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The Spin Dependence of Diffractive Processes and Implications for the Small x Behaviour of g_1 and the Spin Content of the Nucleon

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Abstract

We show that if the Lorentz transformation properties of diffraction are other than scalar, the $x \to 0$ behaviour of $g_1(x, Q^2)$ can grow. We compare with new data on g_1^{p} from SMC, assess implications for sum rules and for future studies of sea polarisation.

Introduction

The measurement of the net quark spin content of the proton and neutron by deep inelastic polarised leptoproduction requires an integral over the structure function $g_1(x, Q^2)$. This includes an extrapolation to high energies, or equivalently $x = 0$, which has tended to be based on Regge theory and the assumed dominance of an a_1 trajectory. In this case

$$g_1 \approx x^{-\alpha_{a_1}}, \quad x \to 0 \tag{1}$$

where α_{a_1} is the intercept of the a_1 Regge trajectory. This has been assumed to lie in the range $-0.5 < \alpha_{a_1} < 0$ and errors on the extrapolation have incorporated this range of values for the intercept.

The current value inferred for the net spin, based on all measurements with a proton target [1, 2, 3] is

$$\Delta q = 0.30 \pm 0.07(stat) \pm 0.10(syst) \tag{2}$$

This is consistent with the historically measured values though the central value has increased significantly from the original [4] estimate of a value consistent with zero.

A significant part of the increase in the inferred value (which is today some two standard deviations below the naive quark model expectation in the absence of strange quark and/or gluon polarisation) is due to the increase in the magnitude of the measured or inferred data on $g_1(x)$ at small x. An important ingredient in this is the fact that the $g_1(x)$ is constructed from a measured polarisation asymmetry which has to be multiplied by the unpolarised structure function, F_2,

$$g_1(x, Q^2) = A_1(x, Q^2)F_2(x, Q^2)/2x(1 + R(x, Q^2)) \tag{3}$$

and F_2 is now known to grow in magnitude at small x [5, 6] as well as being intrinsically larger in overall normalisation than believed originally [7].

A superficial glance at the SMC [2] data hints that $g_1^p(x)$ may be rising for $x < 0.01$ (which is a result of $A(x)$ being roughly constant while the unpolarised structure function is growing). If this trend is confirmed, and if it continues to smaller values of x, then the naive Regge pole extrapolation will be inadequate.

This leads us to the main point of this paper: *what empirical knowledge or theoretical constraints are there on the high energy behaviour (or small x behaviour) of spin dependent total cross sections (polarised structure functions)?* It seems to us that the literature allows the possibility of considerable polarisation dependence in the diffractive region out to large energies and small values of x. We shall consider four examples: an empirical study by Martin [8], a generalisation of Froissart's heuristic derivation of high energy dependence to spin dependence [9], the $x \to 0$ behaviour of $g_1(x)$ in the double log approximation (DLA) of QCD, and a specific model of the Pomeron following from ideas of Donnachie and Landshoff [10]. We then compare these and other models with the data at the smallest x values and evaluate their consequences for the sum rule. We close by assessing what future possibilities there are of improving on the empirical evaluation of the polarisation at small x.

Limits from proton-proton scattering

First, it is worth noting that the measurement of g_1 is unique in that it is the only measurement of a high energy spin dependent total cross section in hadron physics. Martin [8] has shown that one can place a limit on the polarisation dependence for high energy pp scattering since the p-p total cross sections are measured in colliders by combining two of the three quantities

1) the luminosity $\mathcal{L}$

2) the total number of events per second $\mathcal{L}\sigma(total)$

3) the extrapolated number of elastic events per second at $t = 0$ i.e. $\mathcal{L}\frac{d\sigma}{dt}|_{t=0}$. If spin effects are unimportant this is related to $\mathcal{L}\sigma^2_{tot}$ once the real part is known; conversely a difference between these arises if spin effects are large.

