Improved QCD Sum Rule Estimates of the Higher Twist Contributions to Polarised and Unpolarised Nucleon Structure Functions

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Improved QCD sum rule estimates of the higher twist contributions
to polarised and unpolarised nucleon structure functions

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Abstract

We re-examine the estimates of the higher twist contributions to the integral of $g_1$, the polarised structure function of the nucleon, based on QCD sum rules. By including corrections both to the perturbative contribution and to the low energy contribution we find that the matrix elements of the relevant operators are more stable to variations of the Borel parameter $M^2$, allowing for a meaningful estimate of the matrix elements. We find that these matrix elements are typically twice as large as previous estimates. However, inserting these new estimates into the recently corrected expressions for the first moments of $g_1$ leads to corrections too small to affect the phenomenological analysis. For the unpolarised case the higher twist corrections to the GLS and Bjorken sum rules are substantial and bring the estimate of $\Lambda_{QCD}$ from the former into good agreement with that obtained from the $Q^2$ dependence of deep inelastic data.

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The recent measurements of the polarised nucleon structure functions \( g_{1p,n,d}^{p,n,d} \) [1], [2], [3] raise the question of the consistency of the three measurements \( I_{p,n,d} = \int_0^1 dx g_{1p,n,d}(x) \). In particular since the three measurements are at different values of \( <Q^2> \), namely 10.7, 2 and 4.6 GeV\(^2\) for \( p, n \) and \( d \), it is crucial to understand the \( Q^2 \) dependence of the first moment of \( g_1 \) in order to test the Bjorken sum rule [4] or to extract an estimate of the nucleon’s spin content, \( \Delta q \), consistent with all three experiments.

The higher order corrections to the leading twist expressions have been calculated to \( O(\alpha_s^2) \) for the non-singlet quantities and to \( O(\alpha_s) \) for the singlet contribution but it is the power corrections \( \frac{1}{Q^2} \) from the higher twist operators which have recently [5], [6] been shown may play an important role in a consistent analysis of the data. The magnitudes of the reduced matrix elements of the relevant higher twist operators \( U^S, U^{NS}, V^S, V^{NS} \) were extracted from a QCD sum rule calculation by Balitsky, Braun and Kolesnichenko (BBK) [7] and used in the analysis of ref [5]. The aim of the present paper is to sharpen the results of BBK by re-examining the computation of \( \ll U^{NS} \gg, \ll V^{NS} \gg \), including a contribution to the perturbative QCD side of the sum rule which was dropped in the Borel transformation and explicitly retaining the continuum term to the nucleon pole on the low-energy side of the sum rule. This leads to a significant improvement in the stability of the extracted value of the reduced matrix elements and hence to a more reliable estimate of the higher twist contribution to the integral of \( g_1 \).

Moreover the estimated values of the matrix elements are significantly larger than previously but since the coefficient of the twist three piece in the first moment moment of \( g_1 \) has very recently been corrected [8], it turns out that the net higher twist contribution to the moment is minimal. The improvement in stability and increased magnitude of the matrix elements is also true for the unpolarised case and we find that the correction to the Gross-Llewellyn Smith (GLS) sum rule [9] is sufficient to affect the extracted value of \( \Lambda_{\overline{MS}} \) substantially.

Following the procedure of BBK we consider the quantity \( \Gamma_\mu(p) \), if we are interested in the operators \( U^{S,NS} \), given by

\[
\Gamma_\mu(p) = i^2 \int dx \, e^{ipx} \int dy \, \langle T[\eta(x)U_\mu(y)\bar{\eta}(0)] \rangle \tag{1}
\]

where \( \eta \) is the nucleon current. Expanding \( \Gamma_\mu(p) \) in powers of \( \frac{1}{p^2} \) gives

\[
\Gamma_\mu(p) = p_\mu p_\gamma \left[ A p^4 \ln^2(\frac{\mu^2}{-p^2}) + B \ln(\frac{\mu^2}{-p^2}) + C(\frac{1}{-p^2}) \ln(\frac{\mu^2}{-p^2}) + D(\frac{1}{-p^2}) \right] \tag{2}
\]

