + k^2/k_D^2\right)^2} \ . \tag{37}$$

We shall make use of the above results in Sec. III, where we derive the dispersion relations for small amplitude waves in random dusty plasmas. A discussion of the effects of an interaction between the charged grains is given in Sec. IV.

III. THE DISPERSION RELATION

Consider small amplitude high-frequency (in comparison with the ion plasma frequency) waves propagating in the dusty background described in Sec. II. The total electric field seen by the plasma particles is now

$$\vec{E}_T = \vec{E} - \vec{\nabla}\phi\left(\vec{r}, \left\{\vec{R}\right\}\right) \ , \tag{38}$$

and the electron distribution function in the presence of the wave electric field $\vec{E}(\vec{r},t)$ is written as

$$f\left(\vec{r},\vec{v},t\right) = F^0\left(\vec{r},\vec{v}\right) + \delta f\left(\vec{r},\vec{v},t\right) \ , \tag{39}$$

where F^0 is given by (8) and δf is the (small) perturbation due to the propagating wave. Since the frequency ω of the latter is assumed to be much larger than the ion plasma frequency, the immobile ions, which form the neutralizing background, do not respond to the wave fields.

The linearized Vlasov equation governing the perturbed distribution function reads

$$\frac{\partial}{\partial t}\delta f + \left(\vec{v}\cdot\vec{\nabla}\right)\delta f - \frac{e}{m_e}\vec{E}\cdot\frac{\partial}{\partial\vec{v}}F^0 = -\frac{e}{m_e}\vec{\nabla}\delta\phi\cdot\frac{\partial}{\partial\vec{v}}\delta f \ , \tag{40}$$

where the Lorentz force (for the case of electromagnetic waves) gives no contribution because, as mentioned in Sec. II, the unperturbed distribution function F^0 is an isotropic function of velocity. Equation (40) is solved (in Fourier space) by iterations up to the second order in $\delta\phi$ using the expansion (8) (for details, e.g. Ref. 2), and subsequently the electron current density can be obtained in the form

$$\vec{j}_{k\omega} = -e \int \vec{v} \delta f d^3 v = \vec{j}^{(0)}_{k\omega} + \vec{j}^{NL}_{k\omega} , \tag{41}$$

where the linear current density is given by

$$\vec{j}^{(0)}_{k\omega} = -\frac{i\omega}{4\pi} \left(\varepsilon_{k\omega} - 1\right) \vec{E}_{k\omega} , \tag{42}$$

where

$$\varepsilon^t_{k\omega} = 1 - \frac{\omega_p^2}{\omega} \lambda \int \frac{f_{0e}(v)}{\omega - \vec{k} \cdot \vec{v}} d^3 v \tag{43}$$

and

$$\varepsilon^l_{k\omega} = 1 - \omega_p^2 \lambda \int \frac{f_{0e}(v)}{\left(\omega - \vec{k} \cdot \vec{v}\right)^2} d^3 v , \tag{44}$$

are (for $\lambda = 1$) the plasma dielectric functions for transverse and longitudinal waves, respectively. Here

$$\omega_p^2 = \frac{4\pi \overline{n}_e e^2}{m_e} \tag{45}$$

is the plasma frequency corresponding to the actual mean number of the electrons in the system. Furthermore, the correction term to the dielectric functions:

$$\lambda = 1 - \frac{1}{2} \frac{e^2}{T^2} \overline{\delta\phi^2(r)} \tag{46}$$

comes from the expansion (8) of the background distribution function. From (31) we have

$$< \overline{\delta\phi^2(\vec{r})} >=< \overline{\delta\phi(\vec{r})\, \delta\phi(\vec{r})} >= \frac{(4\pi e Z_g)^2}{8\pi k_D} n_g , \tag{47}$$

so that the dielectric function $< \varepsilon_{k\omega} >$ contains a correction due of the order of $n_g Z_g^2$ due to $< \lambda >$, which is associated with the grains.

