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Magnetic Critical Phenomena

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Contents

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1. Prologue

In the vicinity of a continuous phase transition (sometimes referred to as a second-order phase transition, and distinguished from a first-order phase transition by the absence of both hysteresis and a discontinuous change in the order parameter) fluctuations in the order parameter - magnetization of a ferromagnet - become extremely large. For an order-disorder transition, these fluctuations can be readily visualized as arising from the system locally making excursions between the two phases. So for a ferromagnet very close to the critical temperature (T_c) a small volume of the magnet fluctuates between an ordered, ferromagnetic state and a paramagnetic state. Such a process entails a very large number of spin excitations (normal modes or degrees of freedom) and this number is infinite at T_c.

A continuous phase transition is experimentally characterized by the divergence at the critical point of the mean-square fluctuation in the order parameter, for this quantity, apart from some benign factors, is a response function. In the case of a ferromagnet the appropriate response function is the magnetic susceptibility, χ, and if ΔM is the fluctuation in the magnetization ($<\Delta M> = 0$, where <....> denotes a thermal average)

$$\chi \;\; \alpha \;\; <(\Delta M)^2>.$$

An external probe that couples to the order parameter displays a pronounced signature of the phase transition due to the divergence of χ at T_c. The latter is usually a power law; for a ferromagnet it is customary to use the notation,

$$\chi \, \alpha \, (T - T_c)^{-\gamma} \; ; T > T_c,$$

and the critical exponent $\gamma \sim 1.39$ for a simple, isotropic ferromagnet. Table I contains an abbreviated list of critical exponents, relations among the exponents (so-called scaling relations) and theoretically derived numerical values.

The foregoing comments refer to static properties of a system near a continuous phase transition. Equally dramatic effects are observed in the dynamic properties of the order parameter, namely, the associated relaxation time increases when the sample approaches the critical point; this effect is usually referred to as the critical slowing down of fluctuations. A simple, intuitive argument by which to interpret critical slowing down stems from considering the reversal of one spin. Near the critical point the susceptibility is very large, so the perturbation on the magnetic system caused by the spin reversal influences a large volume and hence a large number of spins (the perturbation in the magnet at a large distance from the reversed spin depends on distance (r) like $\{\exp(-r/\xi)/r\}$, to a good approximation, where ξ is the correlation length, Table I). It naturally takes a very long time for thermal fluctuations to restore equilibrium by reversing each one of the large number of perturbed spins. A simple model of spin relaxation, which neglects couplings between the spin modes, predicts a relaxation time whose temperature dependence is provided by $1/\chi$, and this vanishes at T_c.

At the time of writing, it is fair to say that static critical phenomena is well understood. From the viewpoint of theoretical work, there exists a complete formalism (renormalization group and conformal field theory) for the calculation of response functions and their associated critical exponents; a thorough account of this work is provided by Ma (1976) and Zinn-Justin (1990). Hence, given a relatively simple model it is possible to confidently calculate static critical properties to a given accuracy. The same is not true of dynamic critical phenomena; at present, there is no complete computational formalism by which to obtain response functions observed in experiments using, say, neutron and photon beams. Indeed, it is a topic of research in which there is still much to be done; neutron beam (spin-echo, and scattering) and muon beam techniques have much to offer. This paper is largely focused on the dynamic properties of simple magnets in the vicinity of the critical point.

A topic which is not addressed is the nature of the phase transition, and response functions, in spin glasses. It is fairly well established that, there exists a sharp transition in three-dimensional Ising spin glasses. Theoretical results for 3d Heisenberg spin glasses are so far less conclusive (Fischer and Hertz, 1991).

2. Perspective

Introductions to phase transitions and critical phenomena are given by Stanley (1971), Als-Nielsen and Birgeneau (1977) and Cardy (1993). An introduction and more advanced material, together with a survey of experimental results on magnetic systems, is provided by Collins (1989). Detailed accounts of theoretical work can be found in various articles in the series of books edited by Domb and Lebowitz, and the monographs by Ma (1976), Patashinskii and Pokrovskii (1979) and Zinn-Justin (1990).

The nature of the strong critical scattering observed at a continuous phase transition depends on the spatial dimensionality of the magnetic system and the spin dimensionality. Indeed the existence of a phase transition depends on these parameters; for example, a two-dimensional magnet with three-dimensional spins (planar Heisenberg) does not show long-range magnetic order at any finite temperature, in a contrast to a two-dimensional Ising-spin model which orders at a temperature whose magnitude is of the order of the exchange parameter. The behaviour of the susceptibility, and other static response functions, at the phase transition can be classified according to the space and spin dimensionality, and this feature of static critical phenomena is called universality. The latter is more complicated for dynamic critical phenomena. The term universality is justified by the fact that different physical systems with the same space and spin - or the variable(s) that corresponds to spin-dimensionality - display similar critical properties. A third feature that determines the universality class of a system is the nature of the interactions, and specifically their spatial range. For static critical phenomena it seems that just the three features of spin dimension, spatial dimension, and the range of the spin interactions completely determine the universality class.