The coefficients \( A, \ldots, D \) may be read off from eq(8) of BBK corresponding to the QCD evaluation of \( \Gamma_\mu \), including non-perturbative effects due to QCD condensates. The next step is to Borel
transform the coefficient of \( p_\mu \phi \gamma_5 \) in eq(2) which gives

\[
\begin{align*}
p_\mu \phi \gamma_5 \left[ 2AM^2 \int_0^{s_0} ds \ e^{-s/M^2} s^2 \ln\left( \frac{\mu^2}{s} \right) + BM^4 \{ 1 - e^{-s/M^2} \} - \tilde{C}M^2 \{ C_E + \ln\left( \frac{\mu^2M^2}{M^2} \right) \} + CM^2 + D \right]
\end{align*}
\]

which differs from eq(11) of BBK since we have explicitly carried out the \( p^2 \) integration of the \( -\frac{1}{p^2} \ln\left( \frac{\epsilon^2}{p^2} \right) \) contribution instead of neglecting it.

To complete the sum rule the quantity \( \Gamma_\mu(p) \) must also be determined in terms of the nucleon and continuum contributions. Balitsky et al.\cite{7} use the form

\[
\begin{align*}
\Gamma_\mu(p) = -p_\mu \phi \gamma_5 \left[ \frac{2\lambda_2^2 \ll U \gg}{(p^2 - m^2)^2} + \frac{X}{(p^2 - m^2)} \right]
\end{align*}
\]

where the first term is the pure nucleon pole contribution and the single pole term is added to allow for the interference of the pole term with a continuum contribution. The Borel transform of the coefficient of \( p_\mu \phi \gamma_5 \) in eqn(4) is

\[
\begin{align*}
\frac{2\lambda_2^2}{M^2} \frac{e^{-m^2/M^2}}{\ll U \gg} - e^{-m^2/M^2} X
\end{align*}
\]

and the QCD sum rule results from equating eq(3) to eq(5) to give

\[
\ll U \gg - XM^2 = \left[ \frac{e^{-m^2/M^2}}{2\lambda_2^2} \right] \times \left[ 2AM^2 \int_0^{s_0} ds \ e^{-s/M^2} s^2 \ln\left( \frac{\mu^2}{s} \right) + BM^4 \{ 1 - e^{-s/M^2} \} - \tilde{C}M^2 \{ C_E + \ln\left( \frac{\mu^2M^2}{M^2} \right) \} + CM^2 + D \right]
\]

In this paper we are particularly concerned to estimate of the errors in determining the operator matrix elements from the QCD sum rules. Thus we will consider in detail the effects of each of the terms in this expansion and the inclusion of further terms in the Borel expansion in \((M^2)^n/n!\). On the rhs of eq(2) the first term not included is \( \sim 1/p^6 \) which, after Borel transformation contributes a term \( \sim 1/2!M^2 \) to the rhs of eq(6). The coefficient of this term should be no bigger than the coefficient of \( Y \) if the perturbative expansion in \( 1/(n!(M^2)^n) \) is acceptably convergent. By adding such a term and fitting its coefficient we may check this over some range of \( M^2 \) and hence establish over what range of \( M^2 \) (if any) the sum rule converges.

The corrections to the lhs of eq(6) are somewhat more difficult to determine. Further resonance contributions with mass \( m_R^2 > m^2 \) can be included by adding a resonance pole

\footnote{Balitsky et al. argue that with the natural choice, \( \mu^2_{MS} \sim -p^2 \sim 1GeV^2 \), this term vanishes. However the Borel transform integrates over \( p^2 \) so this is, at best, an approximation. Since the effect of the term can be explicitly included we choose not to make this approximation.}
After Borel transformation this gives an additional term \( M^2 e^{(m^2 - m_R^2)/M^2} Y \) on the lhs of eq(6). However the dominant correction to the lhs of eq(2) is not expected to come from a nucleon resonance excitation but from the \((N + n\pi)\) continuum which has a threshold at \( E = \sqrt{m^2 + m_N^2} \), very close to the nucleon pole. As far as we know this contribution has not been considered explicitly even though it is potentially very large. However, we will demonstrate that this contribution does not substantially degrade the accuracy with which we can determine the operator matrix elements provided the resonance term discussed above is added to eq(6).