The nonlinear current density $\vec{j}^{NL}$ is the sum of the contributions to the first and the second order in $\delta\phi$, as given by Forlani et al.[2]. The wave equations for transverse and longitudinal waves are given, respectively, by

$$\left(\frac{k^2c^2}{\omega^2} - \varepsilon^t_{k\omega}\right) \vec{E}_{k\omega} = \frac{4\pi i}{\omega} \vec{j}^{NL^{(t)}}_{k\omega} \ , \tag{48}$$

and

$$k\varepsilon^l_{k\omega} E_{k\omega} = -\frac{4\pi i}{\omega} \vec{k} \cdot \vec{j}^{NL^{(l)}}_{k\omega} \ . \tag{49}$$

From the results of Forlani et al.[2], the ensemble average of the wave equations, using the expressions for $\vec{j}^{NL}$, gives

$$\left(\frac{k^2c^2}{\omega^2} - < \varepsilon^t_{\omega k} >\right) < E^n_{\omega k} > = \iint \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} < \delta\phi_{\vec{k}_2} \delta\phi_{\vec{k}-\vec{k}_1-\vec{k}_2} > \\ \Lambda^{nm}(kk_1k_2) < E^m_{\omega k_1} > \ , \tag{50}$$

and

$$< \varepsilon^l_{\omega k} >< E_{\omega k} > = \iint \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} < \delta\phi_{\vec{k}_2} \delta\phi_{\vec{k}-\vec{k}_1-\vec{k}_2} > \\ \Lambda(kk_1k_2) < E_{\omega k_1} > \tag{51}$$

for the transverse and longitudinal waves, respectively. Here

$$\Lambda^{nl}(kk_1k_2) = \frac{1}{\frac{|\vec{k}_1+\vec{k}_2|^2c^2}{\omega^2} - \varepsilon^{t(0)}_{\omega\vec{k}_1+\vec{k}_2}} \beta_{nm}\left(\vec{k}; \vec{k}_1 + \vec{k}_2\right) \beta_{ml}\left(\vec{k}_1 + \vec{k}_2; \vec{k}_1\right) \\ +\gamma_{nl}(kk_1k_2) \ , \tag{52}$$

and

$$\Lambda(kk_1k_2) = \frac{1}{\varepsilon^{l(0)}_{\omega\vec{k}_1+\vec{k}_2}} \beta\left(\vec{k}_1 + \vec{k}_2; \vec{k}_1\right) \beta\left(\vec{k}; \vec{k}_1 + \vec{k}_2\right) + \gamma(kk_1k_2) \ , \tag{53}$$

where $\varepsilon_{\omega k}^{(0)}$ is given by (43) or (44) for $\lambda = 1$. Moreover, we have defined

$$\beta_{nm}\left(kk_1\right) = \frac{\omega_p^2}{\omega} \int \frac{d^3v}{\omega - \vec{k}\cdot\vec{v}} v_\perp^n \left[-\frac{e}{T} - \frac{e}{m_e}\left(\vec{k} - \vec{k}_1\right) \cdot \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \vec{k}_1 \cdot \vec{v}} \right] \frac{\partial f_{e0}}{\partial v_m} , \tag{54}$$

and

$$\begin{aligned}\gamma_{nm}\left(kk_1k_2\right) = \frac{\omega_p^2}{\omega} \int \frac{d^3v}{\omega - \vec{k}\cdot\vec{v}} v_\perp^n &\left[\frac{1}{2}\frac{e^2}{T^2} + \frac{e^2}{m_e T}\vec{k}_2 \cdot \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \left(\vec{k} - \vec{k}_2\right)\cdot\vec{v}} \right. \\ + \quad & \left. \frac{e^2}{m^2}\left(\vec{k} - \vec{k}_1 - \vec{k}_2\right) \cdot \frac{\partial}{\partial \vec{v}} \vec{k}_2 \cdot \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \vec{k}_1 \cdot \vec{v}} \right] \frac{\partial f_{e0}}{\partial v_m} ,\end{aligned} \tag{55}$$