Another important concept is that of scaling, which we choose to illustrate for quantities of direct relevance to the interpretation of scattering experiments. The fundamental information is that certain quantities are homogeneous functions of a few reduced variables. This information comes from the renormalization group method, although it can also be derived from a less rigorous line of reasoning. We restrict ourselves to consideration of the correlation length, Gibbs potential (G) and the spin correlation function ($T \geq T_c$),

$$C_o(\mathbf{R}) = \langle \Delta S(\mathbf{R} + \mathbf{R}_o) \cdot \Delta S(\mathbf{R}_o) \rangle,$$

where the spin deviation at site **R** is,

$$\Delta S(\mathbf{R}) = S_{\mathbf{R}} - \langle S_{\mathbf{R}} \rangle.$$

Concerning the definition of $C_o(\mathbf{R})$, it is independent of the arbitrary site vector $\mathbf{R}_o$, and,

$$\sum_{\mathbf{R}} C_o(\mathbf{R}),$$

is proportional to the mean-square fluctuation in the magnetization considered in §1. For large values of $R = |\mathbf{R}|$, C_o is essentially independent of the direction of **R**.

The fundamental information is that ξ, G and C_o considered as functions of the reduced temperature $\tau = \{(T - T_c)/T_c\}$ and reduced magnetic field $h \propto (H/T_c)$ are homogeneous functions of the form,

$$\xi(\tau, h) = \lambda\, \xi(\lambda^y \tau, \lambda^x h) \qquad (1)$$

$$G(\tau,h) \;=\; \lambda^{-d}\, G(\lambda^{y}\tau,\lambda^{x}h) \tag{2}$$

and

$$C_o(R,\tau,h) \;=\; \lambda^{2(x-d)}\, C_o(\lambda^{-1}R,\lambda^{y}\tau,\lambda^{x}h)\ . \tag{3}$$

Here, λ is a scaling variable, and the exponents x and y are related to those listed in Table I, as we will see. It is useful to follow the type of argument that yields these relationships, and scaling laws, i.e. relations between the critical exponents. First, let us establish $\nu y = 1$ for $h = 0$. The definition of ν is $\xi \sim (1/\tau)^{\nu}$, cf. Table I. Turning to equation (1), we set $h = 0$ and choose $\lambda^{y}\tau = 1$, or $\lambda = (1/\tau)^{1/y}$; then,

$$\xi(\tau,0) = (1/\tau)^{1/y}\, \xi(1,0) \tag{4}$$

and the definition of ν leads immediately to the relation $\nu y = 1$. In equation (4) note that, $\xi(1,0)$ is a number which is independent of the temperature.

Having expressed the exponent y in terms of one of the critical exponents listed in Table I, let us focus on the exponent x; we shall establish the relation,

$$\gamma = \nu(2x-d), \tag{5}$$

which leads to the scaling relation cited in Table I between γ, η and ν. To obtain (5) we use the standard thermodynamic relation for the isothermal susceptibility (χ) as the second derivative of G with respect to h evaluated h = 0, namely,

$$\chi = -\frac{\partial^2}{\partial h^2} G(\tau,h)\big|_{h=0} \;=\; -(-1/\tau)^{\nu(2x-d)} \frac{\partial^2}{\partial h^2} G(-1,h)\big|_{h=0}. \tag{6}$$

The second equality follows from the choice,

$$-\tau = \lambda^{\nu},$$

in which the sign of τ is appropriate for the ordered magnetic phase. If we assume that the critical exponent γ is the same on both sides of the critical point, the relation (5) follows immediately from equation (6). (Exponents above and below T_c are usually the same but the coefficients of proportionality, or critical amplitudes, are different, i.e. G(1,h) is not expected to be the same as G(-1,h))

In equation (3) we choose $R = \lambda$. For $h = 0$, we have already established,

$$R^y \tau \sim (R/\xi)^y,$$

so we can write,

$$C_o(R,\tau,0) = R^{2(x-d)} C(R/\xi), \tag{7}$$

where C is a function of the dimensionless variable (R/ξ) which contains the temperature dependence. The standard definition of the exponent η is ($T = T_c$)

$$C_o(R,0,0) \sim R^{2-d-\eta}, \tag{8}$$

and it follows that (5) leads to the scaling relation,

$$\gamma = \nu(2 - \eta). \tag{9}$$

The relations (1) - (3) provide other scaling laws involving α and β but we do not pause to obtain them.