The reason is that the \( \pi N \) contribution is well described by the terms proportional to \( X \) in eq(4) together with the resonance term proportional to \( Y \). To demonstrate this we will estimate the contribution due to the \( \pi N \) intermediate state, fig.2. This gives the term

\[
I_{\pi N} = \frac{g_{\pi NN}^2 \lambda^2}{m^2} \int \frac{f(q^2)^2[-2(p + q)\gamma_\mu(p + q)\gamma_5 \ll U \gg]d^4q}{(2\pi)^4(q^2 - m^2_N)(m^2 - (p + q)^2)}
\]

(7)

Here we have included a form factor \( f(q^2) \) needed to describe the \( \eta N\pi \) coupling far from the pion mass shell (on shell we take \( \langle \eta | N\pi \rangle = \lambda_p g_{\pi NN} \)). The result is insensitive to the particular choice of \( f(q^2) \) provided it provides convergence for large \( q^2 \); here we choose \( f(q^2) = q^2/((q^2 - m^2)(1 - q^2/0.7GeV^2)) \) i.e. chosen to vanish as \( q^2 \to 0 \) and to be the same as the nucleon electromagnetic form factor for large spacelike \( q^2 \). (Since \( f(q^2) \) is needed for spacelike \( q^2 \) we have chosen a form factor with no singularities in the spacelike region.) After taking the Borel transform of the coefficient of \( p_\mu \gamma_5 \) and multiplying by \( M^2 e^{m^2/M^2}/2\lambda^2_p \) we find the resulting contribution to the lhs of eq(6) is accurately reproduced by the form

\[
\alpha + \beta M^2 + \gamma M^2 e^{(m^2 - m_R^2)/M^2}
\]

(8)

with \( \alpha \sim 0.03 \ll U \gg, \beta \sim 0.15 \ll U \gg, \gamma \sim 0.2 \ll U \gg, \) and \( m_R^2 \approx 2m^2 \). The origin of this form is easy to understand. The \( \pi N \) singularity starts at \((m_\pi + m)\) in the energy plane and, after Borel transformation, gives a contribution proportional to the integral of \( e^{(m^2 - m_R^2)} \) over \( m_\pi \), with an appropriate weighting factor. To a good approximation this may be approximated by the sum of two exponentials \( e^{(m^2 - m_1^2)/M^2} \) and \( e^{(m^2 - m_2^2)/M^2} \) with \( m_1 \approx (m_\pi + m) \). Taylor expanding the first term in \( m_\pi \) leads to eq(8). Thus we see that the \( \pi N \) contribution may, to a good approximation, be included in the continuum and interference terms and gives a relatively small correction to the determination of the operator matrix elements. Using similar methods suggests that the contribution to constant term in eq(8) from \( N + n\pi \) intermediate states will also be quite small.
Let us now turn to the phenomenological analysis and first consider the analysis of ref[7]. We have four operators to consider $U^S, U^{NS}, V^S, V^{NS}$ the latter two having $\bar{p}_\mu \gamma_\mu$ replaced by $S_{\nu,\rho} A_{\nu,\rho} p_\mu \gamma_\mu \gamma_5$ where $A, S$ stand for symmetric and antisymmetric combination of indices. So we have four QCD sum rules with coefficients $A \to E$ given by BBK eqs(8,9). In extracting a value of $< U >$ from the sum rule, BBK retained only the first two terms on the lhs and also set $\tilde{C}=0$. They obtained $< U >$ at each value of $M^2$ by applying $(1 - M^2 \frac{d^2}{dM^2})$ to the rhs. The stability of this estimate of $< U >$ relies upon the rhs of eq(6) being approximately linear over a range of $M^2$. The solid lines in fig.1 are the BBK values for the matrix elements extracted by this procedure and the variation with $M^2$ reflects the importance of higher derivatives in $M^2$, making it difficult to arrive at a reliable estimates for $< U, V >$.