where $\vec{v}_\perp$ is the velocity component perpendicular to $\vec{k}$, and

$$\beta\left(kk_1\right) = \frac{\omega_p^2}{kk_1} \int \frac{d^3v}{\omega - \vec{k}\cdot\vec{v}} \left[-\frac{e}{T} - \frac{e}{m_e}\left(\vec{k} - \vec{k}_1\right) \cdot \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \vec{k}_1 \cdot \vec{v}} \right] \vec{k}_1 \cdot \frac{\partial f_{e0}}{\partial \vec{v}} , \tag{56}$$

$$\begin{aligned}\gamma\left(kk_1k_2\right) = \frac{\omega_p^2}{kk_1} \int \frac{d^3v}{\omega - \vec{k}\cdot\vec{v}} &\left[\frac{1}{2}\frac{e^2}{T^2} + \frac{e^2}{m_e T}\vec{k}_2 \cdot \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \left(\vec{k} - \vec{k}_2\right)\cdot\vec{v}} \right. \\ + \frac{e^2}{m_e^2}\left(\vec{k} - \vec{k}_1 - \vec{k}_2\right) \cdot \frac{\partial}{\partial \vec{v}} \vec{k}_2 \cdot & \left. \frac{\partial}{\partial \vec{v}} \frac{1}{\omega - \vec{k}_1 \cdot \vec{v}} \right] \vec{k}_1 \cdot \frac{\partial f_{e0}}{\partial \vec{v}} .\end{aligned} \tag{57}$$

Using the expression (36), we can deduce from (50) and (51) the desired dispersion relations for the transverse and longitudinal waves. We have

$$Det\left\{ \left(\frac{k^2c^2}{\omega^2} - <\varepsilon_{k\omega}^t> \right) \delta_{nl} - \frac{\left(eZ_g\right)^2}{4\pi^2 k_D} n_g \int \frac{d^3q}{\left(2\pi\right)^3} \tilde{C}\left(q\right) \Lambda^{nl}\left(kkq\right) \right\} = 0 , \tag{58}$$

and

$$<\varepsilon_{k\omega}^l> - \frac{\left(eZ_g\right)^2}{4\pi^2 k_D} n_g \int \frac{d^3q}{\left(2\pi\right)^3} \tilde{C}\left(q\right) \Lambda\left(kkq\right) = 0 , \tag{59}$$

where $\varepsilon_{k\omega}^t$ and $\varepsilon_{k\omega}^l$ are the modified dispersion functions given in (43) and (44) and $\tilde{C}\left(q\right)$, for random grains, is given by (37).

The quantities Λ^{nl} and Λ are the velocity integrals to be calculated, for any choice of $f_{e0}\left(v\right)$, in the high-frequency limit $\omega >> |\vec{k}\cdot\vec{v}|$, consistently with our approximation.

It is to be noted that since $\vec{q}$ appears in an integral, terms of the form $\omega - \left(\vec{k}+\vec{q}\right)\cdot\vec{v}$ in the Λ-integrals cannot be expanded and will, in general, represent poles, i.e. the possibility of complex ω or $\vec{k}$.

Equations (58) and (59) constitute the final result of this section. These are the average dispersion relations for transverse and longitudinal waves (electron modes only) in an unmagnetized dusty plasma with randomly distributed charged particulates.

On the other hand, if the interaction between the charged grains are taken into account, then in (50) and (51) we have to use the expression (33) instead of (36). Consequently, $\tilde{C}(q)$ in (58) and (59) is replaced by $\tilde{C}(q)\,S(q)$, where $S(q)$ is the static structure factor of the interacting grains.