When these comparisons are applied to ISR data one finds [8] that the ratio $\sigma^{\uparrow\uparrow}/\sigma^{\uparrow\downarrow}$ could lie anywhere between 3/4 and 4/3. At the $Sp\bar{p}S$ the constraints are much poorer ($\frac{1}{2}$ to 2 in ratio). Thus one may conclude that spin asymmetries $A = \Delta\sigma/\sigma$ could be as large as 0.14 at ISR energies or 0.33 at the $Sp\bar{p}S$. These data offer no reason to require a small asymmetry in either polarised pp or (virtual) photoproduction and highlight the importance of these latter as pioneering measures of high energy spin dependence. They also encourage interest in possible proton polarisation at RHIC and measurment of the energy dependence of the asymmetry.

Asymptotic bounds and log x dependence

Theoretical bounds exist for the rise with energy of total cross sections (unpolarised), namely that [9, 11]

$$\sigma \le \log^2 s \tag{4}$$

Froissart showed how this bound is realised in an heuristic model. Consider two particles scattering via a potential parametrised as

$$V(r) = g s^N \exp(-\mu r) \tag{5}$$

where N is near to unity (as in simple diffractive Pomeron exchange) and μ is an inverse length, or mass, scale. Clearly the effective range will grow as s increases. The scaling behaviour of the effective range, R, with energy follows by setting $V(R) = 1$ and hence

$$R \approx (\log g + N \log s)/\mu \tag{6}$$

in which case the cross section reaches the Froissart bound

$$\sigma = \pi R^2 \approx \log^2 s \tag{7}$$

The spin dependence of the cross section depends on the Lorentz nature of the potential. Only for the case of a *scalar* is there no spin dependence in the diffractive scattering; in this case all spin dependence would follow from the (non-diffractive) processes such as a_1 Regge pole exchange as in the present assumed pole parametrisations [2, 12].

An alternative picture, which may be rooted in ideas from QCD where diffractive scattering is driven by multi gluon exchange, will in general have non trivial Lorentz structure, in particular *vector* exchange. (A particular model of diffractive scattering due to Donnachie and Landshoff [10] makes an analogy between Pomeron and photon such that the Pomeron is assumed to couple via a vector γ_μ [13]).

The effective potential has a non leading spin dependence [14] [15]

$$V(r) + \vec{\sigma} \cdot \vec{\sigma} \nabla^2 V(r)/s \tag{8}$$

which is reminiscent of the hyperfine low energy interaction in atomic hydrogen. If one now includes this in the potential argument above

$$V \approx g(s^N \pm \mu^2 s^{N-1}) exp(-\mu r) \tag{9}$$

and so for large s one finds that eq(6) generalises to

$$R^2 \approx N^2 \log^2 s \pm \frac{2N\mu^2 \log s}{s} \tag{10}$$

implying that the spin asymmetry can behave as

$$A \approx \frac{1}{s \log s} \tag{11}$$

or equivalently that

$$\Delta\sigma \approx \frac{\log s}{s} \tag{12}$$

If one is allowed to identify s with $1/x$ then these imply that g_1 is limited by

$$g_1(x \to 0) \approx -\log x \tag{13}$$

Of course there is no reason to expect the Froissart bound to be saturated but since the new small x data on both F_2 and g_1 are interestingly large we need to examine what limits can be set on the behaviour of g_1 in this region. An explicit calculation of the spin dependent diffractive scattering in the Landshoff Donnachie model (which does not saturate the Froissart bound in the unpolarised case) does manifest the $\log x$ behaviour, even at the presently attainable values of x viz [16]

$$g_1(x) \sim (1 + 2\log x) \tag{14}$$

It is interesting to consider what would occur if the potential transformed as an *axial vector*. In this case there is spin dependence in leading order [15] and the scattering is attractive only in one spin state (parallel or antiparallel depending on the overall sign). In this case the limiting behaviour is extreme

$$x g_1 \sim \log^2 x \tag{15}$$

in which eventuality the integral (spin sum) diverges. Physically this would imply that the sea is produced in one polarisation state only. This may appear artificial and lies outside known QCD mechanisms; we shall not pursue this possibility further even though it is allowed a priori.