Instead, we turn to the wave vector dependent isothermal susceptibility, defined for a classical system as the spatial Fourier transform of $C_o(R,\tau,h)$. Denoting this susceptibility by $\chi(k)$, equation (7) leads to the result ($h = 0$)

$$\chi(k) = k^{\eta-2}\, \tilde{C}(k\xi) \tag{10}$$

where $\tilde{C}$ is proportional to the spatial Fourier transform of C. In particular, at T_c, where $k\xi \to \infty$,

$$\chi(k) \sim k^{\eta-2}\,;\, T = T_c. \tag{11}$$

In the limit $k \to 0$, $\chi(k)$ tends the bulk of susceptibility introduced in §1. From equation (10) it follows that, in this limit,

$$\tilde{C}(k\xi) \sim (k\xi)^{2-\eta}$$

and the identification of χ with $\chi(0)$ reproduces the scaling relation (9).

Let us now consider some dynamical properties of spin fluctuations in the vicinity of T_c. Results are less surely footed because there is not the solid base of information that (1) - (3) provide for static critical phenomena, i.e. the renormalization group method is less useful. There is an equally important method for dynamical properties called the mode-coupling approximation, which amounts to a certain recipe for obtaining self-consistent solutions to the full equation of motion for spin fluctuations. The accumulated evidence is that results obtained from the mode-coupling approximation are consistent with information derived from the renormalization group method. Moreover, the mode-coupling approximation has been successful in the interpretation of experimental data. We will say more about this approximation in the next section, but for now let us turn to results for the relaxation rate.

The time-dependent generalization of the spatial Fourier transform of the spin correlation function $C_0(R)$ is denoted by F(k,t). The equation of motion for F(k,t), usually referred to as a generalized Langevin equation (see for example, Lovesey, 1986), shows that F(k,t) is a homogeneous function that satisfies,

$$F(k,t) = F(\lambda^a k, \lambda^{1/2} t) \, , \qquad (12)$$

but the exponent (a) is not determined. To find a value for this exponent it is necessary to invoke an approximation that closes the equation of motion. The aforementioned mode-coupling approximation achieves closure, and produces an integro-differential equation for F(k,t) which is self-consistent, i.e. the dynamics are expressed solely in terms of F(k,t). One can view a self-consistent equation as the result of a perturbation expansion taken to an infinite order. That such a type of approximation is required to generate physically sensible results in the critical region is a consequence of having, inevitably in an approximate manner, to account for the infinite number of degrees of freedom that participate in critical fluctuations (there is a useful analogy with the even more difficult, and as yet unsolved, problem of fully developed turbulence, e.g. Leslie, 1973).

The standard form of the mode-coupling approximation for a Heisenberg model of a ferromagnet gives the result a = - 1/5, and the relation

$$F(k,t;\xi) = F(\lambda^a k, \lambda^{1/2} t; \lambda^a / \xi). \qquad (13)$$

In view of the relation (13) one is led to seek a self-consistent solution of the form

$$F(k,t;\xi) = Q(k^z \, t \, \Omega(k\xi)). \qquad (14)$$

As in the case of static critical phenomena, useful information can be extracted from (14) without knowledge of the single-variable functions Q and Ω. Such knowledge is required, however, when coupled-mode theory is confronted with the spectrum of spin fluctuations observed by neutron spectroscopy (Cuccoli et al. 1989, and 1990).

Equations (13) and (14) are mutually consistent when,

$$az + \frac{1}{2} = 0.$$

Since a = - 1/5 the dynamic critical exponent z = 5/2. At the critical point $(k\xi)\rightarrow\infty$, and $\Omega(k\xi)$ approaches a constant. In consequence, the relaxation rate for ferromagnetic spin fluctuations $\Gamma(k) \propto k^{5/2}$ at $T = T_c$. The data obtained for EuO at T_c, displayed in Fig. (1), is consistent with the value z = 5/2 over a wide range of wave vectors. By carrying through the self-consistent solution of the mode-coupling equation one finds (h = 0),

$$\Gamma(k) = k^{5/2}(2/3\pi)(T_c c v_o)^{1/2} \, ; T = T_c, \qquad (15)$$

where c and v_o are the exchange stiffness and unit cell volume, respectively (the spin-wave stiffness D = 2c S).