IV. RESULTS AND DISCUSSION

In order to exhibit the importance of the effects described in Sec. III, we shall present here explicit solutions of (58) and (59) for the particular case of the Maxwellian distribution function $f_{e0}(\vec{v})$. The two cases are considered separately.

a) Longitudinal waves:

From (44) and (47) we have

$$< \varepsilon^{l}_{k\omega} >= \varepsilon^{l(0)}_{k\omega} + \frac{1}{2}\frac{e^2}{T^2}\frac{\omega_p^2}{\omega^2}\frac{(4\pi e Z_g)^2}{8\pi k_D} n_g \ , \tag{60}$$

where $\varepsilon^{l(0)}_{k\omega}$ is the well known plasma dielectric constant, given by

$$\varepsilon^{l(0)}_{k\omega} = 1 + \frac{1}{k^2\lambda_D^2} - \frac{1}{k^2\lambda_D^2} I(z_k) \ . \tag{61}$$

Here $I(z_k)$ is the plasma dispersion function and $z_k = \omega/\sqrt{2}kv_T$. The real and imaginary parts of I can be expressed as

$$ReI = 2z_k e^{-z_k^2}\int_0^{z_k} dt e^{t^2} \ , \tag{62}$$

$$ImI = -\sqrt{\pi} z_k e^{-z_k^2} . \tag{63}$$

We note that the imaginary part of I (which leads to the electron Landau damping) is exponentially small for high-frequency ($\omega >> kv_T$) waves.

Accordingly, for $z_k >> 1$, we simply take

$$\varepsilon_{k\omega}^{l(0)} \simeq 1 - \frac{\omega_{pe}^2}{\omega^2}\left(1 + 3k^2\lambda_D^2\right) . \tag{64}$$

Our next task is to work out the second term of equation (59). First, we calculate $\Lambda(kkq)$ from (53), (56) and (57) for the Maxwellian distribution function $f_{e0}(\vec{v})$, assuming that $\omega >> |\vec{k}\cdot\vec{v}|$ and $k\lambda_D << 1$. Integrating by parts in (56) and (57), neglecting $\vec{k}\cdot\vec{v}$ but not $\vec{q}\cdot\vec{v}$ (recall from Eq. 59 that we are in an integral over q and the terms with $q > k$ will contribute to the integral) we find

$$\beta\left(\vec{k}, \vec{k}+\vec{q}\right) = \beta\left(\vec{k}+\vec{q}, \vec{k}\right) = \frac{\omega_p^2}{\omega^2}\frac{e}{T}\frac{1}{k|\vec{k}+\vec{q}|}\left[k^2 + \vec{k}\cdot\vec{q} I\left(z_{\vec{k}+\vec{q}}\right)\right] , \tag{65}$$

and

$$\gamma(kkq) = -\frac{\omega_p^2}{\omega^2}\left(\frac{e}{T}\right)^2\left\{\frac{1}{2} - \vec{k}\cdot\vec{q}\frac{\vec{k}\cdot\left(\vec{k}-\vec{q}\right)}{k^2|\vec{k}-\vec{q}|^2}\left[I\left(z_{\vec{k}-\vec{q}}\right) - 1\right]\right\} . \tag{66}$$

The presence of the poles ($ImZ \neq 0$) is due to the fact that $\vec{q}$ can be large enough such that the resonant condition

$$\omega - \left(\vec{k}+\vec{q}\right)\cdot\vec{v} = 0 \tag{67}$$

can also be satisfied for $\omega >> |\vec{k}\cdot\vec{v}|$, and the Landau damping is negligible. This can be physically interpreted as the non-linear Landau damping or the induced scattering: the original $(\omega, \vec{k})$ wave interacts with the spectrum of zero frequency waves $(o, \vec{q})$ representing the density modulations due to the presence of the charged grains, forming beat waves $(\omega, \vec{k}+\vec{q})$ and some of these (for large $\vec{q}$) can have phase velocities much lower than the one of the propagating wave

($\omega/k >> v_T$) and can, therefore, resonate with the electrons in the bulk of the distribution (Eq. 67) producing a very effective damping of the wave.

The efficiency of the process depends on how low are the phase velocities of the beat waves, i.e. how large $\vec{q}$ can be, and this depends on the weight factor in the integral (59) given by $\tilde{C}(q)$ for random grains (which decreases as q^{-4} at high q producing an effective cut-off at some value of the phase velocity).

If the grains' interactions are taken into account the weight factor in (59) is replaced by $S(q)\tilde{C}(q)$ (see equation 33); as $S(q)$ approaches zero rapidly for large $\vec{q}$ (corresponding to the pair correlation function $g(r)$ going to zero as $r \to 0$) inclusion of interactions makes the damping process less effective giving an higher cut-off for the phase velocities of the beat waves.