Let us see how the situation changes with the addition of the proposed continuum term $M^2 e^{(m^2 - m^2_R)} / M^2$ and the term proposal to $\tilde{C}$ in eq(6). As we will demonstrate these are necessary to describe adequately the $M^2$ behaviour of the rhs over a reasonable range. We first fit the rhs of eq(6) by the form on the lhs with four parameters $< U >, X, Y$ and $m^2_R$ (the last corresponding to an effective threshold for the continuum) over a wide range $0.5 \text{ GeV}^2 < M^2 < 2.0 \text{ GeV}^2$. Then, with $m^2_R$ fixed (typically around 2 GeV$^2$), we perform a three parameter fit over each interval of $0.2 \text{ GeV}^2$ in $M^2$. The dashed line in fig.2 shows the resulting values of $< U >$ and we immediately see a marked improvement in the stability compared to neglecting the continuum term which results in a more meaningful estimate of the matrix elements. At the same time we notice that the actual value obtained is considerably different. At $M^2 = 1 \text{ GeV}^2$ for example the new estimate is roughly twice the old value. If instead we choose to describe the continuum by $e^{(m^2 - m^2_R)} / M^2$ (i.e. a double resonance pole) there is a similar improvement in stability and the extracted values of the matrix elements are within 15% of those in fig.1.

Using these results we will now try to sharpen the estimates of the higher twist contributions further by considering possible additional terms to the rhs of eq(6). We will consider the sensitivity of $< U, V >$ to an additional term $\sim \frac{1}{\mu^2}$ on the rhs of eq(2), i.e. a term $\sim \frac{1}{2M^2}$ on the rhs of eq(6). Thus we may have an additional term $e^{m^2 / M^2} \frac{1}{2M^2}$ on the lhs of eqn (6) which can be used in fitting the rhs and help determine the range of $M^2$ over which the sum rule is valid. As a result of fitting the curves (with $\mu^2 = 1 \text{ GeV}^2$) in fig.1 with the constraint that the coefficient of the additional term is no bigger than the coefficient $Y$ of the continuum term lead to an acceptable range $0.8 \leq M^2 \leq 1.75 \text{ GeV}^2$. 

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In fact, the magnitude of the continuum is correlated, as one might expect, with the value of the hadronic continuum parameter $s_0$ on the rhs. Our results correspond to $s_0 = 2.25 \text{ GeV}^2$ as in refs[7, 10, 11]. The dependence on $s_0$ is weak however and careful fitting reveals that this dependence is absorbed by the explicit continuum contributions, leaving the magnitude of $\ll U, V \gg$ practically invariant as $s_0$ varies in the range 1.8 to 5 GeV$^2$.

For the above range in $M^2$, the resulting uncertainties in the values of $\ll U, V \gg$ from the fitting procedure are comparable to the uncertainties expected from varying $\mu^2$ in the range 0.33 to 3 GeV$^2$. The values of the reduced matrix elements obtained are

$$
\ll U^S \gg = 0.046 \pm 0.010 \text{GeV}^2 \quad \ll U^{NS} \gg = 0.317 \pm 0.010 \text{GeV}^2 \\
m^2 \ll V^S \gg = -0.292 \pm 0.010 \text{GeV}^2 \quad m^2 \ll V^{NS} \gg = 0.605 \pm 0.030 \text{GeV}^2
$$

(9)

From the values in eq(9) we compute the coefficients $a_p$ and $a_n$ of the $1/Q^2$ contributions to the integrals of $g_1^p$ and $g_1^n$, using the corrected formulas of ref[8]

$$
a_p + a_n = -\frac{8}{9} \frac{5}{18} \left[ \ll U^S \gg - \frac{1}{4} m^2 \ll V^S \gg \right] \\
a_p - a_n = -\frac{8}{9} \frac{1}{6} \left[ \ll U^{NS} \gg - \frac{1}{4} m^2 \ll V^{NS} \gg \right]
$$

(10)

which gives

$$a_p = -0.029 \pm 0.002, \quad a_n = -0.002 \pm 0.002.
$$

(11)

The errors in eq(9) are based simply on the small variation of the values of $\ll U, V \gg$ with $M^2$ and $\mu^2$ and are typically 5%. These errors are in addition to the underlying uncertainties arising from the factorisation assumption for the vacuum condensates which are typically 20%[12]. Thus a realistic estimate of the errors in eq(11) is more like 0.010.