We conclude this section by considering the influence of dipolar forces between the atomic magnetic moments. Such forces are present in all magnetic materials, of course, and produce the effect of demagnetization. A measure of the strength of dipolar forces is the wave vector q_d defined by,

$$(2\pi/v_o)(g\mu_B)^2 = c q_d^2 \ ,$$

in which g is the gyromagnetic factor. For the moment, it is useful to note the values of q_d for EuO and EuS, namely, 0.15Å^{-1} (EuO) and 0.26Å^{-1} (EuS), from which it is seen that dipolar forces are larger in EuS than EuO. Experiments by Böni et al. (1991) on EuS are largely consistent with theoretical results derived by Frey and Schwabl (1988) from a mode-coupling approximation applied to a Heisenberg Hamiltonian including dipolar forces.

The inclusion of dipolar forces produces spatial anisotropy. For $T \geq T_c$ it is logical to speak of fluctuations parallel (longitudinal) and perpendicular (transverse) to the wave vector **k** associated with the spin fluctuations under observations. One effect of the dipolar forces between atomic moments is to prevent the longitudinal fluctuations from diverging in the critical region, $k \rightarrow 0$ and $T \rightarrow T_c$, so the phase transition is dominated by transverse fluctuations. The relaxation rates are found to be

$$\Gamma_\alpha(k, q_d) \alpha\, k^{5/2}\, \gamma_\alpha(x, y)\, ; \alpha = L, T,$$

where the dimensionless variables x, y are,

$$x = (1 / k\xi), \text{ and } y = (q_d / k).$$

Table II contains the asymptotic behaviour of the scaling functions $\gamma_\alpha(x,y)$, in which the notation is: D, dipolar; I, isotropic; C, critical; H, hydrodynamic. The four limiting regions are then: DC, $x<<1$, $y>>1$; IC,$x<<1$, $y<<1$; DH, $x>>1$, $y>>x$; IH, $x>>1$, $y<<x$.

For the dynamic critical exponent one finds for the longitudinal relaxation rate a cross-over from $z = 5/2$ in the isotropic critical region, considered in previous paragraphs, to $z = 0$ in the dipolar critical region. The situation for transverse fluctuations is a cross-over from $z = 5/2$ to $z = 2$. A numerical solution of the coupled-mode equations shows that for transverse fluctuations the dynamic cross-over is shifted with respect to the static one, $k \sim q_d$, to a wave vector $\sim q_d/10$. Longitudinal fluctuations are not anomalous and the dynamic and static cross-overs coincide. The dynamic cross-over in the transverse fluctuations has not been clearly demonstrated experimentally.

3. Neutron Scattering

The neutron scattering cross-section is proportional to (α, β label Cartesian components),

$$\sum_{\alpha,\beta} (\delta_{\alpha\beta} - \kappa_\alpha \kappa_\beta / \kappa^2) S^{\alpha\beta}(\kappa, \omega),$$

where the so-called van Hove response function for a sample with N spins on a lattice is,

$$S^{\alpha\beta}(\kappa, \omega) = \frac{1}{2\pi N} \int_{-\infty}^{\infty} dt\, e^{-i\omega t} \sum_{\mathbf{R},\mathbf{R}'} \exp\{i\kappa.(\mathbf{R} - \mathbf{R}')\} < \Delta S^\alpha(\mathbf{R}, 0) \Delta S^\beta(\mathbf{R}', t) >. \quad (16)$$

Recall that the spin fluctuation at the site labelled **R** satisfies $< \Delta S^\alpha(\mathbf{R}) > = 0$, while $\Delta S(\mathbf{R},t)$ is a Heisenberg operator at the time t. Inspection of (16) reveals that

$S^{\alpha\beta}(\kappa,\omega)$ is the spatial and temporal Fourier transform of spin autocorrelation functions.

Let us now make contact between the measured response function and quantities introduced in previous sections. First, the integral of $S^{\alpha\beta}(\mathbf{k},\omega)$ with respect to ω is proportional to the isothermal susceptibility, cf. equation (10) that applies to an isotropic system for which the spin autocorrelation function vanishes for $\alpha\neq\beta$, while for $\alpha=\beta$ it is independent of the value of the Cartesian label. The experimental realization of integrating over ω, to measure the total scattering, is discussed by Collins (1989) and Als-Nielsen (1973) in Vol. 5a of the series edited by Domb and Lebowitz.