The case of random grains will, therefore, establish an upper limit to the magnitude of the damping process.

As the important contribution comes from large q, we shall neglect $k\lambda_D << 1$ with respect to $q\lambda_D$ in (65) and (66) and take

$$z_{\vec{k}+\vec{q}} = z_{\vec{k}-\vec{q}} = \frac{\omega}{\sqrt{2}qv_T} \simeq \frac{\omega/\omega_p}{\sqrt{2}q\lambda_D} \equiv \frac{\omega/\omega_p}{\sqrt{2}x} \tag{68}$$

where $x = q\lambda_D$. We can then express

$$J \equiv ImI(x) \simeq -\sqrt{\frac{\pi}{2}}\frac{\omega}{\omega_p}\frac{1}{x}e^{-\frac{1}{2x^2}\frac{\omega^2}{\omega_p^2}} , \tag{69}$$

and

$$\varepsilon^l_{\omega,\vec{k}+\vec{q}} \simeq 1 + \frac{1}{x^2}\left[1 - I(x)\right] . \tag{70}$$

From (65) to (70), taking θ as the angle between $\vec{k}$ and $\vec{q}$ and defining $\mu = \cos\theta$, we finally obtain

$$\Lambda(kkq) = \frac{\omega_p^2}{\omega^2}\left(\frac{e}{T}\right)^2\left[\frac{1}{2} + \mu^2(I-1) + \frac{\omega_p^2}{\omega^2}\frac{k^2 + \mu^2 x^2 I^2 + 2\mu k x I}{1 + x^2 - I}\right] . \qquad (71)$$

where k is normalized by $1/\lambda_D$. Inserting (71) into (59) and using (37) for random grains, we performe the θ integral to obtain the dispersion relation in the form

$$1 - \frac{\omega_p^2}{\omega^2}(1 + 3k^2) + \frac{\omega_p^2}{\omega^2}4\pi n_g \lambda_D^3\left(\frac{e^2 Z_g}{\lambda_D T}\right)^2 [1 - F(k)] = 0 , \qquad (72)$$

where

$$F(k) = \frac{1}{3\pi(2\pi)^3}\int_0^\infty \frac{x^2 dx}{(1+x^2)^2}\left[\frac{3k^2 + 2x^2 I^2}{1 + x^2 - I} + 2I\right] . \qquad (73)$$

The changes in the real part of ω (from $ReF(k)$) can be ignored, and we obtain from (72)

$$Re\omega = \omega_k = \omega_p\left(1 + \frac{3}{2}k^2\right) . \qquad (74)$$

For the imaginary part of $F(k)$, we have

$$ImF(k) = -\frac{2}{3\pi(2\pi)^3}\sqrt{\frac{\pi}{2}\frac{\omega_k}{\omega_p}}\int_0^\infty \frac{x}{(1+x^2)^2} e^{-\frac{\omega_k^2}{\omega_p^2}\frac{1}{2x^2}}$$
$$\left\{1 + \frac{x^2\left[2(1+x^2)R - |I|^2\right]}{|1 + x^2 - I|^2}\right\} dx , \qquad (75)$$

where $R = ReI(x)$ and we have ignored k^2 with respect to x^2 in the integrand.

We note that the two terms in the curly bracket in (75) come, respectively, from the γ and $\beta\beta$ terms in (53). A numerical evaluation of the two integrals for $\omega_k \approx \omega_{pe}$, gives 0.27 for the first term and 0.04 for the second term. Clearly, only the contribution from the γ term is important and this had already been found by de Angelis et al.[1]. Thus, ignoring the second term in the following and introducing the integral

$$f_k = \int_0^\infty \frac{x dx}{(1+x^2)^2} e^{-\frac{\omega_k^2}{\omega_p^2}\frac{1}{2x^2}}; \quad f_k(\omega_k \approx \omega_p) = 0.27 , \qquad (76)$$

we find from (72) the damping coefficient:

$$\gamma^l = -2Im\omega = \omega_p \frac{1}{3\sqrt{2}\pi^{5/2}} n_g \lambda_D^3 \left(\frac{e^2 Z_g}{\lambda_D T}\right)^2 f_k \ . \tag{77}$$