In general we note that if the elastic scattering potential transforms other than as a Lorentz scalar, this could enable the diffractive scattering to exhibit spin dependence at high energies and undermine the Regge (nondiffractive) folklore that $g_1(x \to 0) \sim$ const. as has been commonly assumed in the experimental analyses.

$g_1(x)$ in the DLA of QCD

In the DLA, the leading $\log \frac{1}{x}$ behaviour of $F_2(x)$ is driven by the leading behaviour of the gluon-gluon splitting function at small z, $P_{gg}(z) = 2/z$ and leads to the well-known result $F_2(x \to 0) \sim \exp(k\sqrt{\ln \frac{1}{x}})$.

The helicity structure of the three-gluon vertex leads to a similar behaviour for Δg, Δq driven by $\Delta P_{gg} = 4$ and hence g_1 (if we neglect complications from the anomaly term). This yields $g_1 \sim \exp(\sqrt{2}k\sqrt{\ln\frac{1}{x}})$ and hence the relation

$$g_1 \sim [F_2]^{\sqrt{2}} \tag{16}$$

The precise behaviour will depend on the input polarised gluon distribution $\Delta G(x)$ which, in general, is expected to be non-zero [17]. This provides an example of a naturally generated growth for g_1 at small x in QCD.

Empirical situation

In $F_2(x, Q^2)$ the diffractive behaviour becomes dominant when $x \lesssim 0.1$. It is reasonable to assume that this is true also for $g_1(x)$; certainly for $x \geq 0.1$ the valence quark model gives good predictions for $A_1(x)$ [18] and there is no compelling reason to suspect that the valence - sea transition occurs at radically different kinematic regions in the different helicity states.

Now let us turn to the problem of using the assumed small x behaviour of $g_1^p(x)$ to extract a value for the integral $I_p(0,1) = \int_0^1 dx g_1^p(x)$ at $Q^2 = 10$ GeV^2 from the data. In Fig.1 the values extracted from the asymmetry measurements by SMC [2] and EMC [1] are shown. These values assume that $A_1(x, Q^2)$ is independent of Q^2 and take recent fits [19] for F_2 and R to extract g_1^p according to eq(3). When the data on A_1 become more precise the proper analysis should include the small Q^2 dependence expected from the evolution equations[20]. The new SMC data on the asymmetry A_1^p continue to support the predictions of valence quark models (VQM) for 'large' x and we can use these to estimate the integral $I_p(0.135, 1)$ reliably. The VQM curves in fig.1 give $I_p(0.135, 1) = 0.080 \pm 0.008$.

To get an estimate of the low x integral, we consider various possibilities including those discussed above. The naive assumption $g_1(x) \sim$ constant is not supported by the low x SMC data, but the best fit of this type, $g_1^p(x) = (0.35 \pm 0.05)$,($x < 0.135$) (see fig.1) leads to $I_p(0,1) = 0.127 \pm 0.010$, ($\Delta q = 15 \pm 9\%$, if no higher twist present), to O(α_s). Next we consider three examples where $xg_1(x)$ rises logarithmically as $x \to 0$. For the $\log x$ behaviour given by eq(13) the fit $xg_1^p(x) = (-0.14 \pm 0.02)\ln x$,($x < 0.135$) leads to $I_p(0,1) = 0.137 \pm 0.011$,($\Delta q = 24 \pm 10\%$). The two-gluon Pomeron prediction [16] of eq(14) gives a good fit to the low x data with a coefficient -0.085 ± 0.01 which is close to the preferred value of -0.09. This gives $I_p(0,1) = 0.138 \pm 0.011$,($\Delta q = 25 \pm 11\%$).