It is now almost standard practice to decompose $S^{\alpha\beta}(\mathbf{k},\omega)$ in the following manner,

$$S(\mathbf{k},\omega) = T\chi(\mathbf{k})F(\mathbf{k},\omega), \tag{17}$$

where for simplicity we omit the Cartesian labels on $S(\mathbf{k},\omega)$, $\chi(\mathbf{k})$ and $F(\mathbf{k},\omega)$, and the latter is the time Fourier transform of $F(\mathbf{k},t)$ introduced in §2.

Near T_c, and neglecting the critical exponent η, the susceptibility of an isotropic spin system is well represented by the form adopted by Ornstein and Zernike in which

$$\chi(k) \sim \{(k\xi)^2 + 1\}^{-1}. \tag{18}$$

The expression appropriate in the presence of dipolar forces is given by Frey and Schwabl (1988).

A simple derivation of the coupled-mode equation for $F(\mathbf{k},t)$, together with an examination of its predictions, is given by Lovesey (1986). Complete solutions to the equation, obtained by numerical integration, are applied by Cuccoli et al. (1989, 1990) to the interpretation of neutron scattering data obtained for $T \geq T_c$ and a wide range of wave vector. The confrontation of theory and data provides good evidence for the success of the coupled-mode approximation for the description of paramagnetic and critical spin fluctuations. A similar finding is reached by Frey and Schwabl (1988) in their discussion of effects due to dipolar forces.

Experimental results on the relaxation rate $\Gamma(k)$ are usually obtained by fitting a model $S(\mathbf{k},\omega)$ to the data. This is most often accomplished by use of the susceptibility (18) and

$$\pi \Gamma F(k,\omega) = (\omega^2 + \Gamma^2)^{-1}, \tag{19}$$

where $\Gamma = \Gamma(k)$ is discussed in §2. Ideally, one would like to use a more firmly based approximation for $F(\mathbf{k},\omega)$. Certainly, it is required to test results for $\Gamma(k)$ for their dependence on the assumed shape of $F(\mathbf{k},\omega)$. At present, the computer time involved in solving the coupled-mode equation for $F(\mathbf{k},t)$ is too large for a direct fitting scheme to be a practical exercise, and so some form of parameterization, such as (19), seems unavoidable.

4. **Muon spin relaxation**

The depolarization of a beam of positive muons incident on a magnetic sample increase with time, and it is usually well described by an exponential form exp $(-\lambda t)$ where λ is the relaxation rate. The time interval that can be investigated extends up to about $5\tau_\mu$, where the radioactive lifetime of a muon τ_μ = 2.2 μs. Here we wish to illustrate the connection between λ and the response function measured by neutron scattering.

The Larmor frequency of a muon is 0.57 μeV T^{-1}. Hence, an implanted muon has a Larmor frequency small compared to (electronic) spin-wave frequencies even if it is subject to a static magnetic field as large as 5.0T. In fact, the implanted muon might well occupy an interstitial site at which the dipolar field from the atomic moments is identically zero, e.g. an interstitial site in an fcc magnet (Ni, EuO). Also, the experiments do not require an applied magnetic field, in contrast to neutron beam techniques that utilize polarization analysis. For all these reasons, it is usually quite sensible to analyse muon relaxation experiments using $\omega_\mu \sim 0$.

The microscopic origin of muon depolarization is the fluctuations in the magnetic field ($\mathbf{B}$) experienced by implanted muons. The interaction between the muon magnetic moment and the field is $g_\mu \mu_N \mathbf{I}.\mathbf{B}$. On inserting this interaction in Fermi's Golden Rule for transition rates one obtain,

$$\lambda = \tfrac{1}{3}(g_\mu \mu_N / \hbar)^2 \int_{-\infty}^{\infty} dt < \mathbf{B}.\mathbf{B}(t) >. \tag{20}$$

This result applies to the case when it is appropriate to average λ over the orientations of the muon polarization with respect to the crystal axes of the sample.

The field **B** has two main sources. One is the dipolar interaction between the muon and atomic moments (even if the average dipolar force at the muon is zero, the fluctuating dipolar force can contribute to λ) and the other is the hyperfine interaction (Lovesey et al., 1992 and Yaouanc et al., 1993).