The integrals $I_{p,n,d}$ can be written

$$
I_p = I_3 + I_5 + I_0 + a_p/Q^2 \\
I_n = -I_3 + I_5 + I_0 + a_n/Q^2 \\
I_d = I_3 + I_0 + (a_p + a_n)/Q^2
$$

(12)

where

$$
I_3 = \frac{1}{12} [F + D] \left[ 1 - \frac{\alpha_s}{\pi} - 3.58(\frac{\alpha_s}{\pi})^2 \right] \\
I_5 = \frac{1}{36} [3F - D] \left[ 1 - \frac{\alpha_s}{\pi} - 3.58(\frac{\alpha_s}{\pi})^2 \right] \\
I_0 = \frac{1}{9} \Delta q \left[ 1 - \frac{\alpha_s}{3\pi} \right]
$$

(13)
Using the measured values of the polarisation asymmetries from refs [1, 2, 3], the values of \( I_{p,n,d} \) at values of \( Q^2 = 10.7, 2, 4.6 \text{GeV}^2 \) were extracted in ref [6] and determined to be \( 0.134 \pm 0.012, -0.023 \pm 0.005, 0.041 \pm 0.016 \) respectively. The estimates for the coefficients \( a_p, a_n \) from our improved QCD sum rule analysis, eq(11), when inserted into eqs(12,13) yield estimates for the nucleon spin content (for \( F/D = 0.575 \pm 0.016 \))

\[
\Delta q = 0.24 \pm 0.11 \text{ from } p \\
= 0.53 \pm 0.07 \text{ from } n \\
= 0.27 \pm 0.15 \text{ from } d
\]  

(14)

The value of \( \Delta q \) obtained from the relatively low \( Q^2 \) neutron data from SLAC is still out of line with the values obtained form CERN on the proton and deuterium. If the neutron higher twist coefficient \( a_n \) had come out large and positive, around 0.04 or so, then \( \Delta q \) would decrease a value close to the proton and deuteron estimates. In fact the analysis of Ellis and Karliner[5] used such a value based on the BBK[7] estimates of \( \ll U, V \gg \) but with the (now known to be) incorrect formulas for twist three contribution to the first moment. Thus despite the fact that we claim considerably larger estimates for the matrix elements, the corrected formulas[8] lead to a small neutron correction, thereby ruling out higher twists as a way of reconciling the three experiments. Bag model estimates for \( a_n \) give a zero value[13].

The improvement to our understanding of the QCD sum rule estimates of \( \ll U, V \gg \) has led to a more meaningful determination of the higher twist corrections to the integrals of the polarised structure function \( g_1 \). We recall that two corrections to the sum rule – the evaluation of the \( \frac{1}{p^2} \ln(\mu^2) \) contribution on the rhs and the inclusion of the continuum contribution – resulted in more stable estimates of the reduced matrix elements. It is therefore natural to ask if similar corrections apply to other sum rules e.g. the matrix elements \( \ll O^{S,NS} \gg \) which determine the \( 1/Q^2 \) corrections to the GLS[9] and Bjorken (\( F_1 \))[14] sum rules [10]. The first correction does not apply since \( \frac{1}{p^2} \ln(\mu^2) \) terms cancel for the unpolarised operator \( p_\mu p^\mu \) but the continuum correction should be included.