This is of the same order as found by de Angelis et al.[1]: see Fig. 1 of that paper where $Im(\omega/\omega_p)/\mu^2$ is given, but there is an error in the expression for μ^2 (cf. Eq. 51 of Ref. 1) where N_g should be replaced by $N_g^{1/2}$.

b) Transverse waves:

For this case, the dispersion relation is given by (58). Again, for the case of the Maxwellian distribution function f_{e0} and $\omega >> |\vec{k}\cdot\vec{v}|$ the matrices (54) and (55) are given by

$$\begin{aligned}\beta_{nm}\left(\vec{k},\vec{k}+\vec{q}\right) =& \frac{\omega_p^2}{\omega^2}\frac{e}{T}\left\{\frac{1}{v_T^2}\int d^3v f_{e0} v_n^{\perp} v_m + q_n^{\perp}\int d^3v \frac{v_m}{\omega-\vec{q}\cdot\vec{v}} f_{e0} + \frac{\vec{k}\cdot\vec{q}}{\omega}\int d^3v \frac{v_n^{\perp} v_m}{\omega-\vec{q}\cdot\vec{v}} f_{e0}\right\} \\ =& \beta_{nm}\left(\vec{k}+\vec{q},\vec{k}\right) \ ,\end{aligned} \tag{78}$$

and

$$\begin{aligned}\gamma_{nm}(kkq) =& \frac{\omega_p^2}{\omega^2}\left(\frac{e}{T}\right)^2\left\{-\frac{1}{2}\left(\delta_{nm}-\frac{k_n k_m}{k^2}\right)\right. \\ &\left. + \int d^3v \frac{v_m q_n^{\perp}}{\omega+\vec{q}\cdot\vec{v}} f_{e0} + \frac{\vec{k}\cdot\vec{q}}{\omega}\int d^3v \frac{v_n^{\perp} v_m}{\omega+\vec{q}\cdot\vec{v}} f_{e0}\right\} \ .\end{aligned} \tag{79}$$

Expressing the velocity integrals in terms of the plasma dispersion function $I(z_q)$ (see equations 62 to 64), choosing $\vec{k} = (0,0,k)$ and neglecting terms of order (ω^{-4}), the matrices become 2×2 and the matrix Λ from (52) is given by $(i,j = 1,2)$

$$\begin{aligned}\Lambda ij(kkq) =& \frac{\omega_p^2}{\omega^2}\left(\frac{e}{T}\right)^2\left\{\omega_p^2 \frac{\delta_{ij}-\frac{q_i q_j}{q^2}+\left[1+\left(1+\frac{\vec{k}\cdot\vec{q}}{q^2}\right)(I-1)\right]\frac{q_i q_j}{q^2}}{|\vec{k}+\vec{q}|^2 c^2+\omega_p^2 I-\omega^2} + \frac{1}{2}\delta_{ij} + \right. \\ &\left. - \left(1+\frac{\vec{k}\cdot\vec{q}}{q^2}\right)(I-1)\frac{q_i q_j}{q^2}\right\} \ .\end{aligned} \tag{80}$$

Inserting (80) in (58) and using again (37) for $C(q)$, the dispersion relation can be put in the form

$$\mathrm{Det}\left[\left(\frac{k^2c^2}{\omega^2} - \varepsilon\frac{t(0)}{k\omega}\right)\delta_{ij} - H_{ij}\right] = 0 \ , \tag{81}$$

where

$$H_{ij} = \frac{2}{\pi}\left(\frac{eZg}{\lambda_D}\right)^2 n_g\lambda_D^3 \int \frac{d^3x}{(2\pi)^3}\frac{\Lambda_{ij}(kkq)}{(1+x^2)^2} \ , \tag{82}$$

and $\vec{x} = \lambda_D\vec{q}$.

