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## Two-jet hadroproduction as a measure of the gluon at small $x$

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### Abstract

We investigate the proposal of the CDF collaboration that same-side two-jet production in  $p\bar{p}$  collisions may be used to determine the gluon distribution at small  $x$ .



The gluon is the least well constrained of all the parton distributions of the proton, although it dominates at small  $x$ . There are essentially only two reliable and precise constraints [1]. First the measurements of deep-inelastic lepton-nucleon scattering determine the total fraction of the proton's momentum that is carried by the gluon. Secondly the WA70 measurements [2] of the prompt-photon reaction,  $pp \rightarrow \gamma X$ , determine the gluon in the region  $x \approx 0.35$ .

The behaviour of the gluon at small  $x$  is particularly important phenomenologically, but it is also interesting in its own right. The resummation of soft gluon emission, as embodied in the Lipatov (or BFKL) equation [3], implies that  $xg \sim x^{-\lambda}$  as  $x \rightarrow 0$  with  $\lambda \approx 0.5$ . Of course as  $x$  decreases we will reach a stage where this growth is suppressed by gluon shadowing effects, and eventually we enter the confinement region where perturbative QCD ceases to be valid.

The recent preliminary measurements [4] of the structure function  $F_2(x, Q^2)$  for deep-inelastic electron-proton scattering at HERA do show evidence of a Lipatov-type growth for  $x \sim 10^{-3}$ . This hints that the sea quark distribution has the behaviour  $x\bar{q} \sim x^{-\lambda}$ , and thus, implicitly, also the gluon (if the sea quarks are, as we expect, driven by  $g \rightarrow q\bar{q}$ ). Although the above conclusion is plausible, it is clear that a direct measurement of the gluon at small  $x$  is urgently needed. Extrapolations of partons to small  $x$  give widely differing gluon distributions. For the purposes of illustration we take the  $D'_0$  and  $D'_-$  parton distributions of ref. [5] which both give equally acceptable descriptions of fixed target deep-inelastic and related data. Although  $D'_-$  is favoured by the new preliminary HERA measurements, recall that the data test  $x\bar{q} \sim x^{-0.5}$  and not the gluon. At  $Q = 5$  GeV the  $D'_0$  and  $D'_-$  gluons differ by about a factor of 2 at  $x \sim 10^{-3}$ . When evolved up in  $Q^2$  the two distributions become more similar as can be seen from the results for  $Q = 15$  GeV that are shown in Fig. 1. It is interesting to note that the Lipatov growth of  $xg$  of  $D'_-$  must be compensated by a cross-over with the  $D'_0$  gluon so that the total momentum carried by the gluon is essentially the same for both distributions. In this note we study the possibility of using 2-jet production in  $p\bar{p}$  collisions to determine the behaviour of the gluon at small  $x$  and, in particular, of distinguishing between the distributions in Fig. 1.

The two-jet cross section may be written to leading order in terms of the sum of  $i+j \rightarrow k+l$  partonic subprocesses

$$\frac{d^2\sigma}{dy_1 dy_2 dp_T^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,\ell} \frac{f_i(x_1, \mu)}{x_1} \frac{f_j(x_2, \mu)}{x_2} \overline{\sum} |\mathcal{M}(ij \rightarrow k\ell)|^2 \quad (1)$$

where  $f_i$  are the parton densities of type  $i = g, u, \bar{u}, d, \dots$  evaluated at momentum scale  $\mu$ , and  $y_1, y_2$  are the laboratory rapidities of the outgoing partons each of transverse momentum  $p_T$ .