Let the position of the j'th. atomic moment with respect to the implanted muon be denoted by $\mathbf{R}_j$. The magnetic field experienced by the muon has spatial components,

$$B_\alpha = \sum_j (A_j / g_\mu \mu_N) S^\alpha(j) + g\mu_B \sum_{j\beta} S^\beta(j)(\delta_{\alpha\beta} - 3\hat{R}_j^\alpha \hat{R}_j^\beta) / R_j^3,$$

where A_j is the hyperfine constant, and the second contribution is the classical dipole field created by the atomic moments. For $T<T_c$ and $\omega_\mu = 0$ the most significant contributions to $<\mathbf{B}.\mathbf{B}(t)>$ arise from longitudinal spin fluctuations (along the preferred magnetic axis), since single spin-wave events, generated by transverse components, are energetically unfavourable (if there is a gap in the spectrum they do not conserve energy). We then find,

$$\int_{-\infty}^{\infty} dt < \mathbf{B}.\mathbf{B}(t) > = (2\pi\hbar v_o) \int d\mathbf{k}\, S(\mathbf{k},0) \{|A(\mathbf{k})|^2 + \sum_\alpha D^{\alpha z}(\mathbf{k}) D^{\alpha z}(-\mathbf{k})$$

$$+ 2\mathrm{Re.}(D^{zz}(\mathbf{k}) A^*(\mathbf{k}))\} \qquad (21)$$

where,

$$A(\mathbf{k}) = (1/g_\mu \mu_N) \sum_j A_j \exp(i\mathbf{k}.\mathbf{R}_j)$$

and

$$D^{\alpha\beta}(\mathbf{k}) = (g\mu_B) \sum_j \exp(i\mathbf{k}.\mathbf{R}_j)(\delta_{\alpha\beta} - 3\hat{R}_j^\alpha \hat{R}_j^\beta) / R_j^3 .$$

The results (20) and (21) provide the explicit relation between the muon relaxation rate and the longitudinal spin response function observed in neutron scattering.

For simple ferromagnetic spin-waves with a dispersion ($h + Dk^2$) we find,

$$S(k,0) = \left(\frac{Tv_o}{16\pi^2 D^2 k}\right)(\exp[(h + \tfrac{1}{4}Dk^2)/T] - 1)^{-1}. \qquad (22)$$

If the magnetic field h = 0, the integral of S(k,0) is dominated by its behaviour at the centre of the Brillouin zone. In consequence, the factor multiplying S(k,0) in (21) can be evaluated for $k \rightarrow 0$. For the dipolar sums it is probably sufficient to use,

$$D^{\alpha\beta}(\mathbf{k}) \rightarrow (4\pi g\mu_B / 3v_o)(3\hat{k}^\alpha\hat{k}^\beta - \delta_{\alpha\beta}).$$

We then find that the cross-terms in (21) between the hyperfine interaction and dipolar terms vanish. Assembling results, one finds with use of (22),

$$\lambda = (v_o^2 T^2 \hbar / 12\pi^3 D^3)\{\Gamma_o + 2(3\pi/16)^2 \Gamma\} \ell n\{1 - \exp(-h/T)\}^{-1}. \qquad (23)$$

Here, the quantities Γ_o and Γ, which have dimension $(1/\text{time})^2$, are

$$\Gamma_o = \tfrac{1}{2}(N_o A / \hbar)^2$$

and

$$\Gamma = 8\,(16 g g_\mu \mu_B \mu_N / 9\hbar v_o)^2$$

where N_o is the number of moments that have a hyperfine constant A.

The result (23) shows that, for a simple ferromagnet with $T \ll T_c$ the relaxation rate is essentially quadratic in the temperature. This result follows immediately from the fact that we have focused on two spin-wave events, and each event is weighted by a Bose factor which is proportional to T in the limit of long wavelengths.

In the absence of an external magnetic field it is necessary to take account of the dipolar interaction between the atomic spins. A tedious calculation shows that, in this case, $\lambda \propto T^3$.

Fig. (2) summarizes the expected behaviour of λ for a ferromagnet. Analogous results for antiferromangets are given by Lovesey (1992).

For the simple ferromagnet EuO one finds $\Gamma = 0.055.10^6\ \mu s^{-2}$. If the muon is at an interstitial site, and the hyperfine interaction is created by the four neighbouring atomic moments $\Gamma_o = 8\ (A/\hbar)^2$. (In a magnetic salt one expects $\Gamma \gg \Gamma_o$, while the reverse situation is likely in a metallic magnet). An estimate of the longitudinal response function in the critical region leads to,

$$\lambda \sim 3.0.10^{-6}\mu s\, \xi^{3/2}\, (\Gamma_o + 0.69\Gamma),$$

where the correlation length ξ is measured in units of Å.If $\Gamma_o \sim 0$, and the temperature is 1% above T_c, one finds $\lambda \sim 25\ \mu s^{-1}$.