The matrix H_{ij} is diagonal and $H_{11} = H_{22}$. Thus, the dispersion relation becomes

$$\frac{k^2c^2}{\omega^2} - \varepsilon\frac{t(0)}{k\omega} = H_{11} \ . \tag{83}$$

Using the polar coordinates and performing the φ-integration, we find that

$$H_{11} = \frac{2}{(2\pi)^3}\left(\frac{e^2Z_g}{\lambda_D T}\right)^2 n_g\lambda_D^3\frac{\omega_p^2}{\omega^2}h(k) \ , \tag{84}$$

where

$$h(k) = \int_0^\infty \frac{x^2dx}{(1+x^2)^2}\int_0^\pi d\theta\sin\theta\left\{\cos^2\theta\frac{1+\left[\left(1+\frac{k}{x}\cos\theta\right)(I-1)\right]^2}{\frac{c^2}{v_T^2}\left(k^2+x^2+kx\cos\theta\right)+I-\frac{\omega^2}{\omega_p^2}} + \frac{1}{2} + \right.$$
$$\left. - \left(1+\frac{k}{x}\cos\theta\right)(I-1)\cos^2\theta\right\} \ , \tag{85}$$

and $k = k\lambda_D$.

We are interested in the imaginary part of H_{11} in (83) and, therefore, we only need to calculate the $Imh(k)$. We have

$$Imh(k) = \int_0^\infty \frac{x^2dx}{(1+x^2)^2}\int_0^\pi d\theta\sin\theta\cos^2\theta J(x)$$
$$\left\{\frac{2(R-1)\frac{c^2}{v_t^2}\left(1+\frac{k}{x}\cos\theta\right) - 1 + \left(1+\frac{k}{x}\cos\theta\right)^2\left[(R-1)^2 - J^2\right]}{\left[\frac{x^2c^2}{v_t^2}\left(1+\frac{k}{x}\cos\theta\right)+R\right]^2 + J^2} - 1\right\} \ , \tag{86}$$

where $c^2k^2/v_T^2 \approx \omega^2/\omega_p^2$ has been used. Here, $RandJ$ are the real and imaginary parts of $I(z_q)$. Specifically, we have

$$J(x) = -\sqrt{\frac{\pi}{2}}\frac{\omega}{\omega_p}\frac{1}{x}e^{-\frac{\omega^2}{\omega_p^2}\frac{1}{2x^2}} . \tag{87}$$

As $J(x)$ is a factor of the whole integrand, for $\omega >> \omega_p$ that is $\omega_k \simeq kc$ and $k >> v_T/c$, only $x >> 1$ will contribute to the integral and in this limit $R << 1$; expanding the integrand and performing the angular integration, we finally obtain

$$Imh(k) = \frac{\omega}{\omega_p}\frac{2}{3}\sqrt{\frac{\pi}{2}}\int_0^\infty \frac{xdx}{(1+x^2)^2}e^{-\frac{\omega_k^2}{\omega_p^2}\frac{1}{2x^2}} \equiv \frac{\omega}{\omega_p}\frac{2}{3}\sqrt{\frac{\pi}{2}}f_k . \tag{88}$$

From (83), (84), and (88) we find then

$$\gamma^t = -2Im\omega = \omega_p\frac{1}{6\sqrt{2}\pi^{5/2}}\left(\frac{e^2Z_g}{\lambda_D T}\right)^2 n_g\lambda_D^3 f(k) . \tag{89}$$

For $\omega_k \approx \omega_p$, the result is 1/2 of γ^l (see 77), whereas for $\omega_k >> \omega_p$ the effect becomes smaller.

In the foregoing, we have considered the temporal damping of waves and have solved the dispersion relations for real k and complex ω. Since the plasma is nonuniform in the presence of density perturbations $\delta n(r)$ associated with the charged grains, the possibility of spatial damping also exists if at some point the number density is larger than the critical density for the wave propagation. In what follows, we look for solutions of the dispersion relations with real ω and complex k.