$\overline{\sum}|\mathcal{M}|^2$  represents the sub-process matrix elements squared averaged over initial, and summed over final, parton spins and colours. For the moment we assume we can identify the outgoing jets with the outgoing partons. The observed jet rapidities can be used to determine the laboratory rapidity of the two-parton system ( $y_{boost}$ ) and the equal and opposite rapidities ( $\pm y^*$ ) of the two jets in the parton-parton c.m. frame

$$y_{boost} = \frac{1}{2}(y_1 + y_2), \quad y^* = \frac{1}{2}(y_1 - y_2). \quad (2)$$

The longitudinal momentum fractions of the incoming partons are then given by

$$x_{1,2} = \frac{2p_T}{\sqrt{s}} \cosh(y^*) \exp(\pm y_{boost}). \quad (3)$$

The CDF collaboration [6] have emphasized that their observation of a pair of same-side jets with large and equal rapidities  $y_1 = y_2$  can give a valuable determination of the gluon density at small  $x$ . For example for same side jets with  $y_1 = y_2 = 2.5$  and  $p_T = 35$  GeV at  $\sqrt{s} = 1.8$  TeV we have

$$x_1 = 0.47, \quad x_2 = 0.0032. \quad (4)$$

For these  $x$  values the jet-pair will originate from  $q_{val}(x_1)g(x_2)$  and so an accurate measurement of same side jet production will be a valuable determination of the gluon at small  $x$ , a region in which it is so far unmeasured. The idea is similar to that proposed [7] for forward  $Z^0$  production, but has the added advantages of a variable dijet mass and higher statistics.

Before such a method can be employed we must address two problems. On the experimental side there are uncertainties arising from normalization, jet trigger efficiency and energy smearing etc. On the theoretical side there are ambiguities associated with the choice of scale  $\mu$ . To overcome the experimental problem the CDF collaboration [6] normalise the signal to the production of a pair of identical jets but with opposite rapidities,  $y_1 = -y_2$ . That is they measure the ratio

$$R_1(y, p_T) \equiv \frac{\sigma_{SS}}{\sigma_{OS}} = \frac{\text{No. of same-side jets (with } y_1 = y_2 = y)}{\text{No. of opposite-side jets (with } y_1 = -y_2 = y)} \quad (5)$$

From (2) we see that  $\sigma_{SS}$  is built up from parton cross sections with  $y_{boost} = y$  and  $y^* = 0$ , whereas  $\sigma_{OS}$  corresponds to  $y_{boost} = 0$  and  $y^* = y$ . Hence the opposite-side jets originate from partons each with  $\bar{x} = x_1 + x_2$  where  $x_1$  and  $x_2$  are the momentum fractions of the partons giving the same-side jets.

To relate the measured ratio  $R_1(y, p_T)$  to the parton distributions we must choose the scale  $\mu$  in (1). Since we wish to use  $R_1(y, p_T)$  to distinguish between gluon distributions with

$g(x \sim 0.003, \mu \sim p_T)$  which differ by about 30%, this is clearly an important issue. To study the scale dependence we draw on the work of Ellis, Kunszt and Soper [8] on 2-jet production at  $O(\alpha_s^3)$ . The  $O(\alpha_s^3)$  calculation reduces the dependence on the choice of scale. Inter alia, Ellis et al. determine the scale  $\mu$  for which the Born or lowest-order ( $O(\alpha_s^2)$ ) calculation approximately reproduces the less scale dependent  $O(\alpha_s^3)$  result. They find

$$\mu \approx \frac{\cosh(y^*)}{\cosh(0.7y^*)} \frac{p_T}{2} \equiv k(y^*) \frac{p_T}{2}, \quad (6)$$

so for same-side jets  $\mu = p_T/2$  whereas for opposite side jets  $k$  increases from 1 to 2.4 as  $y^*$  goes from 0 to 3. In terms of partons we therefore have

$$R_1(y, p_T) = \frac{\sum_{i,j} f_i(x_1, \frac{1}{2}p_T) f_j(x_2, \frac{1}{2}p_T) \alpha_s^2(\frac{1}{2}p_T) \hat{\sigma}_{ij}(0, p_T)}{\sum_{i,j} f_i(\bar{x}, \frac{1}{2}kp_T) f_j(\bar{x}, \frac{1}{2}kp_T) \alpha_s^2(\frac{1}{2}kp_T) \hat{\sigma}_{ij}(y, p_T)} \quad (7)$$

where we have extracted the  $\alpha_s^2$  factors from the subprocess cross sections  $\hat{\sigma}(ij \rightarrow 2 \text{ partons})$ . Here  $\bar{x} = x_1 + x_2$  and  $k(y)$  is given by (6).