Finally we consider an extreme point of view where a rapid rise at small x is expected. General theorems on the high energy behaviour of the spin dependent total cross sections show that *if* negative signature cuts reach $J = 1$ at $t = 0$ there can be a leading contribution to $xg_1(x) \sim 1/\log^2 x$ [21, 22, 23]. Such a behaviour was discussed in an analysis of the first EMC results [24]. Allowing such a rapid rise has been criticised [25] but there seems to be no compelling argument for the decoupling of such non-factorisable contributions to the amplitude. Isoscalar $t-$channel exchanges with axial-vector quantum numbers, as listed in eqs(4.1,4.2) of ref.[26], do include the possible contributions from the negative signature cuts of refs[21, 22, 23]. We are unaware of any general theorems based on symmetry principles, angular momentum etc. that forbid the above behaviour although it may be that the magnitude of such contributions is indeed small or even vanishing in specific dynamic models.

Phenomenologically it is worth noting that the SMC data may be even more severe than the $1/x\log^2 x$ behaviour – see fig. 1. Our analysis [24] of the initial EMC data suggested the small x region was consistent with $xg_1^p(x) = 0.135/\ln^2 x$. The combined SMC and EMC data prefer a parametrisation $xg_1^p(x) = (0.17\pm0.03)/\ln^2 x, (x < 0.135)$ which leads to $I_p(0,1) = 0.165\pm0.010$ $(\Delta q = 50 \pm 16\%)$.

Given the debatable nature of $g_1(x)$ as $x \to 0$, one could attempt to estimate the integral $I_p(0,1)$ by simply fitting the small x data to an arbitrary power law plus a conventional constant term in order to assess the range of uncertainty. Even then the answer depends critically on the range of x over which the fit is performed. For example taking $x < 0.135$ again, the fit gives $g_1 \sim$const$+x^{-2}$ which leads to a divergent value for $I_p(0,1)$.

In any event this range of possibilities serves

(i) to illustrate that our limited understanding of the small x region does allow for an estimate of the integral of $g_1^p(x)$ which is entirely consistent with the original Ellis-Jaffe sum rule [27] whose value, including $O(\alpha_s^2)$ corrections, is 0.172±0.009 at $Q^2 = 10$ GeV^2.

(ii) as a challenge for future experiments to eliminate.

The resulting values inferred for Δq vary considerably and so highlight the importance of being able to discriminate between, at least, a roughly constant or falling a_1 pole (non-diffractive or Lorentz scalar diffraction) on the one hand and a (logarithmic) growth on the other.

Possible routes for resolving these questions include the following.

(a) Currently planned experiments [28] giving precision data for $0.01 \leq x \leq 0.1$ which indicate a clear trend over this range and which tightly constrain continuation to the less precise data from SMC at smaller x.

(b) Reduction of the systematic and statistical uncertainties in the SMC data for $x \lesssim 0.05$ to confirm the apparent rise.

(c) Measurement of the sea polarisation directly via semi-inclusive production of fast K^- and π [29,30]

(d) Theoretical understanding of the rise at small x in $F_2(x, Q^2)$ and possible linkage with the Donnachie Landshoff description being extended to a unified description involving spin dependence.

(e) Precise data for the deuteron at small x where, if diffraction dominates, $g_1^d(x)$ would be positive. Present data are not accurate enough to rule out this possibility.

(f) Measurememt of the energy dependence of polarised pp and polarised (real) photoproduction asymmetries.

If any or all of these imply that there is significant non-trivial spin dependence and growth in the diffractive region at small x, then this may stimulate investigation of the possibility of creating longitudinally polarised proton beams at HERA. Polarised electron - proton interactions at HERA could turn out to have significant physics interest.