For longitudinal waves, (72) yields

$$Imk = \sqrt{\frac{3}{2}}\frac{1}{18\pi^{5/2}}\frac{1}{(l-\frac{\omega_p^2}{\omega_k^2})^{1/2}}n_g\lambda_D^3\left(\frac{e^2Z_g}{\lambda_D T}\right)^2 f_k \tag{90}$$

(where k is in the unit of λ_D^{-1}); whereas for electromagnetic waves we obtain from (83) (again taking into account only the imaginary part of H_{11})

$$Imk = \frac{1}{12\sqrt{2}\pi^{5/2}}\frac{\omega_p}{\omega_k}\frac{k}{1-\frac{\omega_p^2}{\omega_k^2}} n_g \lambda_D^3 \left(\frac{e^2 Z_g}{\lambda_D T}\right)^2 f_k \tag{91}$$

For $\omega_k >> \omega_p$ the spatial damping is negligible (recall that $f_k \sim \omega_p^2/\omega_k^2$). However, for electromagnetic waves with frequency $\omega_k \geq \omega_p$, we have

$$Imk \approx \frac{v_T}{c}\frac{1-\omega_p^2/2\omega_k^2}{1-\omega_p^2/\omega_k^2} n_g \lambda_D^3 \left(\frac{e^2 Z_g}{\lambda_D T}\right)^2 , \tag{92}$$

which can be important for $Z_g >> 1$.

IV. CONCLUSIONS

In this paper, we have shown that the presence of charged dust grains in a multicomponent plasma can alter the properties of propagating waves. Specifically, we have demonstrated the possibility of temporal or spatial dampings of high-frequency plasma waves due to the interaction of the wave with the density perturbations associated with massive charged grains.

This effect (temporal damping) has been shown for the first time by de Angelis et al.[1] and has been extended by Forlani et al.[2] by including other terms of the same order in the expansion in the potential perturbations $\delta\phi$, which were neglected in Ref. 1.

With the help of the present explicit calculations we have shown that the new terms that have been included in Forlani et al.[2] are negligible (in the high frequency domain). We have derived and solved the general dispersion relations (both for longitudinal and transverse waves) in order to obtain the temporal and spatial damping coefficients, namely, (77) and (90) for the longitudinal waves and (89) and (91) for the transverse waves.

The temporal damping of the waves, which can be physically interpreted as due to the elastic scattering of electron oscillations with the wave on the electrons in the shielding clouds which have an effective infinite mass, can be effective for longitudinal waves (in the cold plasma regime $\omega >> kv_T$ where the electron Landau damping is exponentially small) and for transverse waves with frequency not much larger than the local electron plasma frequency.

In order to illustrate the importance of the present investigation, we compare the collisionless temporal damping rate with the electron-ion collision frequency. For the Maxwellian plasma, the latter is roughly

$$\nu_{ei} \sim 10^{-5} n_e T_e^{-3/2} \ , \tag{93}$$

where $n_e(T_e)$ is expressed in the unit of $cm^{-3}(eV)$.

Expressing in the same units, we obtain

$$\gamma^l \simeq 10^{-7} \frac{n_g}{n_e} Zg^2 n_e T_e^{-3/2} \tag{94}$$

so that

$$\frac{\nu_{ei}}{\gamma^l} \sim 10^2 \frac{n_e}{n_g Z_g^2} \ . \tag{95}$$

A close examination of the results of de Angelis et al.[1]1, where the grain charge Z_g is plotted as a function of n_g/n_e for several values of the plasma number density and the temperature, reveals that in many parameter ranges one finds that $(n_g/n_e)Z_g \approx 1$. Hence, $\nu_{ei}/\gamma_l \sim 100/Z_g$, and in these regimes the effect of the collisionless damping is, therefore, dominant with respect to collisional damping of high-frequency waves propagating in dusty plasmas.

Finally, we stress that the present investigation assumes that the wave frequency (or the damping rate) is much larger than the frequency of the charge osscillations on the grains. Accordingly, for high-frequency waves, it is justified to assume Z_g as constant. However, for low-frequency modes one should self-consistently include[14] the oscillations of the grain charge.

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