When the  $x_i$  in (7) are small, we would expect the cross section to be dominated by gluon-gluon scattering. Conversely, when the  $x_i$  are large, valence quark scattering will dominate. More quantitatively, we recall the ‘single effective subprocess approximation’ [9], which states that the gluon-gluon, quark-gluon and quark-quark subprocess scattering are approximately in the ratio  $1 : \frac{4}{9} : \frac{4}{9}^2$ . The numerator and denominator in (7) can therefore be approximated by  $F(x_1)F(x_2)\alpha_s^2\hat{\sigma}_{gg}$ , where  $F(x) = g(x) + \frac{4}{9}\sum_q(q(x) + \bar{q}(x))$ . It is then straightforward to identify the dominant subprocesses at given  $y$  and  $p_T$ . In particular, for large  $y$  the observed ratio  $R_1(y, p_T)$  directly measures the gluon distribution  $g(x, \mu)$  at small  $x$ . To be precise

$$R_1(y, p_T) \approx C g(x_2, \frac{1}{2}p_T) \quad (8)$$

where  $x_2 = 2p_T e^{-y}/\sqrt{s}$  and  $C(y, p_T)$  depends on parton distributions at  $x$  values where they are reliably known. This is not quite true because there is some uncertainty in  $C$  from the lack of knowledge of  $g(\bar{x}, \frac{1}{2}kp_T)$ .

The predictions for  $R_1$  obtained using the MRS parton sets  $D'_0$  and  $D'_-$  are compared with a preliminary sub-sample of CDF data in Fig. 2. We see that the data definitely favour the  $D'_-$  small  $x$  behaviour of the gluon in comparison with that for  $D'_0$ , although this should be regarded as an illustrative comparison since further detector corrections to the data have to be made. It is interesting to note that if we were to take  $k = 1$  in the denominator of (7) then the peak values of  $D'_-$  and  $D'_0$  in Fig. 2 would be reduced to 1.8 and 1.4 respectively. This large

reduction demonstrates the importance of the choice of scale. The main problem is that at large rapidity the  $O(\alpha_s^3)$  prediction of the opposite-side jet cross section itself becomes much more scale dependent (see, for example, Fig. 2 of Ellis et al. [8] and note that for  $y = 2$  their variable  $\lambda \equiv \frac{1}{2} \sinh(2y) \simeq 14$ ). This scale dependence of  $\sigma_{OS}$  would appear to make it problematic to use the ratio (5) to definitively measure the gluon to much better than 30%. We should also note that we are applying formula (6) of Ellis et al. outside the kinematic region for which it was established. Thus for  $\sigma_{SS}$  the dijet mass  $M_{jj} = 2p_T$  is too small, while for  $\sigma_{OS}$  for  $y \gtrsim 2$  the value of  $\lambda$  becomes too large. However the general trends are clear.

In an attempt to reduce the scale dependence we introduce an alternative ratio

$$\begin{aligned}
 R_2(y, p_T) &\equiv \frac{\sigma_{SS}(y)}{\sigma_{SS}(0)} = \frac{\text{No. of same-side jets (with } y_1 = y_2 = y \text{)}}{\text{No. of central jets (with } y_1 = y_2 = 0 \text{)}} \\
 &= \frac{\sum_{i,j} f_i(x_1, \frac{1}{2}p_T) f_j(x_2, \frac{1}{2}p_T) \hat{\sigma}_{ij}(0, p_T)}{\sum_{i,j} f_i(\hat{x}, \frac{1}{2}p_T) f_j(\hat{x}, \frac{1}{2}p_T) \hat{\sigma}_{ij}(0, p_T)} \quad (9)
 \end{aligned}$$