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Figure captions

1. The relaxation rate for EuO at T_c obtained by neutron spin echo and scattering experiments; from Böni et al. (1987)

2. Anticipated behaviour of the muon relaxation rate for a simple ferromagnet.

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Table I

Magnetization $\sim (T_c - T)^{\beta}$

Susceptibility, $\chi \sim (T - T_c)^{-\gamma}$

Correlation length, $\xi \sim (T - T_c)^{-\nu}$

Specific heat $\sim (T - T_c)^{-\alpha}$

Fisher exponent, η : $\gamma = \nu(2 - \eta)$

$$\left.\begin{aligned} \alpha &= 2 - \nu d \\ 2\beta &= \nu(d - 2 + \eta) \end{aligned}\right) \quad d = \text{spatial dimension}$$

Isotropic ferromagnet, d = 3: critical exponents derived from a renormalization group calculation (from Zinn-Justin, 1990, table 25.6)

$$\beta = 0.368 \pm 0.004$$
$$\gamma = 1.390 \pm 0.010$$
$$\nu = 0.710 \pm 0.007$$
$$\eta = 0.040 \pm 0.003$$

Table II

	γ_T	γ_L
DC	$y^{1/2}$	$y^{5/2}$
IC	1	1
DH	$x^2 y^{1/2}$	$y^{5/2}$
IH	$x^{1/2}$	$x^{1/2}$

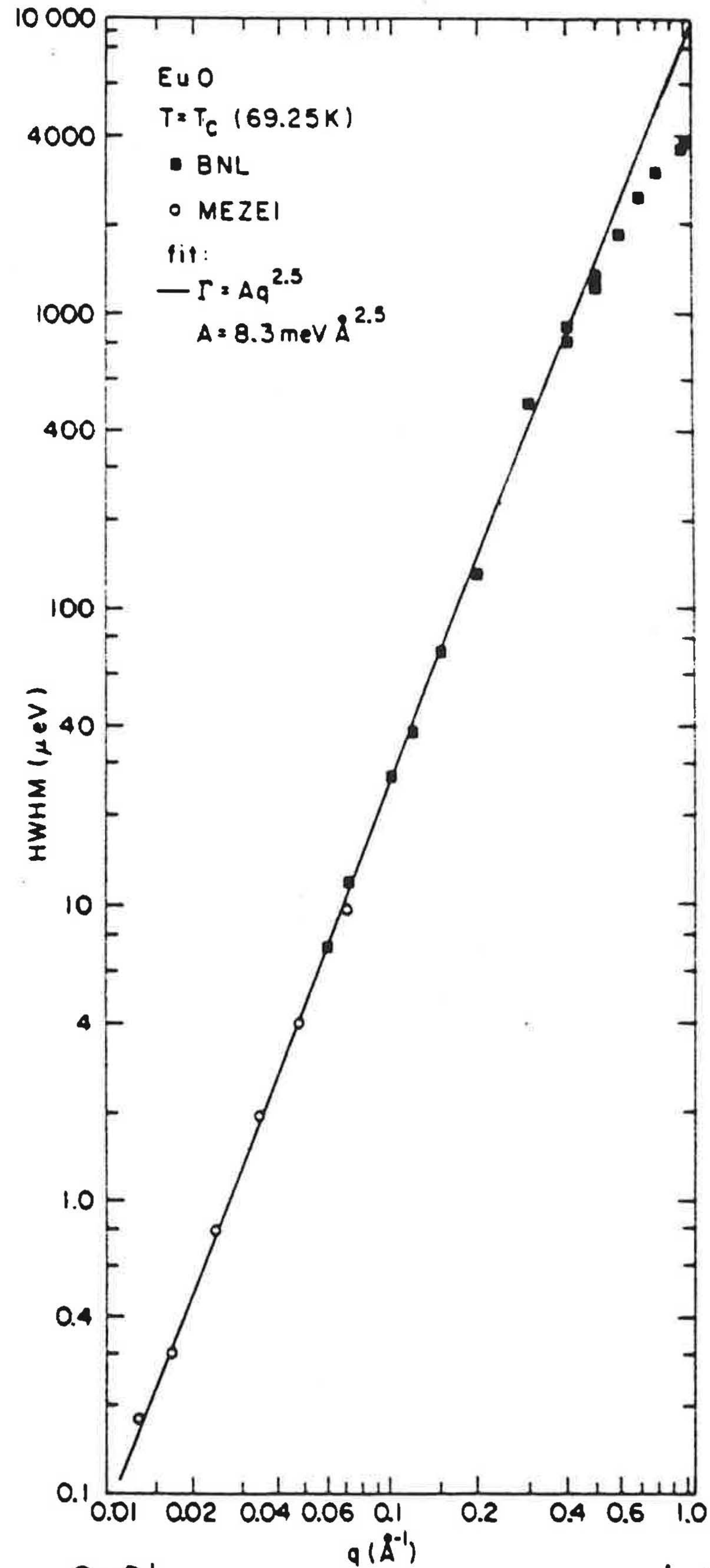
Asymplotic behaviour of the scaling functions for the relaxation rates in a paramagnet in which the dipolar energy is included: After Frey and Schwabl (1988).