Fig.3 shows the corresponding determinations of \( \ll O^{S,NS} \gg \). In fig.3(a) we note the strong variation of \( \ll O^S \gg \) with \( M^2 \), again indicating the inadequacy of the nucleon pole terms alone in eq(4). Carrying out an analogous fitting procedure as in the polarised case, the resulting estimate for \( \ll O^S \gg \) is far less sensitive to the value of \( M^2 \) indicated by the dashed line in fig.3(a). Interestingly, the estimated magnitude of both matrix elements \( \ll O^{S,NS} \gg \) increases
when a realistic continuum term is included. In particular, the estimate used in the analysis of Chyla and Kataev[15] was that of ref[10]

\[ \langle O^S \rangle = 0.33 \pm 0.16 \text{GeV}^2 \]  

(15)

which led to a value of \( \Lambda_{\overline{MS}}^{(4)} \) extracted from the data on \( xF_3(x, Q^2 = 3 \text{ GeV}^2) \) on the GLS sum rule of

\[ \Lambda_{\overline{MS}}^{(4)} = 318 \pm 23(\text{stat}) \pm 99(\text{syst}) \pm 62(\text{twist}) \text{MeV}. \]  

(16)

If \( I_{\text{GLS}} \) is the measured value of the GLS sum rule, then

\[ 1 - \frac{I_{\text{GLS}}}{3} = \frac{\alpha_s(Q^2)}{\pi} + \frac{8}{27} \frac{\langle O^S \rangle}{Q^2} \]  

(17)

and we see that an increased estimate of \( \langle O^S \rangle \) leads to a lower value of \( \Lambda_{\overline{MS}}^{(4)} \). We estimate the larger and more precise value

\[ \langle O^S \rangle = 0.53 \pm 0.04 \text{GeV}^2 \]  

(18)

which leads to

\[ \Lambda_{\overline{MS}}^{(4)} = 232 \pm 23(\text{stat}) \pm 99(\text{syst}) \pm 17(\text{twist}) \text{MeV} \]  

(19)

which is more in accord with estimates got from studying the \( Q^2 \) dependence of deep inelastic data[16], \( \Lambda_{\overline{MS}}^{(4)} = 230 \pm 55 \text{ MeV} \). Again, the error in eq(18) does not include the intrinsic uncertainty (\( \sim 20\% \)) associated with the factorisation assumption; including this raises the final error in eq(19) from 17 to 45 MeV.

In summary, we have shown that there are corrections to QCD sum rules which have not been included in previous determinations of the relevant reduced matrix elements. These corrections lead to significant improvement in the stability of the extracted values with respect to the range of the Borel parameter \( M^2 \). As an example, we have applied these corrections to the case of the polarised structure function \( g_1 \) of the nucleon and found that the size of the the higher twist contributions is now determined with better precision. We have considered in some detail the corrections to the sum rule. The \( N\pi \) correction, potentially very significant, has been shown to have little effect on the determination of the reduced matrix elements. The remaining contributions are under control for a reasonable range of \( M^2 \) allowing for a realistic determination of the error. Our final estimates for the matrix elements, both for polarised and unpolarised structure functions are more reliable and, moreover, significantly larger than previous estimates. Nevertheless because of the recent corrections to the term multiplying the
twist three contribution, the resulting impact on the phenomenology of moments of $g_T^{p,n,d}$ is reduced. For the analysis of the GLS sum rule however, the resulting value of $\Lambda^{(4)}_{\overline{MS}}$ is more in line with other deep inelastic scattering phenomenology.

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References


Figure Captions

Fig. 1 Values (in GeV$^2$) of the matrix elements $\langle U, V \rangle$ derived from fitting the rhs of eq(6) in bins of 0.2 GeV$^2$ in $M^2$ with just the two terms on the lhs (solid lines) and together with the continuum term $M^2e^{(m^2-m_\pi^2)/M^2}Y$ (dashed lines).

Fig. 2 $\pi N$ contribution, eq(7), to the correlation function.

Fig. 3 Values (in GeV$^2$) of the matrix elements $\langle O^{S,NS} \rangle$ derived from fitting the rhs of eq(39) of BK[10] with the nucleon double and single pole terms only (solid lines) and together with a continuum term (dashed lines), as in fig.1.
Fig. 1

$m^2 \ll V^{NS}$

$m^2 \ll V^S$

$\ll U^{NS}$

$\ll U^S$
Fig. 3