where now  $\hat{x} = \sqrt{x_1 x_2} = 2p_T/\sqrt{s}$ . For  $R_2$ , the subprocess cross sections in the numerator and in the denominator are evaluated at exactly the *same* centre-of-mass kinematics, and therefore we would expect almost all of the scale dependence uncertainty to cancel. In fact, only a weak dependence on the factorization scale would remain. Although this ratio also partially removes the experimental uncertainties, to reliably measure the ratio of forward to central 2-jet production presents more of a challenge than  $R_1$  of (5), since the jet pairs at large  $y$  are close to the beam. At large  $y$ , a measurement of  $R_2$  determines

$$R_2(y, p_T) \approx C' \frac{g(x_2, \frac{1}{2}p_T)}{g^2(\hat{x}, \frac{1}{2}p_T)} \quad (10)$$

where  $C'(y, p_T)$  is known,  $\hat{x} = 2p_T/\sqrt{s}$  ( $= 0.039$  for  $p_T = 35$  GeV and  $\sqrt{s} = 1.8$  TeV) and  $x_2 = \hat{x}e^{-y}$  ( $= 0.0056$  for  $y = 2$  for example). In Fig. 3 we compare the predictions of  $R_2$  obtained from parton sets  $D'_-$  and  $D'_0$  [5]. The  $D'_-$  prediction for  $R_2$  is more than 30% larger than that for  $D'_0$  for  $y \approx 2$  due partly to the numerator of (10), but mainly due to the quadratic dependence in the denominator. Both factors increase  $R_2(D'_-)/R_2(D'_0)$  as can be seen from the different behaviours of  $xg$  for  $D'_0$  and  $D'_-$  shown in Fig. 1, with a cross-over at  $x \sim 0.009$  required to maintain the total momentum carried by the gluon.

As compared to  $R_1(y, p_T)$ , the ratio  $R_2(y, p_T)$  has the advantages of (i) much less scale dependence and (ii) large differences due to the different small  $x$  behaviours of the gluon at more moderate values of  $y$ , that is  $1.5 \lesssim y \lesssim 2$ . We conclude that measurements of same-side 2-jet production at the FNAL  $p\bar{p}$  collider offer the possibility of determining the small  $x$  behaviour

of the gluon, but that the uncertainties in the choice of scale need to be carefully considered. In general the scale dependence can be reduced by considering jets of higher  $p_T$ . This would offer a valuable constraint on the gluon at somewhat higher values of  $x$ . The preliminary sub-sample of CDF 2-jet data appear to favour MRS parton set  $D'_-$ , as compared to  $D'_0$ ; a result also found from measurements of  $F_2(x, Q^2)$  at HERA [4]. When final CDF data are available, a precise determination of the behaviour of the gluon at small  $x$  should be possible.

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## References

- [1] A.D. Martin, W.J. Stirling and R.G. Roberts, RAL preprint-93-027, to be published in Proc. of Durham Workshop on ‘HERA – the new frontier for QCD’, J. Phys. G.
- [2] WA70 collaboration: M. Bonesini et al., Z. Phys. **C38** (1988) 371.
- [3] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP **45** (1977) 199; Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.
- [4] H1 collaboration: A. De Roeck, to be published in Proc. of Durham Workshop on ‘HERA – the new frontier for QCD’, J. Phys. G.
- [5] A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. **306B** (1993) 145.
- [6] CDF collaboration: contribution to the EPS Conference on High Energy Physics, Marseilles, July 1993, preprint FERMILAB-Conf-93/203-E.
- [7] A.D. Martin and W.J. Stirling, Phys. Lett. **B243** (1990) 443.
- [8] S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. Lett. **69** (1992) 1496.
- [9] B. Combridge and C.J. Maxwell, Nucl. Phys. **B239** (1984) 429.

## Figure Captions

- Fig. 1 The gluon  $xg(x, Q = 15 \text{ GeV})$  corresponding to the  $D'_-$  and  $D'_0$  set of partons of ref. [5].
- Fig. 2 The curves show the same-side/opposite-side dijet ratio  $R_1$  predicted from (7) using the  $D'_-$  and  $D'_0$  partons of ref. [5] for  $27 < p_T < 60 \text{ GeV}$ . The data points are the preliminary CDF measurements [6] of the ratio for jets with  $27 < E_T < 60 \text{ GeV}$ . The measured  $E_T$  values have not been corrected for CDF detector effects and therefore do not correspond directly to the true jet transverse energies.
- Fig. 3 The ratio  $R_2$  of (9) calculated using the  $D'_-$  and  $D'_0$  partons of ref. [5] for  $27 < p_T < 60 \text{ GeV}$ .