After Böni et al. (1987) Phys. Rev. B35, 8449

EuO, $T=T_c$

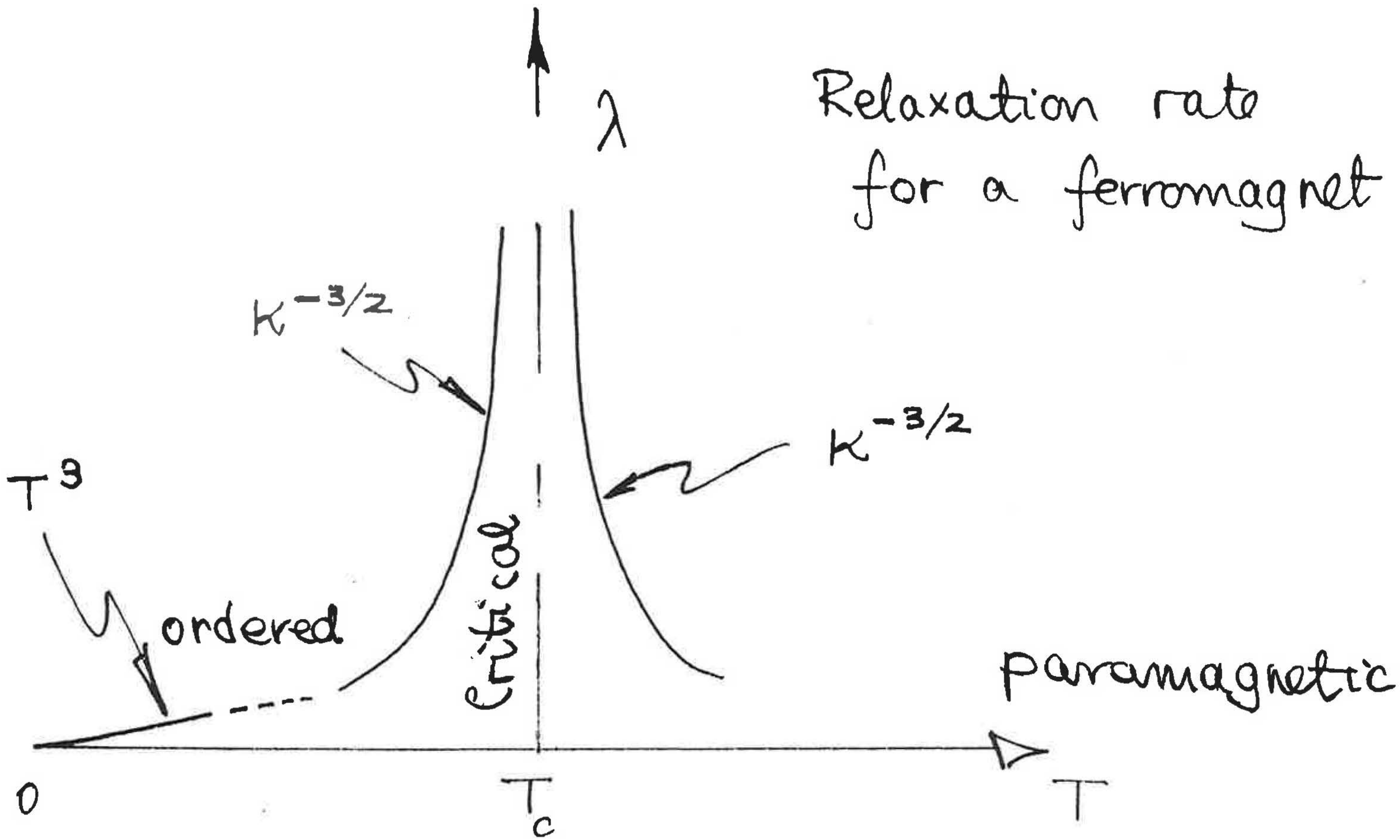
HWHM

$\sim q^{5/2}$



0.01 q (Å^{-1}) ⟶ 1.0

Fig. (1)



Inverse correlation length,
$\kappa \propto |(T - T_c)|^{\nu}$.

Fig. (2)