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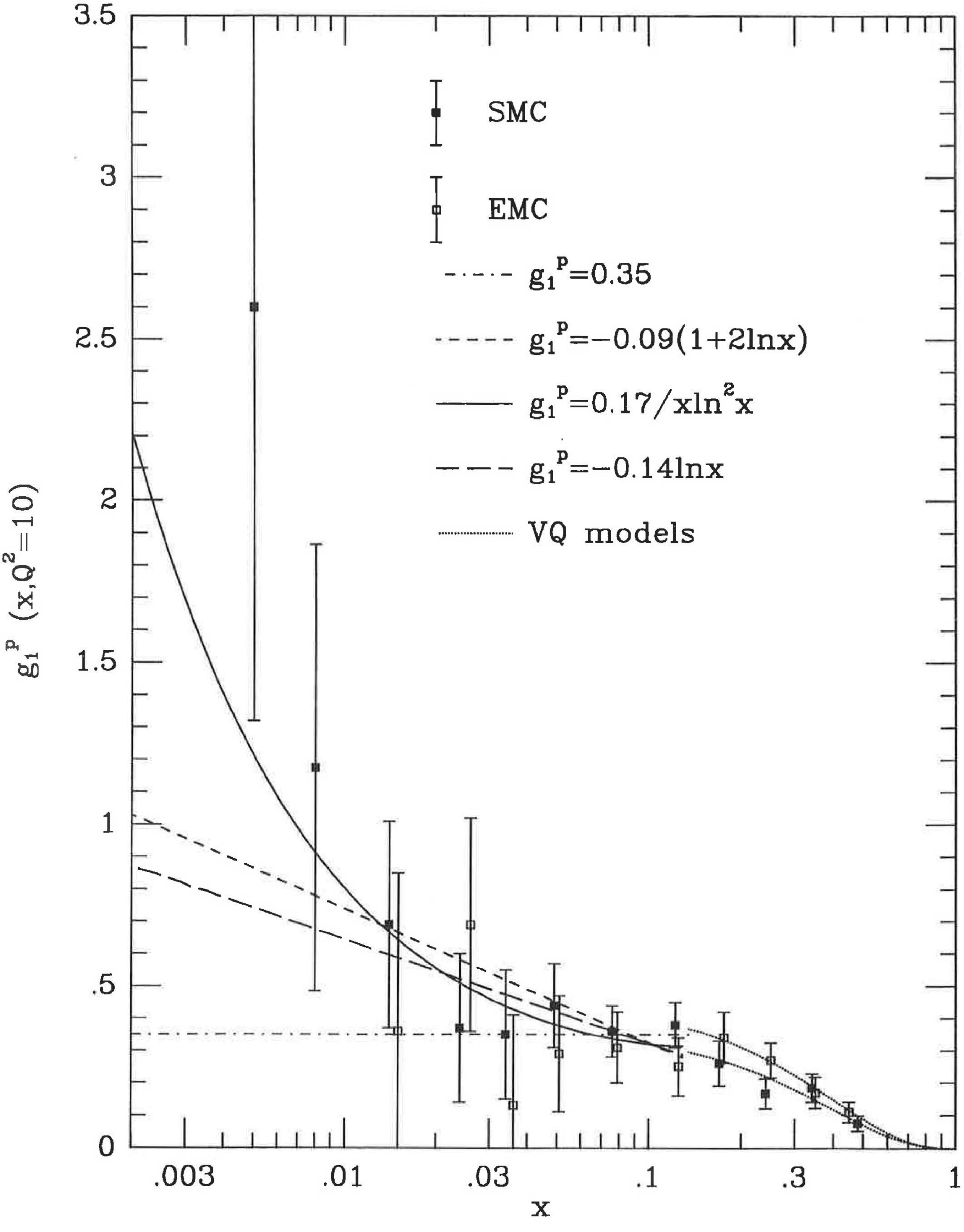
Figure Caption

Fig. 1 $g_1^p(x)$ at $Q^2 = 10$ GeV^2. Data are from refs [1, 2].



SMC
EMC
g_1^p=0.35
g_1^p=−0.09(1+2lnx)
g_1^p=0.17/xln^{2}x
g_1^p=−0.14lnx
VQ models
g_1^p (x,Q^2=10)
3.5
3
2.5
2
1.5
1
.5
0
.003
.01
.03
.1
.3
1
x

Fig. 1