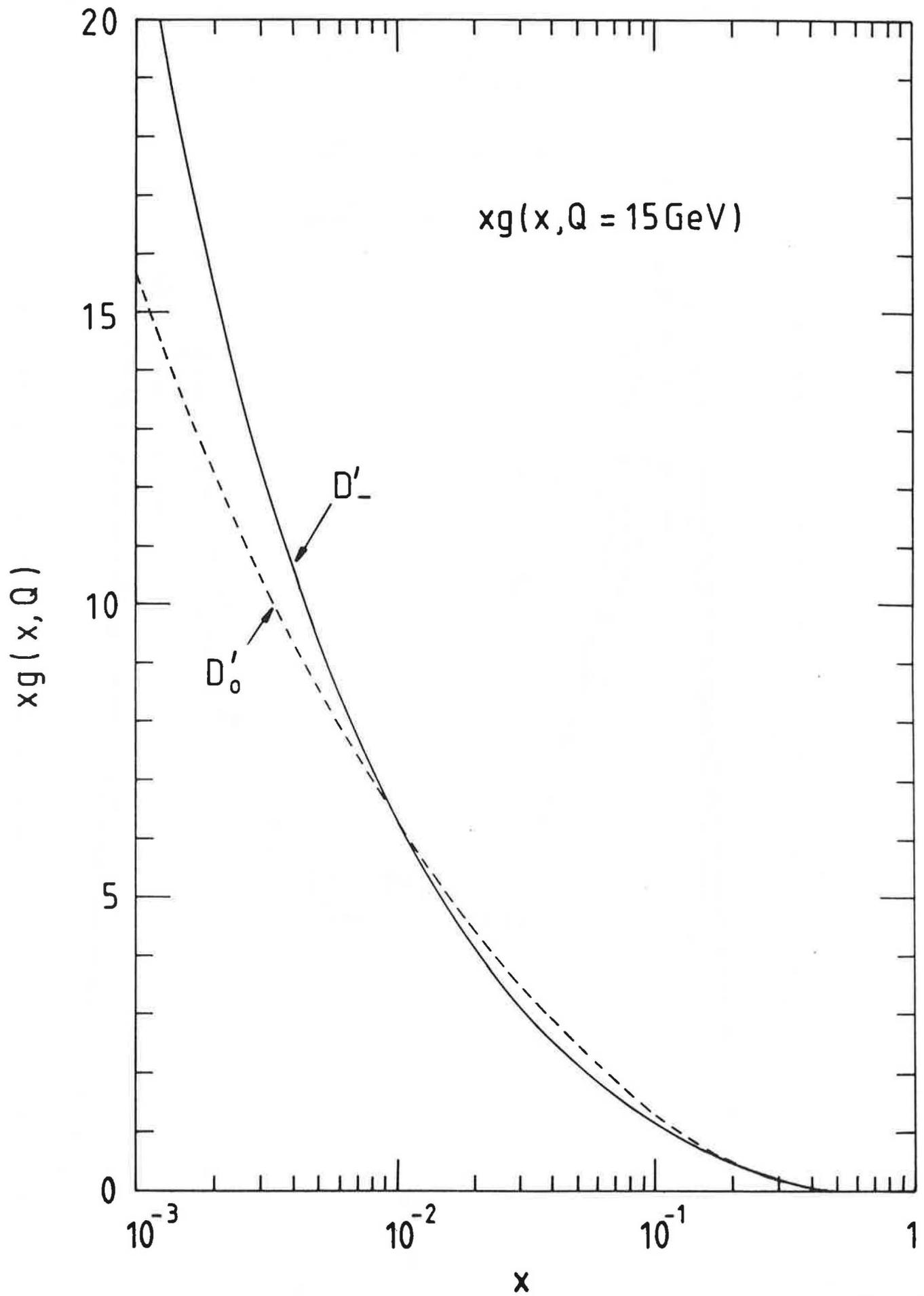


Fig. 1

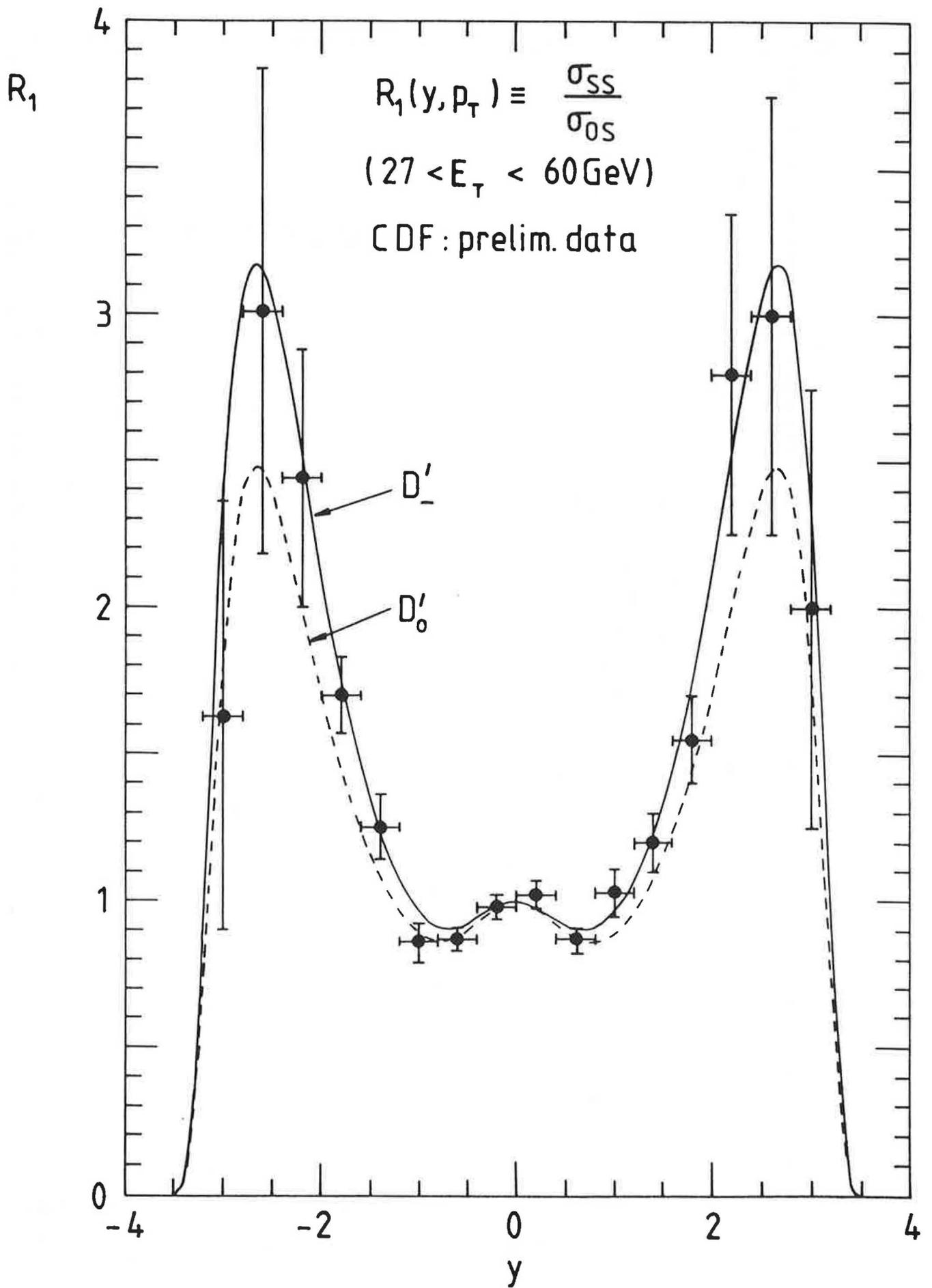


Fig. 2

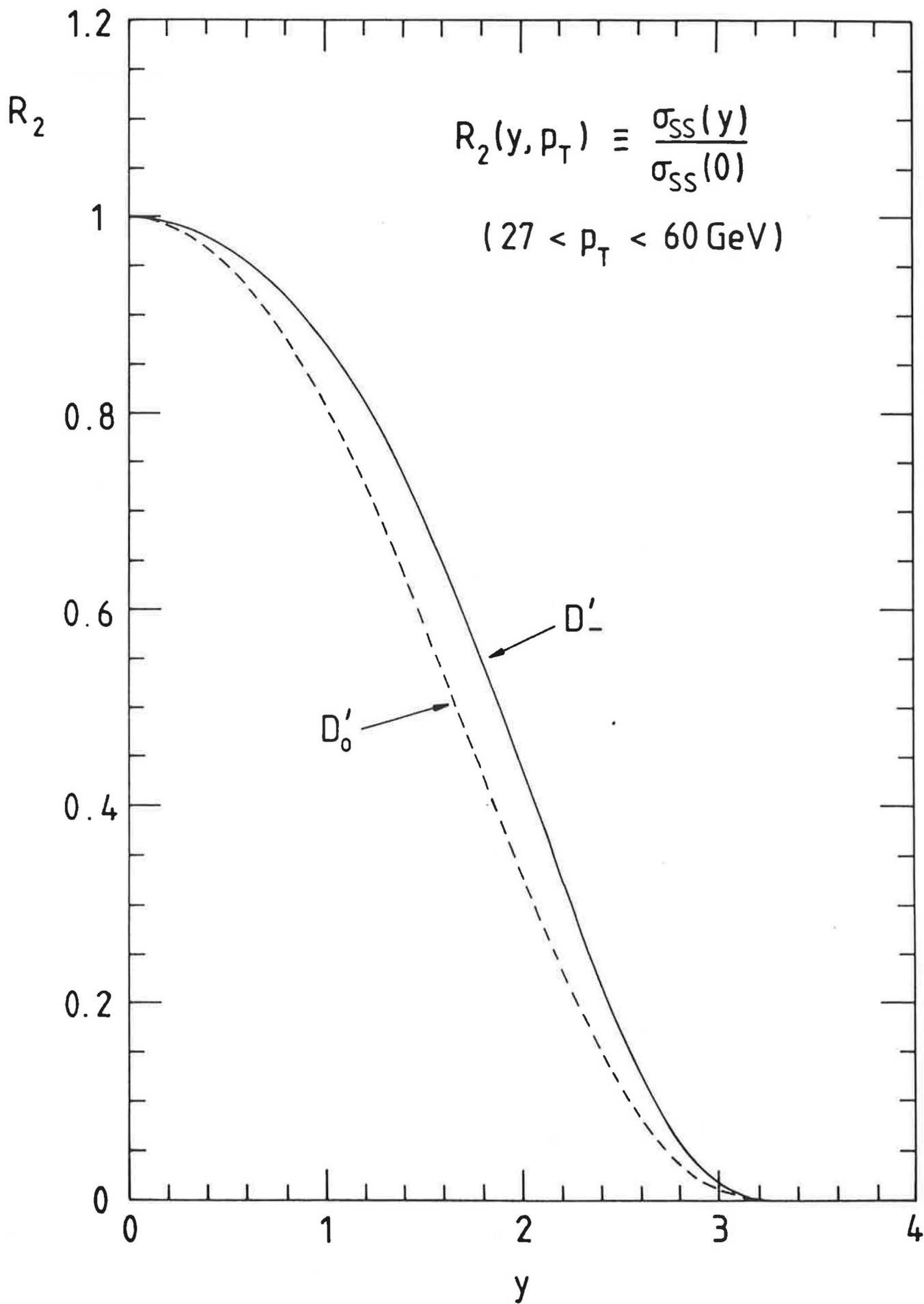


Fig. 3





